



Whitney Block Estimation for Flexural Strength of UHPC Beams

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Abstract

Despite the development of modern building materials, including Ultra High Performance Concrete (UHPC) and the development of their resistance, the calculations of the flexural strength of structural members remained somewhat complicated and far adequate accuracy. The aim of this study is to present a simple accurate practical model for calculating the flexural strength by using compressive and tensile equivalent stresses with strain compatibility inclusion. Compressive equivalent stress was derived in a manner similar to Whitney Block in normal concrete by converting trapezoidal stress distribution into rectangular stress and providing an equation for both ratios of equivalent height to neutral axis (β) and the ratio of rectangular and trapezoidal stress intensity (k_3). The tensile stress was also modeled as equivalent block. Nine UHPC beams were fabricated and casted to study three main variables, which are compressive strength, ratio of longitudinal steel bars and volumetric fraction of steel fibers. The present flexural strength equation gave very similar results to the experimental.

Keywords: Flexural Strength, Strain Compatibility, Whitney Block, Equivalent Stress Block, UHPC.

1. Introduction

Ease of dealing with the analysis of structural elements with advanced materials in simple and accurate ways that allow researchers to shed light and create efficient structural elements. Variation in compressive stress and its distribution along the depth of the compression area play a significant rule to develop the strength. The yield stress of steel fibers, aspect ratio of their length to diameter, volumetric fraction, modes of failure, and distribution of tensile stress diagram either softening or hardening all of them affect on the tensile strength of the section. Many researchers as well as many international codes do not agree to some extent on the full representation of the stress and tension curve generated in UHPC.

The three lines of relations that adjusted the compressive strain and stress was considered. The first line originated up to eighty-five percentage of compressive strength to a specific strain while middle line was constant with the same magnitude of the first line but up to 0.0004 strain. The last line was falling down to a corresponding strain of 0.007 [1].

Many figures for both service and ultimate compressive and tensile strength were constructed. In ultimate strength, the compressive relation was plotted as bilinear with maximum strain of 0.003 and 1.3 limitation for safety due to manufacturing mistakes [2].



The same approach of AFGC regarding the compressive effects whereas the tensile effects was assumed as uniform intensity as fraction of square root of characteristics compressive strength [3].

The experimental examination for the bending capacity of UHPC beams was introduced. Three contents of steel fibers were included which were entire, half and free. The steps of ACI-318 code were utilized in design of UHPC beam with maximum strain of 0.0035. No any indication for tensile strength was mentioned [4].

The bilinear compressive stress distribution over the compression region of cross-section as trapezoidal was utilized. A Tension Softening Diagram (TSD) for standard tension UHPC and direct tension tests were carried out. The lower bound of 6.8 MPa as tensile stress was adopted [5].

The ductility capacities of 10 beams reinforced with steel bars and 150 MPa of compressive strength of UHPC were studied. The design of all beams was based upon ACI-318 code and KCI 2007 standard 5. The compressive strength was represented as normal concrete of 85 percentage of compressive strength with maximum stain 0.003. The tensile strength was limited to bar component with the absence of a steel fibers contribution. The total length of the beams was 3.1 m with 220 mm in width and 250 mm in depth. The experimental flexural strengths were higher by about 30% than the estimated strength [6].

A comprehensive work for comparison the flexural strength between UHPC and High Strength Concrete (HSC) was done. Nine Beams of UHPC and three HSC were fabricated and tested under pure bending. Various reinforcement ratios and fibers content were utilized. The work included the study of the crack pattern, modes of failure, load-deflection relations, ductility and toughness. For tensile strength, the notched prisms were used to carry out the crack mouth opening displacement (CMOD) test. No design method was declared to know how much difference in estimation the flexural strength [7].

An experimental program of UHPC beams reinforced with steel bars was performed. The study contained the flexural mode failure type as well as the combined flexural-shear modes. For flexural strength, a triangular distributed compressive stress was assumed. The tensile strength was assumed as uniformly distributed along the partial depth underneath the neutral axis. The tensile stress was introduced by multiplying it with 0.47 as mean factor. The suggested model was extended for comparison purposes with previous studies with 0.89 convergence of the flexural ratio [8].

The investigation of the ductile behavior of UHPC beams was done. The beams were reinforced by hybrid steel bars and Carbon Fiber Reinforced Polymer bars (CFRP). The design that based on the steps mentioned in Euro code 2 and 10 MPa as tensile strength was suggested [9].

In tensile region, the ultimate strength that generated by steel fibers was assumed as constant stress of 6.9 MPa along the crack path, while a reduction of twenty-five percentage was suggested to be 8 MPa as a residual tensile stress [10].



The expressions for flexural strength of over, balanced and under reinforced of CFRP bars embedded within reactive powder concrete beams were derived. For both cases of over-reinforced and balanced, the compressive stress was modeled as trapezoidal distribution. For under reinforced case, the triangle stress destitution was used and converted to uniformly distributed by two factors [11].

No researchers had used the equivalent stress block for compressive stresses by converting the trapezoidal distribution to the rectangular as Whitney Block criteria for UHPC and strain compatibility.

3. Whitney Block Derivation

After a review of the above sources, most of the references state the use of the trapezoidal distribution. So, the distribution will use in this study and then converted to equivalent rectangular stress intensity by Whitney criteria. The depth of the uniform of stress intensity (x) can be obtained from the triangles similarity of stain and can be expressed as the following equation.

$$x = \left(1 - \frac{\lambda f_c'}{\varepsilon_c \cdot E_c}\right) \cdot c = \gamma \cdot c \quad (1)$$

Where the ε_c is the concrete's stain at level (x) from top fiber; E_c refers to the modulus of elasticity of UHPC; C is the depth of Neutral Axis (NA) and λ was used as 0.65, the inverse of safety factor of 1.3 multiplying by 0.85.

Then, the compressive forces were listed based on the same criteria of Whitney Block which are 1) the same areas of the stress block intensity and 2) coincide the centroids of the actual and the practical model as, Fig 1:

$$k_3 = \frac{(1+\gamma)}{2\beta} \quad (2)$$

$$\beta = \frac{2}{3} \times \frac{(1+\gamma+\gamma^2)}{1+\gamma} \quad (3)$$

Where, β is the ratio between height of equivalent rectangular concrete compressive stress block and neutral axis depth; k_2 is the ratio of distance between extreme compressive fiber and resultant force of compressive block to that between the same fiber to neutral axis and; k_3 is the ratio of the UHPC strength of the trapezoidal concrete strength to the rectangular compressive stress block.

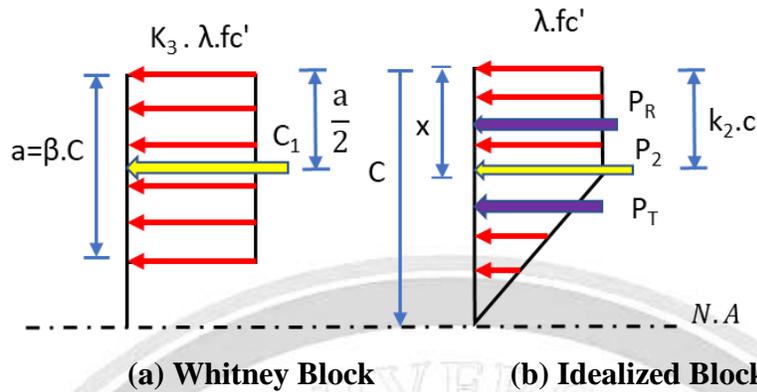


Fig. 1. Equivalent compressive stress.

Then, the compressive forces can be expressed as:

$$C_1 = P_2 = P_R + P_T = k_3 \cdot \lambda \cdot f c' \cdot a \cdot b \tag{4}$$

The tensile forces of the concrete underneath NA that excited by steel fibers was assumed as uniformly distributed stress besides the reinforcing bars. The β and k_3 values were graphically represented for wide ranges of compressive strength by the Figs. 3 and 4.

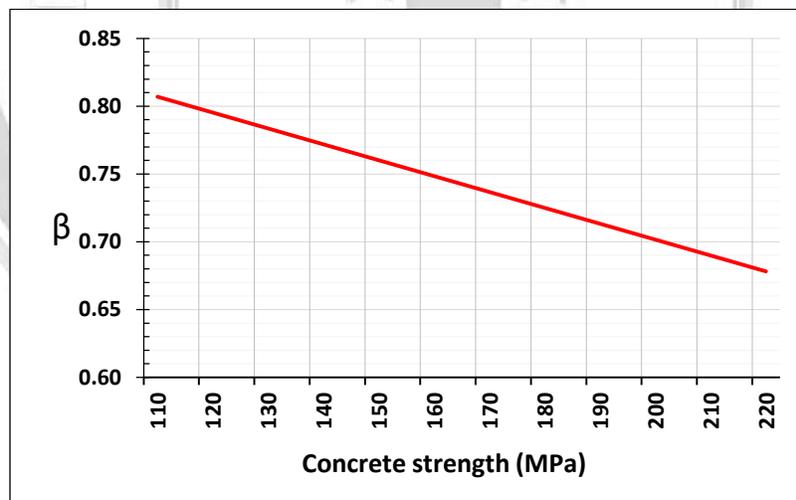


Fig. 3. β value versus UHP concrete strength

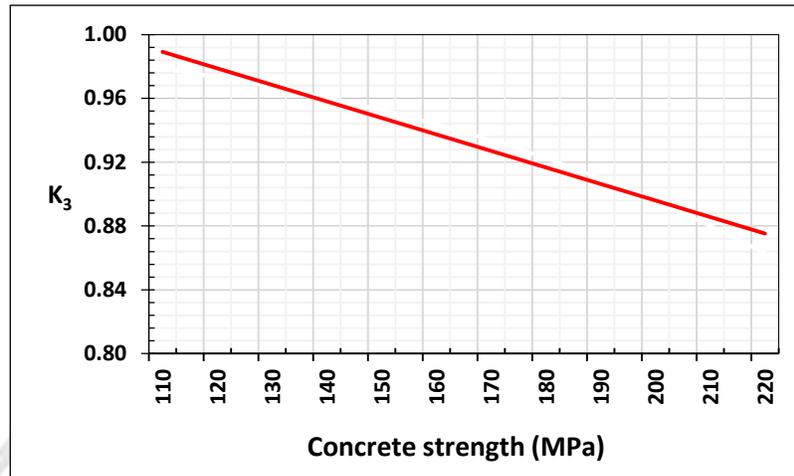


Fig. 3. k_3 value versus UHP concrete strength

3. Fulfilment the Models

Predication of the suggested method for calculation of the ultimate flexural capacity of UHBC beams was carried out over other studies to extend the validity of the current method. Eight beams were subjected to their suitability for beams analysis. Variation in compressive strength, width, depth, modulus of elasticity and ratio of longitudinal reinforcement are the most factors that included. Table 2 illustrates the comparison of the predicated load capacity ($P_{u,f}$) based on the specific reference's data and their experimental load capacities ($P_{u,E}$).

Table 2. Comparison Predication and experimental Load Capacities

Ref.	β	γ	K_3	c (mm)	a (mm)	($P_{u,f}$) (kN)	$P_{u,E}$ (kN)	% ($P_{u,Exp}/P_{u,f}$)
Sung-Woo [6]								
15-BH4-a	0.742	0.398	0.941	78.84	58.52	377.40	360.0	95.39
15-BH4-b	0.742	0.398	0.941	90.68	67.30	431.83	444.2	102.86
15-BH4-c	0.742	0.398	0.941	101.97	75.68	480.24	495.5	103.18
In-Hwan [7]								
UHPC-F20-R1	0.744	0.402	0.943	54.15	40.27	197.24	187.5	95.06
UHPC-F20-R2	0.744	0.402	0.943	52.68	39.17	207.12	224.4	108.34
UHPC-F20-R3	0.744	0.402	0.943	59.1	43.95	237.70	243.5	102.44
Shiming [8], B-4	0.727	0.350	0.928	76.05	55.30	373.20	353.1	94.61
Si-Larbi [9] Beam1	0.733	0.369	0.934	77.8	57.01	203.04	199.0	98.01
Average								99.99



4 The Experimental Work

4.1 UHPC Properties

The materials involved in forming super concrete can be summarized:

- 1- Type I of ordinary Portland cement.
- 2- The fine granular sand of maximum size of 600 μm per BS- No. 882 [12].
- 3- Micro granular silica (small than 0.1 μm) per ASTM C1240-04[13].
- 4- Copper plated steel fibers of 65 aspect ratio (0.2 mm in diameter and 13 mm in length).
- 5- Super-plasticizer PC260.

The proportion of the used mixture for the structural beams is shown in Table 1. For measuring the compressive strength and tensile strength, the cylinders of (200 \times 100) mm were used and prisms of (100 \times 100 \times 500) mm dimensions were utilized to measure the modulus of rupture (tensile strength), Tables 2 and 3.

Table 2. Mix Proportion for UHPC

Compressive Strength (MPa)	Cement (kg/m ³)	Sand (kg/m ³)	Silica Fume (kg/m ³)	HRWR (%)	W/C (%)	Steel Fibers (kg/m ³)
135	1000	1000	250	3	0.2	157
150	1015	900	305	3	0.2	157
160	1035	920	325	3	0.2	157

Table 3. Compressive and Tensile Strengths

Compressive Strength (MPa)	Tensile Strength (Splitting) (MPa)	Tensile Strength (Prism) (MPa)
135	13.4	15.7
150	13.8	16.1
160	14.1	16.9



4.2 Steel Bars

According to ASTM (A615/A615-04a), three samples for each diameter were tested. The obtaining results were the yield stress (f_y), ultimate stress (f_u) as well as the elongations (Table 4).

Table 4. Tensile Strengths for Reinforced Bars

Test Results				ASTM A 615/A615-04a Min. limits		
Bar size (mm)	Yield strength (MPa)	Ultimate strength (MPa)	Elongation (%)	Min. Yield strength (MPa)	Min. Ultimate strength (MPa)	Elongation (%)
10	515	624	31.3	420	620	9
16	571.6	657.7	32.9	420	620	9
25	581.3	673.8	33.7	420	620	8

4.3 Specimens and Parameters

All beams were designed to fail in flexural. Group 1 consists of three beams N-F-1, N-F-2 and N-F-3. Each one was reinforced by 2 Φ 25mm to study the effect of compressive strength on the flexural strength. Group 2 have three beams N-F-4, N-F-5 and N-F-6 with different ratios of tensile longitudinal main bars to investigate the adequation of the present model along the experimental data. Group 3 have three beams N-F-7, N-F-8 and N-F-9 and were reinforced by 2 Φ 25mm. The aim of this group is to examine the effect of the volumetric fraction of steel fibers. Group 2 and Group 3 have the same compressive strength as average value with suitable standard deviation. The beams were designed to fail in flexural and all dimensions are 200 mm in width and 250 mm in depth. The clear span is 2.1 m and the overall length are 2.5 m. The modulus of elasticity of each beam were obtained by divided the specific compressive strength on the assumed ultimate concrete strain (0.0035). The beams were fabricated in such a way to avoid the deep beam limitation. Both shear spans were supported by stirrups of Φ 10 mm each 100 mm, Fig. 5.

Table 5. The Details of Tested Beams

Group	Beam ID	Compressive Strength			E_c (MPa)	Rein. Area (mm ²)	Steel Fibers (%)	
		f_c' (MPa)	Average (MPa)	Standard Deviation				
G-1	N-F-1	133.50	---	---	38143	2 Φ 25(982)	2	
	N-F-2	153.00			43714			
	N-F-3	166.63			47610			
G-2	N-F-4	158.5	160	1.32	45286	3 Φ 16 (603)	2	
	N-F-5	159.4			45543			2 Φ 25 (982)
	N-F-6	161.1			46029			3 Φ 25(1472)
G-3	N-F-7	156.8	161	4.3	44800	2 Φ 25(982)	1	
	N-F-8	161.1			46029		1.5	
	N-F-9	165.4			47257		2	

4.5 Production Process

After completing the wooden mold according to the dimensions mentioned and completing the process of connecting the reinforcing steel structure, the pouring of the fresh UHP concrete was carried out according to the following steps:

Mix each of the cement with the silica Fume first in the mixer for about a minute and a half. Then add fine sand to the mixture while continuing the process of mixing dry materials for an additional minute and a half. The water and the superplasticizer are mixed separately, then half the quantity is added to the dry mixture and the mixing process continues. Finally, half the amount of water with the superplasticizer is added to the mixture while continuing to mix until a good consistency is obtained and transferred to be poured into the form.

The total load capacity of the testing rig is 600 kN. The position of dial gages was at the center of the clear span and center of shear spans. The 1 kN of pre-testing loading was applied to ensure the right position of the beam. The loads are applied in successive increments of 5-10 kN until reaching the failure load.

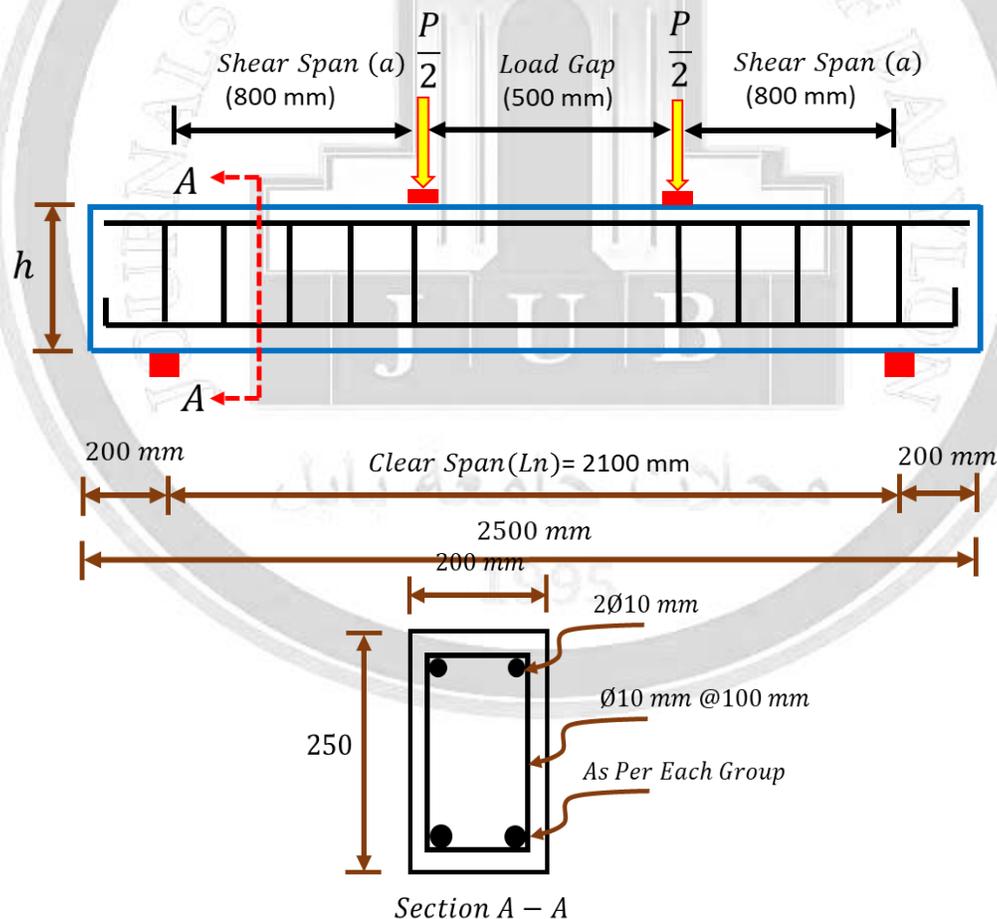


Fig. 4. Layout of Tested Beams

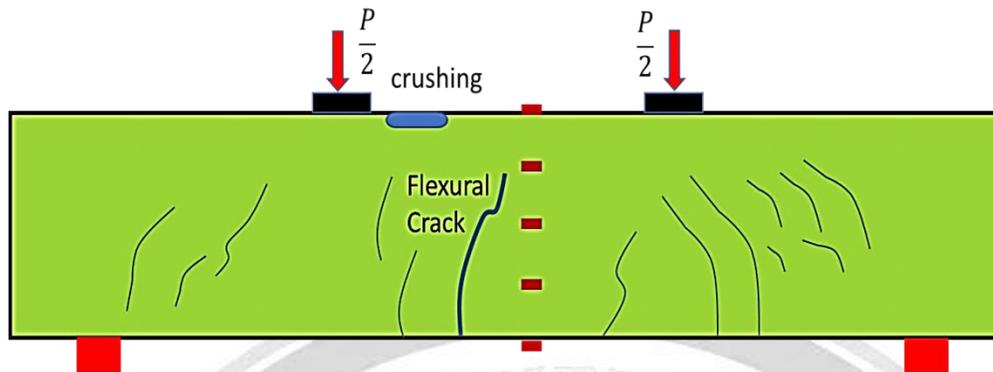


Fig. 5. Flexural Mode Failure

5. Results and Discussion

In group one, the 2 % volumetric fraction of steel fiber was the same for all. That means the steel fibers participation for flexural strength remains constant, Fig. 6 to Fig. 9. Also, the ratio of longitudinal bars is the same while the compressive strength of concrete was variable. When increasing the resistance from 135 MPa in the N-F-1 specimen to 160 MPa in the N-F-3 specimen through the resistance of 150 MPa in the N-F-2 beam, the flexural strength increased. This may be due to the increasing in the compressive force in the upper parts of the section above the neutral axis. The increased compressive force was accompanied by a decrease in the depth of the neutral axis, which caused the internal equilibrium. The ultimate carrying capacity load was almost the same than the tested beams, Table 6. So, the using of residual tensile force of 8 MPa pushed the flexural strength to be in accepted ranges.

Table 6: Proposed and Experimental Ultimate Loads

Group	Beam ID	γ	β	k_3	a (mm)	c (mm)	$P_{u,prd}$ (kN)	$P_{u,Exp}$ (kN)	% ($P_{u,Exp}/P_{u,prd}$)
G-1	N-F-1	0.395	0.741	0.941	43.59	58.80	279.14	293.60	105.18
	N-F-2	0.350	0.727	0.928	39.05	53.70	284.00	294.30	103.63
	N-F-3	0.382	0.737	0.937	37.53	50.92	285.70	295.10	103.29
G-2	N-F-4	0.350	0.727	0.928	29.12	40.05	221.03	230.40	104.24
	N-F-5	0.350	0.727	0.928	37.59	51.70	285.52	294.60	103.18
	N-F-6	0.343	0.725	0.926	46.73	64.45	366.51	385.70	105.24
G-3	N-F-7	0.350	0.727	0.928	32.36	44.50	250.92	250.60	99.87
	N-F-8	0.350	0.727	0.928	33.49	46.06	263.21	270.10	102.62
	N-F-9	0.350	0.727	0.928	36.39	50.05	286.85	295.60	103.05
								Average	103.37

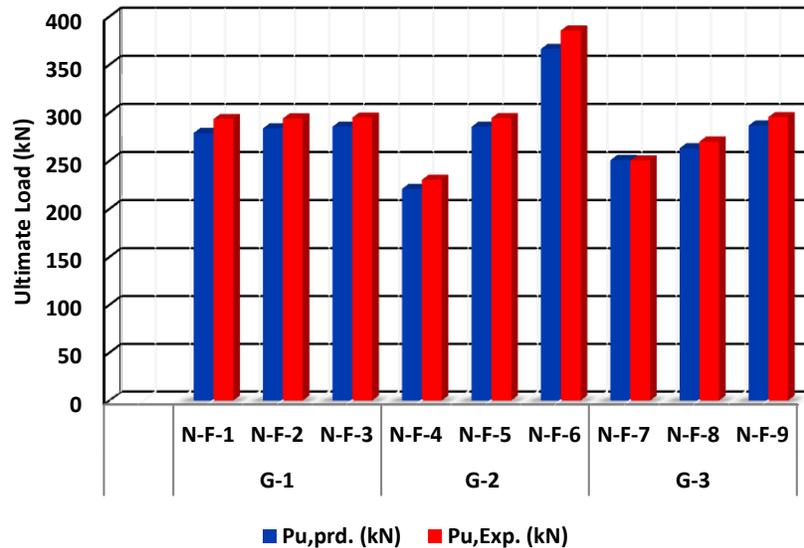


Fig. 6. Ultimate Experimental and Proposed Flexural Load

For Group 2, the volumetric fraction of steel fibers was the same for all beams, 2%. Also, the compressive strength with the suitable standard deviation remained approximately the same too. When the ratio of longitudinal steel increases, the ultimate loads increase. This may be due to the increasing in the flexural strength of rebar part. The neutral axis depth is increased when the ratio of steel bars is increased. The increase in the depth of the neutral axis provided an increase in compressive strength, which works with the increase in steel strength to raise the efficiency of the UHP concrete section. A 70% increase in ultimate load was obtained when the rebars area was increased by 2.4 times, comparing the N-F-4 and the N-F-6. Also, an increase of 200% was obtained in the resistance concerning the rebar component of flexural strength, Fig. 8.

The third group was designed to study the effect of steel fiber content on resistance. The presence of steel fibers leads to a slight increase in compressive strength, but at the same time it plays a major role in the tensile strength below the neutral axis. When the steel fibers content increases, the neutral axis increases slightly and then increases the compressive strength. The value of residual tensile stress (F) is varied due to the variable content of steel fibers.



- The value of ratio between height of equivalent rectangular concrete compressive stress block and neutral axis depth (β) can be estimated as equation in term of concrete strength of $0.8128 - 0.006 f_c'$.
- The ratio of the UHPC strength of the trapezoidal concrete strength to the rectangular compressive stress block (k_3) can be expressed as $0.99-0.0052 f_c'$.
- The flexural strength of the fibers increases by 50% when the volumetric fraction of fibers is increased from 1% to 2%.
- The ultimate load of rebars increased by 70 % when the tensile rebar area is increased by 2.4 times while the flexural strength increased by 200 % at the same increasing of the tensile steel area.
- All beams failed in compression failure type at extreme fibers with the ductile behavior.

Nomenclatures

b	Width of concrete beam, mm
c	Neutral axis
C_1	Compressive Force, kN
f_{cu}	Concrete compressive strength, MPa
f_y	Yield strength of longitudinal reinforcement, MPa
f_R	Residual Tensile Stress, MPa
G	Group
h	Depth of Beam, mm
K_3	Ratio of rectangular and trapezoidal stress intensity
T_1	Tensile Force due to steel fibers, kN
T_2	Tensile Force due to rebars, kN

Greek Symbols

ϕ	Diameter of reinforcement, mm
β	Ratio between height of equivalent rectangular concrete compressive stress block and neutral axis

Abbreviations

B-F	Beam
UHPC	Ultra-High Performance Concrete
HSC	High Strength Concrete

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تقدير كتلة وتني لمقاومة الانحناء لعتبات الخرسانة فائقة الأداء

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الخلاصة

على الرغم من تطوير مواد البناء الحديثة، بما في ذلك الخرسانة فائقة الأداء (UHPC) وتطوير مقاومتها، ظلت حسابات قوة الانحناء للأعضاء الهيكلية معقدة إلى حد ما وذات دقة غير كافية إلى حد بعيد. الهدف من هذه الدراسة هو تقديم نموذج عملي بسيط ودقيق لحساب قوة الانحناء باستخدام الضغوط المكافئة للشد والضغط مع تضمين توافق الانفعال. تم اشتقاق الضغط المكافئ للضغوط بطريقة مشابهة لـ Whitney Block في الخرسانة العادية عن طريق تحويل توزيع الإجهاد شبه المنحرف إلى إجهاد مستطيل وتوفير معادلة لكل من نسب الارتفاع المكافئ إلى المحور المحايد (β) ونسبة شدة الإجهاد المستطيل وشبه المنحرف (k_3). تم تصميم إجهاد الشد أيضًا على أنه كتلة مكافئة. تم تصنيع تسع عتبات خرسانية فائقة الأداء وصيها لدراسة ثلاث متغيرات رئيسية وهي مقاومة الانضغاط ونسبة القضبان الفولاذية الطولية ونسبة الياف الحديد. أعطت معادلة قوة الانحناء الحالية نتائج مشابهة جدًا لنظيراتها العملية.

الكلمات الدالة: مقاومة الانحناء، توافق الانفعال، كتلة وتني، كتلة الاجهاد المتوافق، خرسانة فائقة الأداء.

