

Investigation and Simulation the Kinematic Singularity of Three Links Robot Manipulator with Spherical Wrist

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Abstract:

The kinematics singularity of a manipulator is a central problem in robot control and trajectory tracking since the jacobian matrix rank is decreased. The robot in this paper consists of three links with revolute joints and spherical wrist (six degree of freedom). Danivart Hartenberg convention is used to predicate the forward kinematics position and orientation of the robot end effector. Both geometrical and analytical jacobian are presented to identify and analyze the robot singularities in order to avoid them in trajectory planning and control problems. The investigation shows that the manipulator has three type of singularity (elbow singularity when $\theta_3=0,\pi$, Shoulder singularity occurs at the condition $a_2c_2 + a_3c_{23} = 0$ and Wrist singularity when $\theta_5=0,\pi$). The calculated singular configurations have been verified and simulated with Peter Corke robotics toolbox of MATLAB. The presented approaches can be applied to analyze the singularity of other similar kinds of robot manipulators in order to achieve a suitable solution for tracking trajectories.

Key words: Robot manipulator, DH parameters, Forward kinematics, Jacobian, Singularity.

1- Introduction:

Kinematic singularity can be defined as the robot loss of one or more degree of freedom (DOF), mathematically; at which jacobian matrix is a rank decreasing. Singularity represents configurations from which certain direction of motion may be unattainable, infinite solutions to the inverse kinematics may exist and an end effector small velocity in operational space may cause a large joint velocity [1] Since the forward and inverse kinematics give the relationship between the work space and joint space, the differential of kinematics gives the relationship between the joints velocities and the corresponding end effector linear and angular velocities; this mapping is described by a matrix termed jacobian. Jacobian is one of the most important tool for manipulator characterization. It is useful for finding singularities, determining inverse algorithm, related the joint torques and applied forces, deriving equations of motion and designing operational control schemes[2].

There is a large amount of literatures which discusses the robot kinematics modeling and analysis especially in industrial robots. In general; almost the researchers are modeling the forward kinematics based on Danivart Hartenberg (DH) convention and using the MATLAB as a software program because its capabilities in simulations. [3] Developed a visual software for AL5B with 5 DOF as an educational experimental tool in the practical aspect of robot manipulator. [4] Presented the direct modeling of 5 DOF stationary articulated robot arm where a Labvolt R5150 arm was taken as case study. [5] developed a software for both forward and inverse kinematics of Lynx6 educational robot. [6] discussed the geometric jacobian driving of two 6 DOF robots (smokie robot and Barrett WAM) and driving kinematics singularities through jacobian determinant.

In this work, forward kinematic model will be achieved by DH convention of a 6 DOF robot manipulator as it is shown in figure (1). Jacobian matrix derivation will be presented in both geometrical and analytical methods to analyze the manipulator singularities in order to avoid them in trajectory

planning and control problems. The paper is organized sequentially to explain: forward kinematic, manipulator jacobian, manipulator singularity, results and discussion and conclusions.



Figure (1) Three Link Robot Manipulator 6 DOF [7].

2-Forward Kinematics:

Forward kinematics is the transformation between joint space and the Cartesian space to solve the position and orientation of the robot end effector. Denavit Hartenberg convention is based on attaching coordinate frame system at each joint and specifying the four parameter of DH: α_{i-1} , a_{i-1} , θ_i and d_i where:

a_{i-1} : (link length) is the distance between z_{i-1} and z_i axes along the x_i axis.

α_{i-1} :(link twist) is the required rotation of z_{i-1} to z_i axes about the x_i axis.

d_i : (joint offset) is the distance between x_{i-1} and x_i axes along the z_{i-1} axis.

θ_i : (joint angle) is the required rotation of x_{i-1} to x_i axes about the z_{i-1} axis.

The transformation matrix of frame $\{i\}$ relative to previous frame $\{i-1\}$ is: [8]

$${}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_{i-1} & s\theta_i s\alpha_{i-1} & a_{i-1} c\theta_i \\ s\theta_i & c\theta_i c\alpha_{i-1} & -c\theta_i s\alpha_{i-1} & a_{i-1} s\theta_i \\ 0 & s\alpha_{i-1} & c\alpha_{i-1} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \dots\dots\dots (1)$$

And the transformation matrix of n^{th} coordinate frame to base coordinate frame is:

$${}^0T_n = {}^0T_1 {}^1T_2 \dots\dots\dots {}^{n-1}T_n \quad \dots\dots\dots (2)$$

The first (3x3) matrix represents rotation matrix of frame $\{i\}$ relative to frame $\{i-1\}$ and the fourth column represents the origin of the frame $\{i\}$ position. DH parameters of robot manipulator are defined for the assigned frames as it is shown in figure (2) at table (1).

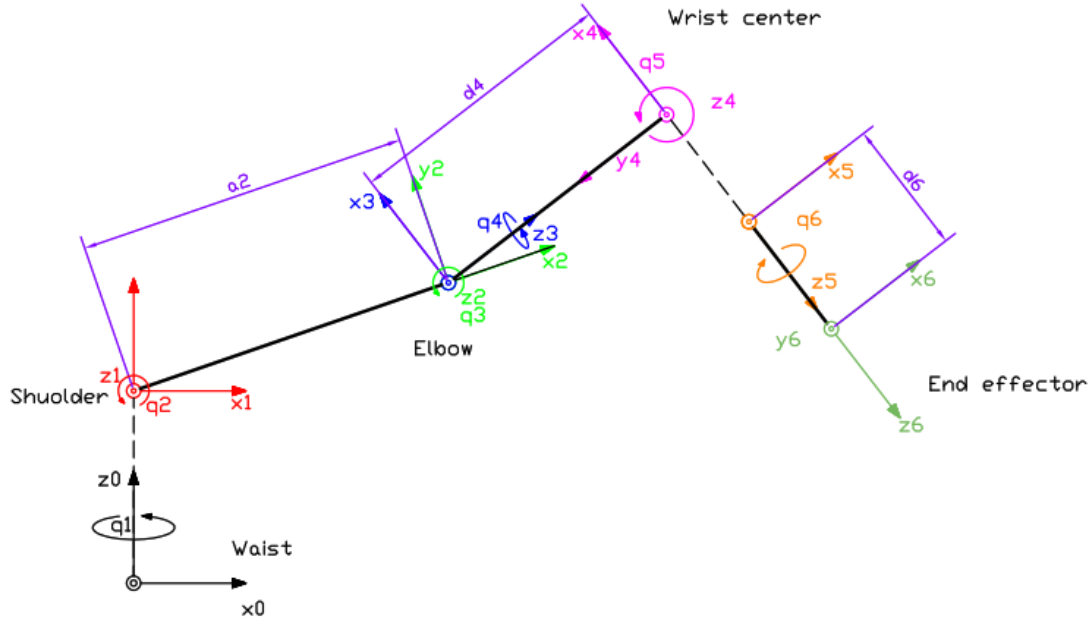


Fig. (٧) The Attached Coordinate Frame Systems.

Table (1) DH Parameters of Robot Manipulator

I	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	90	0	θ_1
2	a_2	0	0	θ_2
3	0	90	0	θ_3
4	0	-90	d_4	θ_4
5	0	90	0	θ_5
6	0	0	d_6	θ_6

3. Manipulator Jacobian:

For n-DOF manipulator; the end effector linear and angular velocities can be defined as:

$$\begin{bmatrix} v_e \\ \omega_e \end{bmatrix}_{(6 \times 1)} = \begin{bmatrix} J_p \\ J_o \end{bmatrix}_{(6 \times n)} \begin{Bmatrix} \dot{q} \end{Bmatrix}_{(n \times 1)} \quad \dots\dots\dots(3)[9]$$

Where:

v_e : (3x1) matrix represents the end effector linear velocity in Cartesian space.

ω_e : (3x1) matrix represents the end effector angular velocity in Cartesian space.

J_p : (3xn) jacobian matrix relates the end effector linear velocity to joints velocities.

J_o : (3xn) jacobian matrix relates the end effector angular velocity to joints velocities.

As it is shown in figure (3) the angular and the linear velocities of i^{th} link in $\{i-1\}$ frame coordinate system can be written in the following vector form:

$${}^{i-1}\omega_i = \dot{\theta}_i \hat{z}_{i-1} \quad \dots\dots\dots(4)$$

$${}^{i-1}v_i = {}^{i-1}\omega_i \times {}^{i-1}r_i \quad \dots\dots\dots(5)$$

Where \hat{z}_{i-1} the z direction unit vector of i^{th} frame represented in base frame.

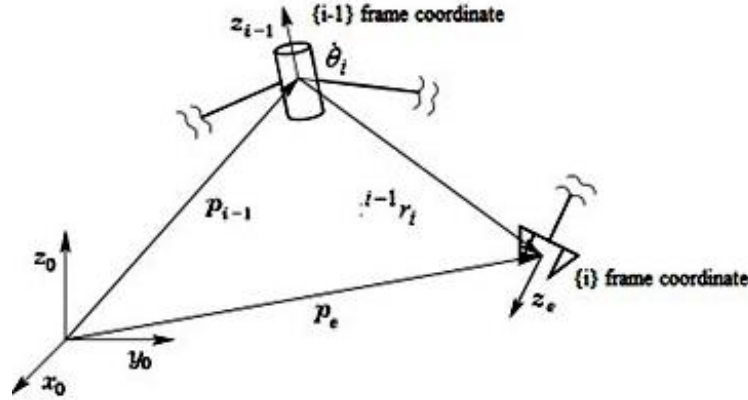


Figure (v) Representation of Vectors Related Revolute Joint and End Effector.[9]

Due to the rotation of frame $\{i\}$ w.r.t. frame $\{i-1\}$, the expression of the angular and the linear velocities of i^{th} link represented in base frame become:

$${}^0\omega_i = {}^0\omega_{i-1} + \dot{\theta}_i \hat{z}_{i-1} \quad \dots\dots (6)$$

$${}^0v_i = {}^0v_{i-1} + \omega_i \times {}^{i-1}r_i \quad \dots\dots(7)$$

Note that the computation of the linear velocity with reference to the origin of end effector frame is:[9]

$$\dot{q}_i J_{p_i} = {}^{i-1}\omega_i \times {}^{i-1}r_e = \dot{\theta}_i \hat{z}_{i-1} \times (p_e - p_{i-1}) \quad \dots\dots (8)$$

Where:

p_e : frame origin of end effector represented in base frame.

p_{i-1} : frame origin of previous i^{th} frame represented in base frame.

Since the vectors of z_{i-1} , p_e and p_{i-1} are all function of the joint variable, in particular:

- z_{i-1} is given by third column of the rotation matrix ${}^0R_{i-1}$.
- p_e is given by the first three elements of fourth column of transformation matrix 0T_e .
- p_{i-1} is given by the first three elements of the fourth column of transformation matrix ${}^{i-1}T_i$

While for angular velocity it is:

$$\dot{q}_i J_{o_i} = \dot{\theta}_i \hat{z}_{i-1} \quad \dots\dots(9)$$

In summery; the geometry column of jacobian matrix for revolute joint is:[9]

$$\begin{bmatrix} J_{p_i} \\ J_{o_i} \end{bmatrix} = \begin{bmatrix} z_{i-1} \times (p_e - p_{i-1}) \\ z_{i-1} \end{bmatrix} \quad \dots\dots(10)$$

Then the jacobian of the robot end effector is:[1]

$$J_e = \begin{bmatrix} z_0 \times (p_e - p_0) & z_1 \times (p_e - p_1) & z_2 \times (p_e - p_2) & z_3 \times (p_e - p_3) & z_4 \times (p_e - p_4) & z_5 \times (p_e - p_5) \\ z_0 & z_1 & z_2 & z_3 & z_4 & z_5 \end{bmatrix} \quad (11)$$

Where:

$$\begin{aligned}
 z_0 &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}, \quad z_2 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}, \quad z_3 = \begin{bmatrix} c_1 s_{23} \\ s_1 s_{23} \\ -c_{23} \end{bmatrix}, \quad z_4 = \begin{bmatrix} -c_1 c_{23} s_4 + s_1 c_4 \\ -s_1 c_{23} s_4 - c_1 c_4 \\ -s_{23} s_4 \end{bmatrix}, \\
 z_5 &= \begin{bmatrix} s_5 (c_1 c_{23} c_4 + s_1 s_4) + c_1 s_{23} c_5 \\ s_5 (s_1 c_{23} c_4 - c_1 s_4) + s_1 s_{23} c_5 \\ s_{23} c_4 s_5 - c_{23} c_5 \end{bmatrix}, \quad p_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad p_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad p_2 = \begin{bmatrix} a_2 c_1 c_2 \\ a_2 s_1 c_2 \\ a_2 s_2 \end{bmatrix}, \\
 p_3 &= \begin{bmatrix} a_2 c_1 c_2 \\ a_2 s_1 c_2 \\ a_2 s_2 \end{bmatrix}, \quad p_4 = \begin{bmatrix} c_1 (d_4 s_{23} + a_2 c_2) \\ s_1 (d_4 s_{23} + a_2 c_2) \\ -d_4 c_{23} + a_2 s_2 \end{bmatrix}, \quad p_5 = \begin{bmatrix} c_1 (d_4 s_{23} + a_2 c_2) \\ s_1 (d_4 s_{23} + a_2 c_2) \\ -d_4 c_{23} + a_2 s_2 \end{bmatrix}, \quad p_e = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}
 \end{aligned}$$

$$s_i = \sin(\theta_i), \quad c_i = \cos(\theta_i), \quad s_{ij} = \sin(\theta_i + \theta_j), \quad c_{ij} = \cos(\theta_i + \theta_j).$$

4. Manipulator singularity:

For manipulators having spherical wrist, it is possible to split the problem of singularity determination into two problems (arm singularity and wrist singularity) and sub divide the jacobian as below[9]:

$$J_e = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \quad \dots\dots(12)$$

Where as it is mentioned before in eq.(11)

$$J_{12} = [z_3 \times (p_w - p_3) \quad z_4 \times (p_w - p_4) \quad z_5 \times (p_w - p_5)]$$

Choosing the end effector origin in wrist center as it is shown in figure (2) $d_6=0$ leads to $p_6 = p_5 = p_4$ and $(p_w - p_3)$ parallel to z_3 then $J_{12}=0$ and matrix block be lower triangular so:

$$\det(J_w) = \det(J_{11}).\det(J_{22})=0 \quad \dots\dots(13)$$

In turn,

- $\det(J_{11}) = 0$ leads to determine the arm singularity .
- $\det(J_{22}) = 0$ leads to determine the wrist singularity .

From eq. (11)

$$J_{11} = \begin{bmatrix} -s_1(a_2 c_2 + d_4 s_{23}) & -c_1(a_2 s_2 - d_4 c_{23}) & d_4 c_1 c_{23} \\ c_1(a_2 c_2 + d_3 s_{23}) & -s_1(a_2 s_2 - d_4 c_{23}) & d_4 s_1 c_{23} \\ 0 & a_2 c_2 + d_4 s_{23} & d_4 s_{23} \end{bmatrix} \quad \dots\dots(14)$$

$$J_{22} = \begin{bmatrix} c_1 s_{23} & -c_1 c_{23} s_4 + s_1 c_4 & s_5 (c_1 c_{23} c_4 + s_1 s_4) + c_1 s_{23} c_5 \\ s_1 s_{23} & -s_1 c_{23} s_4 - c_1 c_4 & s_5 (s_1 c_{23} c_4 - c_1 s_4) + s_1 s_{23} c_5 \\ -c_{23} & -s_{23} s_4 & s_{23} c_4 s_5 - c_{23} c_5 \end{bmatrix} \quad \dots\dots(15)$$

The program flowchart is showed in appendix.

Since the problem is decoupled then for arm (three links only) the kinematic equations of 0_3T can be produced from DH in table(2) as it is shown in figure(4) and the direct differentiation of position vector gives the linear jacobian termed analytical jacobian.

Table (2) DH of Arm

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	90	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

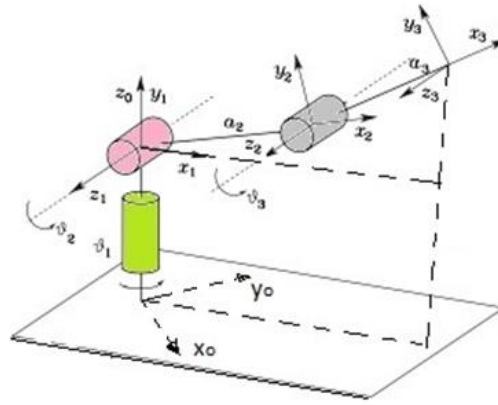


Figure (4) Arm Attached Coordinates System. [1]

Then:

$${}^0_3T = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & c_1(a_2c_2 + a_3c_{23}) \\ s_1c_{23} & -s_1s_{23} & -c_1 & s_1(a_2c_2 + a_3c_{23}) \\ s_{23} & c_{23} & 0 & a_3s_{23} + a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{.....(16)}$$

The analytical jacobian:[2]

$$J = \begin{bmatrix} \frac{\partial f_x}{\partial \theta_1} & \frac{\partial f_x}{\partial \theta_2} & \frac{\partial f_x}{\partial \theta_3} \\ \frac{\partial f_y}{\partial \theta_1} & \frac{\partial f_y}{\partial \theta_2} & \frac{\partial f_y}{\partial \theta_3} \\ \frac{\partial f_z}{\partial \theta_1} & \frac{\partial f_z}{\partial \theta_2} & \frac{\partial f_z}{\partial \theta_3} \end{bmatrix} \quad \text{.....(17)}$$

From eq. (16)

$$f_x = c_1(a_2c_2 + a_3c_{23}) \quad \text{.....(18)}$$

$$f_y = s_1(a_2c_2 + a_3c_{23}) \quad \text{.....(19)}$$

$$f_z = a_2s_2 + a_3s_{23} \quad \text{.....(20)}$$

Then arm jacobian is:

$$J_{Arm} = \begin{bmatrix} -s_1(a_2c_2 + a_3c_{23}) & -c_1(a_2s_2 + a_3s_{23}) & -a_3c_1s_{23} \\ c_1(a_2c_2 + a_3c_{23}) & -s_1(a_2s_2 + a_3s_{23}) & -a_3s_1s_{23} \\ 0 & a_2c_2 + a_3c_{23} & a_3c_{23} \end{bmatrix} \quad \text{.....(21)}$$

5. Results and discussion:

Comparing equations (13) & (20) (note that every $d_4s_{23} = a_3c_{23}$ & $d_4c_{23} = a_3s_{23}$); shows that they are different because the differences in forward kinematics, but there determinates (for both) lead to the same singular configurations with different values of θ_3 because of the rotation offset $\pi/2$.

Programs have been developed to embrace the theoretical work, the programs are able to calculate the forward kinematics and jacobian symbolically math and numerically with MATLAB software, so for equation (20):

$$|J_A| = s_3 a_2 a_3 (a_2 c_2 + a_3 c_{23}) = 0 \quad \dots\dots\dots(22)$$

Either $s_3=0$ then $\theta_3=0, \pi$. The elbow is outstretched or retracted as it is shown in figure (5) termed elbow singularity. Or $a_2 c_2 + a_3 c_{23} = 0$ this configuration occurs when the wrist center intersect the axis of the base rotation z_0 . There will be infinite solutions no matter how θ_1 is rotated; this type of singularity termed shoulder singularity. For our robot with $a_2=30\text{cm}$ and $a_3=28\text{ cm}$ the values of θ_2 and θ_3 are shown in figures (6, 7)

While the wrist singularity occurs when $|J_{22}| = s_5 = 0$ then $\theta_5=0, \pi$ which is not allowed to rotate about orthogonal axis z_4 . Such singularity is unavoidable without imposing a limitation on wrist to prevent z_3 and z_5 from lining up.

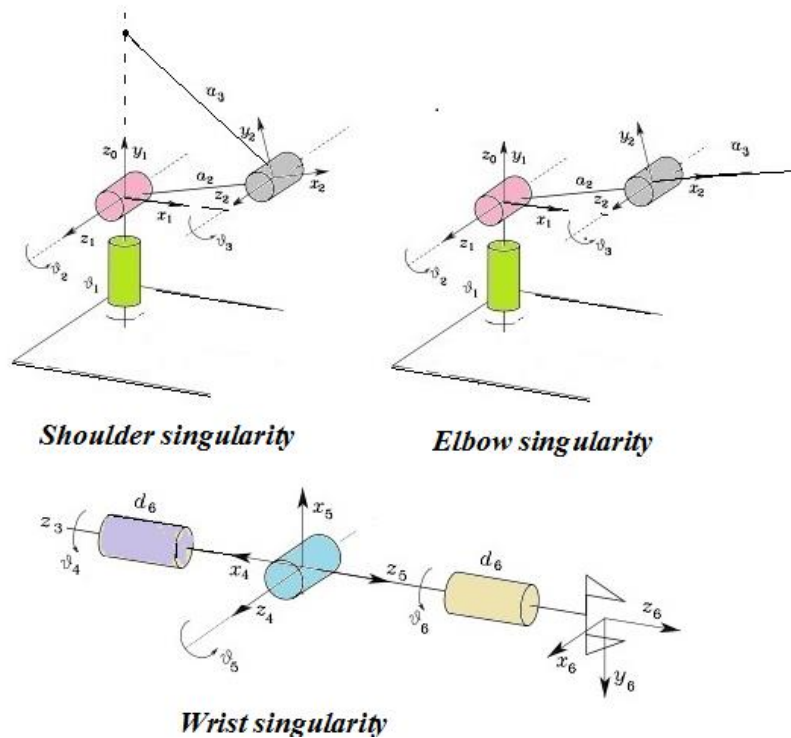


Figure (5) Manipulator Singularities [1]

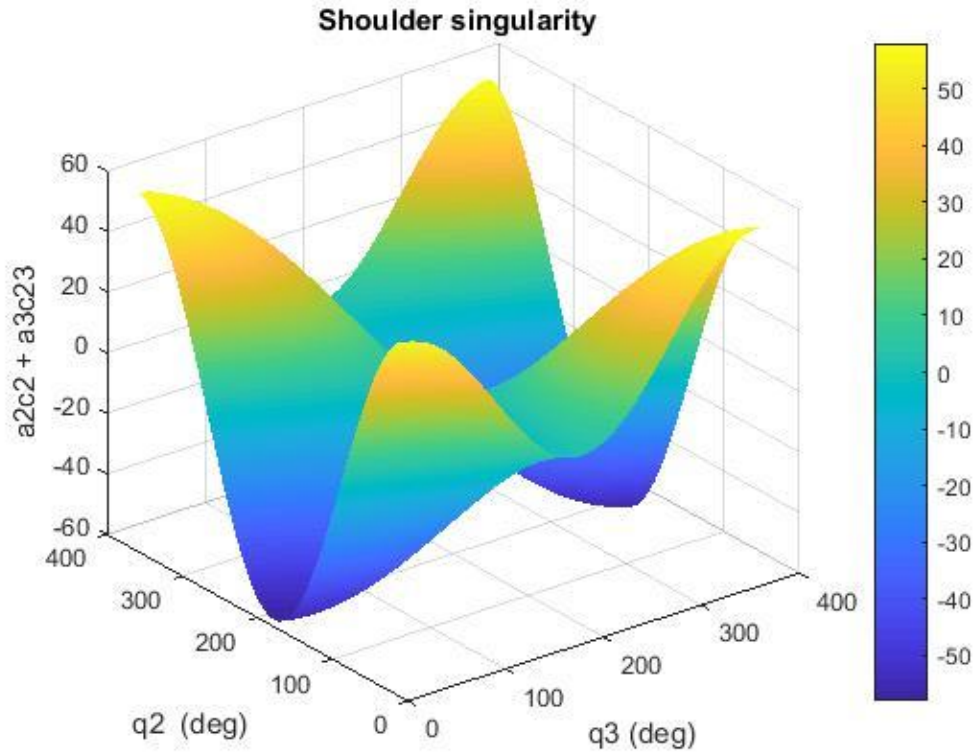


Figure (6) 3D Plot Illustrates the Shoulder Singularity

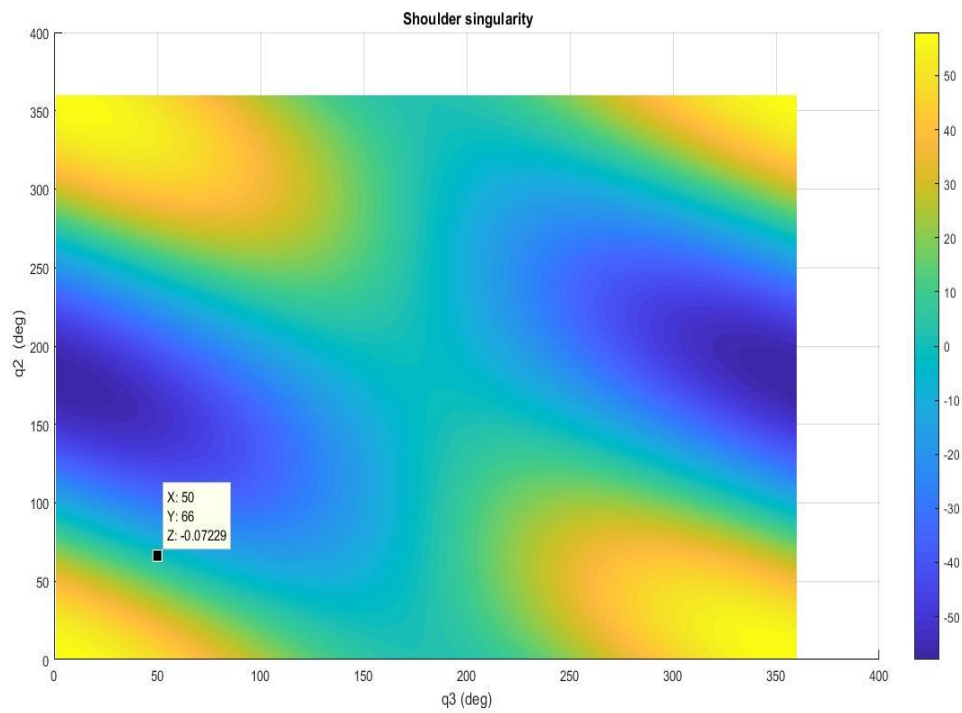


Figure (7) the Shoulder Singularity Locations at Zero Z-Axis

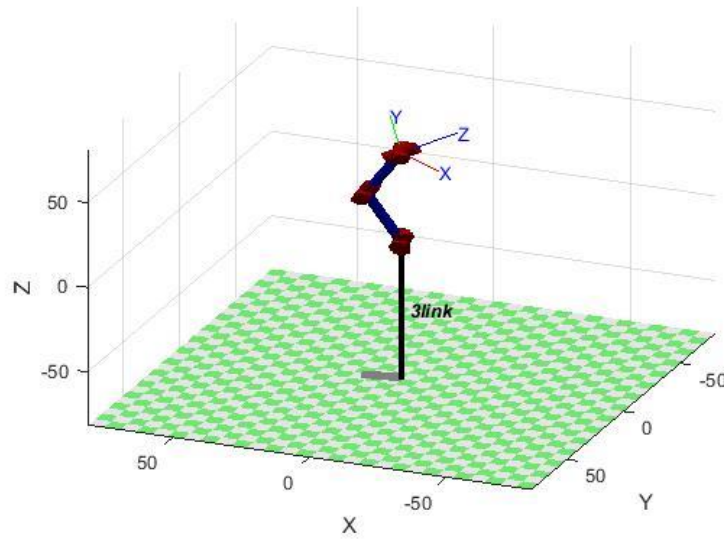
Table (3) Some Results Of the Arm Jacobian Determinant

Alert message appears when det(J11) sign is changed	det(J11)	θ_1	θ_2	θ_3
	-0.7158	10.0000	65.9200	50.0000
	-0.1254	10.0000	65.9210	50.0000
	0.4650	10.0000	65.9220	50.0000
	singularity pass	th1=10	th2=65.922	th3=50

The results show a full agreement with MATLAB robotics toolbox [10]. Where the input angles $([10 \ 65.922 \ 50+90 \ 10 \ 30 \ 23]^* \pi/180)$ for the configuration shown in figure (8) and the output results of jacobian matrix is:

$$J = \begin{bmatrix} 0.004 & 23.3355 & 20.0822 & 0 & 0 & 0 \\ 0.0006 & -18.8251 & -13.8064 & 0 & 0 & 0 \\ 0.0001 & 43.1851 & 13.7873 & 0 & 0 & 0 \\ -0.7275 & 0.5232 & 0.5232 & -0.4603 & 0.3907 & 0 \\ 0.3913 & 0.8478 & 0.8478 & 0.1954 & 0.9205 & 0 \\ 0.5636 & 0.0868 & 0.0868 & 0.8660 & 0.0000 & 1.0000 \end{bmatrix}$$

The determinant is not zero exactly since the chosen inputs are not exactly at singularity but are the nearest because the prices provision need more resolution in step angle and more processing time.

**Figure (8) Ones of the Singular Configurations with MATLAB.**

6. Conclusions:

- 1- The proposed kinematic model makes it possible to control the manipulator to achieve any reachable position and orientation.
- 2- For open chain manipulator with spherical wrist, the singularity problem may be decoupled into two problems; arm singularity and wrist singularity since $J_{12} = 0$.
- 3- There are three type of singularity for the manipulator:
 - Elbow singularity when $\theta_3 = 0, \pi$.
 - Shoulder singularity occurs at the condition $a_2 c_2 + a_3 c_{23} = 0$.

- Wrist singularity when $\theta_5=0, \pi$.

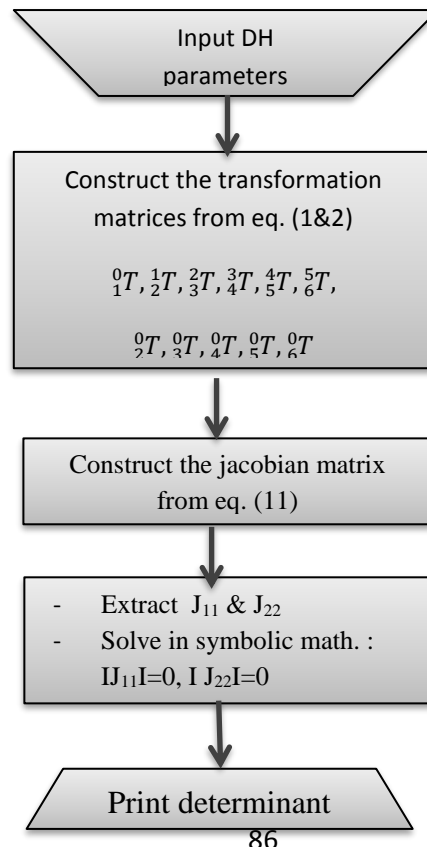
4- The built programs are capable of solving similar kinds of robot manipulators.

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8. Appendix:

Flow chart to calculate the jacobian determinant from forward kinematics



التحقيق والمحاكاة للتفرد الحركي لروبوت مناوّر ذو ثلاث روابط مع معصم كروي

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الخلاصة:

يُعد التفرد الحركي للذراع المناورة الروبوتية مشكلة مركزية في تتبع المسارات بسبب تقليل الرتبة للمصفوفة الجاكوبية. يتكون الروبوت في هذا البحث من ثلاث أذرع ذات مفاصل دوارة ومعصم كروي (ستة درجات حرية) وتم اعتماد طريقة دانفينت – هارتنبيرغ للتحليل الكينماتيكي الامامي لمعرفة الموقع والميلان للنهاية المؤثرة للروبوت. كما استخدمت الطريقة الهندسية والحسابية للمصفوفة الجاكوبية لتشخيص ومعرفة مواقع التفرد لغرض تجنبها في تخطيط المسارات ومشاكل السيطرة. اظهرت الدراسة ان الروبوت يمتلك ثلاث انواع من التفرد (تفرد الكوع $\theta_3=0,\pi$ و تفرد الكتف عند الشرط $a_2c_2 + a_3c_{23} = 0$ و تفرد المعصم $\theta_5=0,\pi$) و تم مطابقة و محاكاة الوضعيات المتفردة مع ادوات الروبوت لبرنامج الماتلاب. وقد كان للطرق المقدمة امكانية تحليل وتشخيص التفرد في الروبوتات المماثلة لغرض تحقيق مسارات مناسبة.

الكلمات الداله: روبوت مناوّر، معاملات دانفينت – هارتنبيرغ، الكينماتك الامامي، مصفوفة جاكوبية، التفرد.