Modeling and Simulation of Linear and Nonlinear Dynamic Model for Double Inverted Pendulum on Cart System

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Abstract:

This paper displays linear and nonlinear model for double inverted pendulum system depended on Lagrange equation and the system response for linear and nonlinear of DIPS are noted and analyzed. The inverted pendulum has been advanced for a laboratory experiments. The essential variance between the linear and nonlinear equation is present. This analysis is useful for progress of effective controller for a DIPS.

Keywords:-double inverted pendulum system; nonlinear model; linearization; simulation.

الخلاصة

يعرض هذا البحث النموذج الخطي والغير خطي لنظام مزدوج البندول المقلوب اعتمادا على معادلة لاكرنج واستجابة النظام الخطي والغير خطي قد تم ملاحظتها وتحليلها . وان البندول المقلوب طور لإجراء التجارب المختبرية. التباين الجوهري بين المعادلة الخطية وغير الخطية تمت ملاحظتها خلال الاشتقاق . هذا التحليل مهم في تطوير المسيطر الفعال لنظام مزودج البندول المقلوب.

الكلمات المفتاحية :- مزدوج نظام البندول مقلوب . النموذج الغير خطي . الخطية . المحاكاة

1.Introduction

inverted pendulum systems (IPS) denote to important type for the systems of mechanical utilized in control systems through a quantity of practical applications. It is clear that a vertical movement for arm of robot and it can modeled by used one type of an "inverted pendula system". The variety for modeled systems is mirrored in the diversity of existing inverted pendula models (Jadlovská *et.al.*, 2011). This paper will show the double inverted pendulum system as display in figure 1. The variety of modeled systems is mirrored in the varied diversity of available inverted pendula models. So we can differentiate between: inverted pendula systems that transfers in a linear manner or in a rotary manner in a horizontal plane and other inverted pendula systems, which differ by the number of pendulum links attached to the base(Sultan *et.al.*,2003).

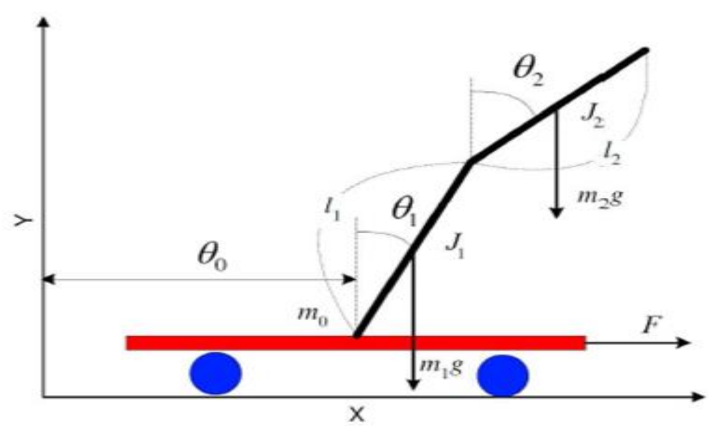


Figure (1) double inverted pendulum systems

2 .The Derivation For Nonlinear Model

In this section we will derivative on the modelling of nonlinear *for double inverted pendula on a cart* is consisted of *two* homogenous, symmetric rods which are joint-bound with each other and related to a stable moving base(Jadlovská *et.al.*,2009). The input for cart is force; The outputs are denoted through cart position and pendula angles . First the cart position is straight influenced by the input force, so any system of inverted pendula is considered to be under-actuated (Demirci, 2004).

The double inverted pendulum on the cart system that display in figure (1) is derived by using Lagrange equations:-

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = Q$$

Lagrangian equation represent by $L = T - P$and Q is represent the vector of force. T is kinetic. energy . and P represent the potential energy.

$T = To + T1 + T2$

$P = Po + P1 + P2$

Where

$T_0 = \frac{1}{2} m_0 \dot{\theta}_o^2$

$T_1 = \frac{1}{2} m_1 \dot{\theta}_o^2 + \frac{1}{2}(m_1 l_1^2 + I_1)\dot{\theta}_1^2 + m_1 l_1 \dot{\theta}_0 \dot{\theta}_1 \cos\theta_1$

$T_2 = \frac{1}{2} m_2 [(\dot{\theta}_0 + L_1 \dot{\theta}_1 cos\theta_1 + l_2 \dot{\theta}_2 \cos\theta_2)^2 + \left(L_1 \dot{\theta}_1 \sin\theta_1 + l_2 \dot{\theta}_2 \sin\theta_2\right)^2 + \frac{1}{2} I_2 \dot{\theta}_2^2$

$$T_2 = \frac{1}{2} m_2 \dot{\theta}_o^2 + \frac{1}{2} m_2 L_1^2 \dot{\theta}_1^2 + \frac{1}{2}(m_2 l_2^2 + I_2)\dot{\theta}_2^2 + m_2 L_1 \dot{\theta}_0 \dot{\theta}_1 \cos\theta_1 + m_2 l_2 \dot{\theta}_0 \dot{\theta}_2 \cos\theta_2 + m_2 L_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$P_0 = 0$

$P_1 = m_1 g l_1 \cos\theta_1$

$P_2 = m_2 g (l_1 \cos\theta_1 + l_2 \cos\theta_2)$

So the system of the Lagrangian is assumed by :-

$$L = \frac{1}{2}(m_0 + m_1 + m_2)\dot{\theta}_o^2 + \frac{1}{2}(m_1 l_1^2 + m_2 L_1^2 + I_1)\dot{\theta}_1^2 + \frac{1}{2}(m_2 l_2^2 + I_2)\dot{\theta}_2^2 + (m_1 l_1 + m_2 L_1)\cos(\theta_1)\,\dot{\theta}_0 \dot{\theta}_1 + m_2 l_2 \cos(\theta_2)\dot{\theta}_0 \dot{\theta}_2 + m_2 L_1 l_2 \cos(\theta_1 - \theta_2)\dot{\theta}_1 \dot{\theta}_2 - (m_1 l_1 + m_2 L_1) g \cos\theta_1 - m_2\, l_2 g \cos\theta_2$$

Where $\dot{\theta}$ and θ Subject to Lagrange equation such as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_0}\right) - \frac{\partial L}{\partial} = u$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0$$

Or explicitly

$$(\sum m_i)\ddot{\theta}_0 + (m_1 l_1 + m_2 L_1)\cos(\theta_1)\ddot{\theta}_1 + m_2 l_2 \cos(\theta_2)\ddot{\theta}_2 - (m_1 l_1 + m_2 L_1)\sin(\theta_1)\dot{\theta}_1^2 - m_2 l_2 \sin(\theta_2)\dot{\theta}_2^2 = u$$

$$(m_1 l_1 + m_2 L_1)\cos(\theta_1)\ddot{\theta}_0 + (m_1 l_1^2 + m_2 L_1^2 + I_1)\ddot{\theta}_1 + m_2 L_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_2 + m_2 L_1 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_2^2 - (m_1 l_1 + m_2 L_1) g \sin\theta_1 = 0$$

$$m_2 l_2 \cos(\theta_2)\ddot{\theta}_0 + m_2 L_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_1 + (m_2 l_2^2 + I_2)\ddot{\theta}_2 - m_2 L_1 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_1^2 - m_2 l_2 g \sin\theta_2 = 0$$

$$D(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = Hu$$

$$D(\theta) = \begin{pmatrix} d_1 & d_2 \cos\theta_1 & d_3 \cos\theta_2 \\ d_2 \cos\theta_1 & d_4 & d_5 \cos(\theta_1 - \theta_2) \\ d_3 \cos\theta_2 & d_5 \cos(\theta_1 - \theta_2) & d_6 \end{pmatrix}$$

$$C(\theta, \dot{\theta}) = \begin{pmatrix} 0 & -d_2 \sin(\theta_1)\dot{\theta}_1 & -d_2 \sin(\theta_1)\dot{\theta}_1 \\ 0 & 0 & d_5 \sin(\theta_1 - \theta_2)\dot{\theta}_2 \\ 0 & -d_5 \sin(\theta_1 - \theta_2)\dot{\theta}_1 & 0 \end{pmatrix}$$

$$G(\theta) = \begin{pmatrix} 0 \\ -f_1 \sin\theta_1 \\ -f_2 \sin\theta_2 \end{pmatrix}$$

$$H = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

The mass centers for the pendulum are in geometrical center of the links, that represents solid rods:

$we\ obtain\ form$ the the matrices of elements $D(\theta), C(\theta, \dot{\theta}) and\ G(\theta)$

$$d_1 = m_0 + m_1 + m_2$$
$$d_2 = \left(\frac{1}{2} m_1 + m_2\right) L_1$$
$$d_3 = \frac{1}{2} m_2 L_2$$
$$d_4 = \left(\frac{1}{3} m_1 + m_2\right) L_1^2$$
$$d_5 = \frac{1}{2} m_2 L_1 L_2$$
$$d_6 = \frac{1}{3} m_2 L_2^2$$
$$f_1 = \left(\frac{1}{2} m_1 + m_2\right) L_1 g$$
$$f_2 = \frac{1}{2} m_2 L_2 g$$

Table 1 Parameters of Inverted Pendulum System

Parameters	**Notation**	**Values**
Mass of cart	m_0	0.3
Mass of lower pen.	m1	0.275
Mass of upper pen.	m2	0.275
Length of pen1	l1	0.5
Length of pen2	l2	0.5
Friction coefficient	δ_0	0.3
Damping constant of pen1	δ_1	0.2
Damping constant of pen2	δ_2	0.1
Moment of inertia of pen1	J1	0.022
Moment of inertia of pen2	J2	0.022
Force on cart	F	0.4

3. Linearised Dynamic Model

The DIPS is a multi-variable for nonlinear system . In order to simplify control strategy, linear the model near the balance point.

$A1. \quad \cos\theta_1 . \ddot{\theta}_1$

$\cos\theta_1 \cong \cos\overline{\theta_1} + \cos\theta_1 \big|_{\theta_1=\overline{\theta_1}} (\theta_1 - \overline{\theta_1})$

$\cos\theta_1 \cong \cos 0 - \sin 0\big(\ddot{\theta}_1 - 0\big) \cong 1$

Hence,

$\cos\theta_1 . \ddot{\theta}_1 \cong \ddot{\theta}_1$

$A2. \quad \cos\theta_2 . \ddot{\theta}_2$

$\cos\theta_2 \cong \cos\overline{\theta_2} + \cos\theta_2 \big|_{\theta_2=\overline{\theta_2}} (\theta_2 - \overline{\theta_2})$

$\cos\theta_2 \cong 1$

Hence,

$\cos\theta_2 . \ddot{\theta}_2 \cong \ddot{\theta}_2$

$A3. \quad \sin\theta_1 . \dot{\theta}_1^2$

$\sin\theta_1 . + \frac{\partial}{\partial\theta_1}\left(\sin\theta_1 . \dot{\theta}_1^2\right)\big|_{\theta_1=\overline{\theta_1}, \quad \dot{\theta}_1=\dot{\bar{\theta}}_1} (\theta_1 - \overline{\theta_1})$

$+ \frac{\partial}{\partial\theta_1}\left(2\cos\theta_1 . \dot{\bar{\theta}}_1^2\right)\big|_{\theta_1=\overline{\theta_1}, \quad \dot{\theta}_1=\dot{\bar{\theta}}_1} \left(\dot{\theta}_1 - \dot{\bar{\theta}}_2\right)$

$\sin\theta_1 . \dot{\theta}_1^2 \cong 0 + 2\cos\theta_1 . \dot{\bar{\theta}}_1^2 (\theta_1 - 0) \cong 0$

$A4. \quad \cos\bar{\theta}_1 . \dot{\theta}_1$

$\cos\theta_1 \cong \cos\bar{\theta}_1 - \cos\theta_1 \big|_{\theta_1=\overline{\theta_1}} (\theta_1 - \overline{\theta_1})$

$\cos\theta_1 \cong \cos 0 - \sin 0\big(\ddot{\theta}_1 - 0\big) \cong 1$

Hence,

$\cos\theta_1 . \dot{\theta}_0 \cong \dot{\theta}_0$

$A5. \quad \sin(\theta_1 - \theta_2)\dot{\theta}_2^2$

$$\sin(\theta_1-\theta_2)\dot{\theta}_2^2 \cong \sin(\bar{\theta}_1-\overline{\theta_2})\,\dot{\bar{\theta}}_2^2+\frac{\partial}{\partial\theta_2}\left(\sin(\theta_1-\theta_2)\dot{\theta}_2^2\right)\Big|_{\theta_1=\overline{\theta_1},\ \ \theta_2=\overline{\theta_2}}(\theta_1-\overline{\theta_1})$$
$$+\frac{\partial}{\partial\theta_2}\left(\sin(\theta_1-\theta_2)\dot{\theta}_2^2\right)\Big|_{\theta_1=\overline{\theta_1},\ \ \theta_2=\overline{\theta_2}}(\theta_2-\overline{\theta_2})$$

$$\sin(\theta_1-\theta_2)\dot{\theta}_2^2 \cong 0-\cos(\bar{\theta}_1-\overline{\theta_2})\dot{\bar{\theta}}_2^2\,(\theta_1-0)+\cos(\theta_1-\theta_2)\dot{\theta}_2^2\ (\theta_2-0)$$
$$+\,2\sin(\theta_1-\theta_2)\dot{\theta}_2^2\ (\theta_2-0)\cong 0$$

$A6.\quad \cos(\theta_1-\theta_2)\ddot{\theta}_2$

$$\cos(\theta_1-\theta_2)\ddot{\theta}_2 \cong \cos(\bar{\theta}_1-\overline{\theta_2})+\frac{\partial}{\partial\theta_1}\cos(\theta_1-\theta_2)\Big|_{\theta_1=\overline{\theta_1},\ \ \theta_2=\overline{\theta_2}}$$
$$+\frac{\partial}{\partial\theta_2}\cos(\theta_1-\theta_2)\Big|_{\theta_1=\overline{\theta_1},\ \ \theta_2=\overline{\theta_2}}(\theta_2-\overline{\theta_2})$$
$$\cos(\theta_1-\theta_2)\ddot{\theta}_2 \cong 1-\sin(\bar{\theta}_1-\overline{\theta_2})\,(\theta_1-0)-\sin(\bar{\theta}_1-\overline{\theta_2})\,(-1)(\theta_2-0)\cong 1$$

$A7.\quad \sin\theta_1$

$$\sin\theta_1 \cong \sin\bar{\theta}_1+\frac{\partial}{\partial\theta_1}\sin\theta_1\Big|_{\theta_1=\overline{\theta_1}}(\theta_1-\overline{\theta_1})\cong\theta_1$$

$A8.\quad \sin\theta_2$

$$\sin\theta_2 \cong \sin\bar{\theta}_2+\frac{\partial}{\partial\theta_2}\sin\theta_2\Big|_{\theta_2=\overline{\theta_2}}(\theta_2-\overline{\theta_2})\cong\theta_2$$

$A9.\quad \sin\theta_2.\dot{\theta}_2^2$

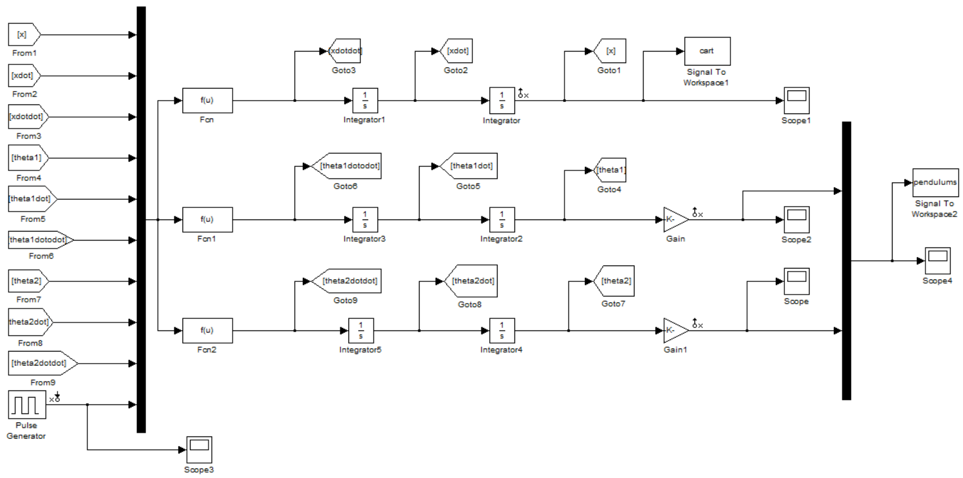
$$\sin\theta_2.\dot{\theta}_2^2 \cong \sin\bar{\theta}_2.\bar{\theta}_2^2+\frac{\partial}{\partial\theta_1}\sin\theta_2.\dot{\theta}_2^2\Big|_{\theta_2=\overline{\theta_2},\ \ \theta_2=\overline{\dot{\theta}_2}}(\theta_2-\overline{\theta_2})$$
$$+\frac{\partial}{\partial\dot{\theta}_2}\sin\theta_2.\dot{\theta}_2^2\Big|_{\theta_2=\overline{\theta_2},\ \ \dot{\theta}_2=\overline{\dot{\theta}_2}}(\theta_1-\overline{\theta_1})$$
$$\sin\theta_2.\dot{\theta}_2^2=0+\cos\bar{\theta}_2.\bar{\theta}_2^2(\theta_2-0)+2\sin\bar{\theta}_2.\bar{\theta}_2^2\cong 0$$

Then get the new equation

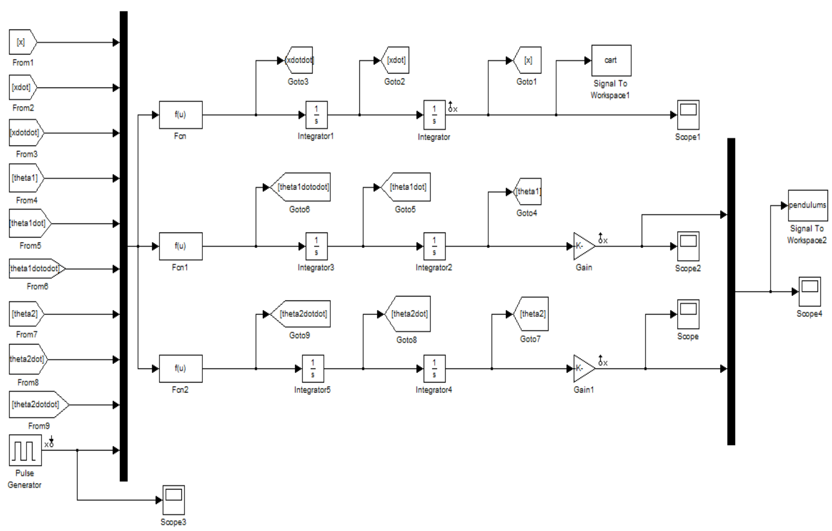
$$\begin{bmatrix} m_0+m_1+m_2 & \frac{1}{2}m_1L_1+m_2L_1 & \frac{1}{2}m_2L_2 \\ \frac{1}{2}m_1L_1+m_2L_1 & \frac{1}{3}m_1L_1^2+m_2L_1^2 & \frac{1}{2}m_2L_1L_2 \\ \frac{1}{2}m_2L_2 & \frac{1}{2}m_2L_1L_2 & \frac{1}{3}m_2L_2^2 \end{bmatrix}\begin{bmatrix}\ddot{\theta}_0\\ \ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}+\begin{bmatrix}\delta_0 & 0 & 0\\ \frac{1}{2}m_1L_1 & \delta_1+\delta_2 & -\delta_2\\ \frac{1}{2}m_2L_2 & -\delta_2 & \delta_2\end{bmatrix}\begin{bmatrix}\dot{\theta}_0\\ \dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$$
$$+\begin{bmatrix}0\\ -\left(\frac{1}{2}m_1+m_2\right)L_1g\\ -\frac{1}{2}m_2L_2g\end{bmatrix}\begin{bmatrix}\theta_0\\ \theta_1\\ \theta_2\end{bmatrix}=\begin{bmatrix}u(t)\\ 0\\ 0\end{bmatrix}$$

4. **Simulation Model**

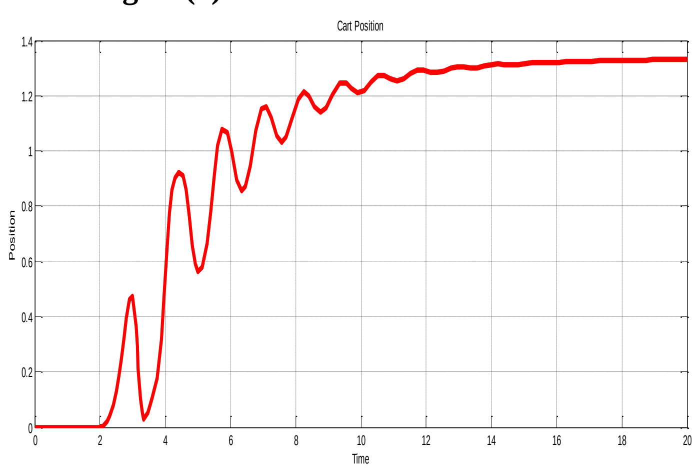
Figure 2 and 3 display the simulation of linear and nonlinear for DIPS . All equations in state space representation were applied in Simulink model and the response of system are detected .Figure 4 represent transient response for cart position of Nonlinear model ,figure (5) represent the pendulum angular response of nonlinear model, figure(6) represent the transient response for cart position of linear model and figure(7) represent the transient response for cart position of linear model .



Figure(2) Simulink for non linear model



Figure(3) Simulink for linear model



Figure(4) transient response for cart position of nonlinear model

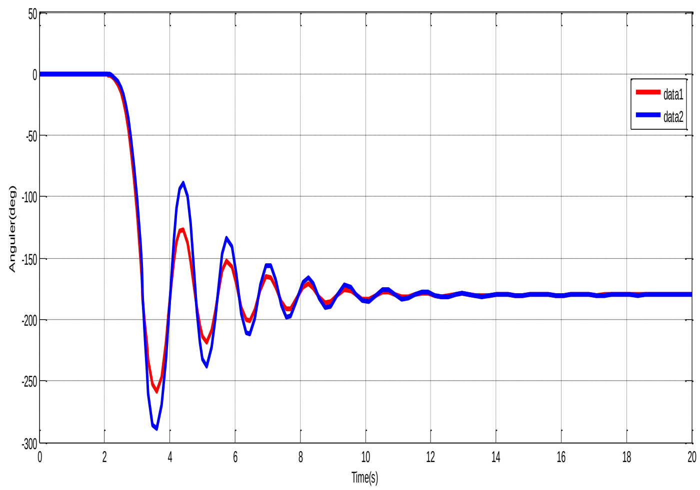


Figure (5) Pendulum angular (lower and upper) response of nonlinear model

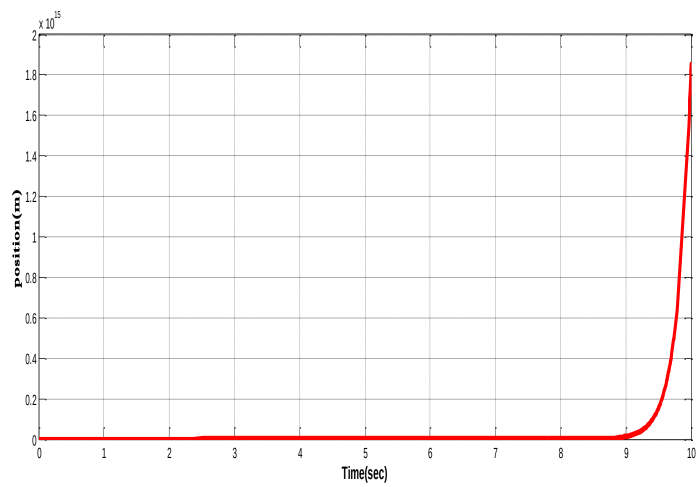


Figure (6) transient response for cart position of linear model

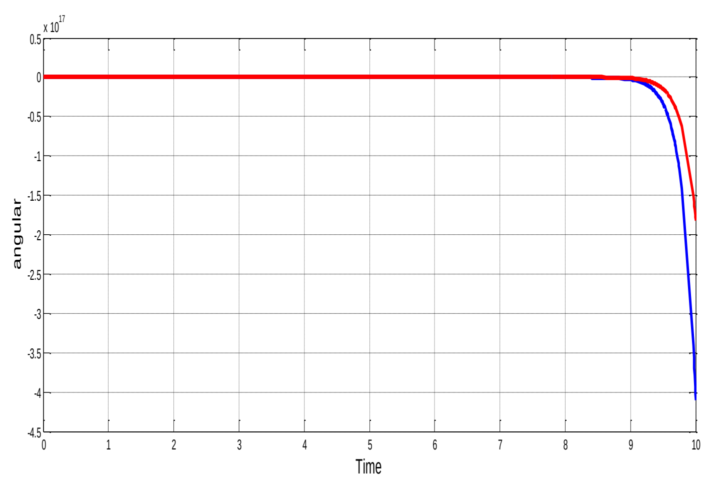


Figure (7) Pendulum angular (lower and upper) response of linear model

5. Conclusions

For designing of controller for linear model is easier form nonlinear model .For double inverted pendulum system, successfully. Derived the nonlinear equations of DIPS. The linear model was derived by the linearization at the specified operating points. Simulation results show that the derived models in equations gives the same responses as the responses presented by the reference paper.

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