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A-density in Soft Topological Spaces

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Abstract

The soft set theory regards one of an important block of the set theory has real role in various fields of mathematics , because that the soft set theory solved several mathematical problems .Since the general topology depends on the statements of the set theory , as the soft topology depends on the soft set theory as well .

In this study, the concentration would be on building several properties with newest aspects in the soft topological space . this work required going through some basic concepts with the needed definitions as well .

Keywords: Soft set , a-soft intersection , a-soft union , a-soft complement ,a-soft neighborhood , a-soft closure , a- soft interior , a-soft dense , a- soft nowhere dense .

الخلاصة

تعتبر (نظرية المجموعات الناعمة) واحدة من أهم فروع (نظرية المجموعات) والتي تعتبر بدورها الاداة الرياضية الاهم في كل فروع المعرفة العلمية ، ,وهذا يؤدي بدوره الى ارتباطهِ ً ببقية العلوم الاخرى وذلك لما يحتويه من ادوات رياضية ذات مرونة عالية في حل العديد من المشكلات العلمية . وحيث ان التوبولوجيا العامة تعتمد اعتماداً كلياً على قوانين (نظرية المجموعات) فأن (التبولوجي الناعم) الذي هو فرع التوبولوجيا العامة يعتمد هو الاخر وبشكلٍ تامٍ على (نظرية المجموعات الناعمة) .

في هذه الدراسة، كان التركيز الرئيس هو بناء وتكوين العديد من الخصائص بسمات وجوانب جديدة في الفضاء التبولوجي الناعم. هذا العمل تطلب منا بعض المفاهيم والتعاريف الاساسية التي تساعدنا في البحث .

الكلمات المفتاحية: المجموعة الناعمة، الجوار الناعم –a ، الانغلاق الناعم–a ، الادخال الناعم – a، الكثافة الناعمة – a ، انعدام الكثافة الناعمة – a.

Introduction

[1], started with the soft set theory as a mathematical tool to deal with problems in which there is some suspicion which the usual mathematical tools are un able to resolve . Because of the necessity of finding new solutions for these problems , this scientist has put many tools to resolve some of problems that are in concern with human life directly or un directly .

In [1] [2] [3] [4] [5] [6] [7] [8], the fact that we must focus on is that all of these researches depending on the following fact when defining the soft set :

If χ is a universal set and A be a subset of a set of parameters E , then a soft set $F_A, F: A \longrightarrow IP(\chi)$ is not the null soft set , (I . e $F_A \neq \tilde{\Phi}_A$ $\forall$ a $\in$ A iff $F(a) \neq \varphi$ $\forall$ a $\in$ A , where $\tilde{\Phi}_A$ is the null soft set $[F(a) = \varphi$ $\forall$ a $\in$ A .

We will proof in the following paper that this condition needs not be true , but it is a special case for the following condition :

If F_A is not the null soft set $a \in A$ such that $F(a) \neq \varphi$.

This means that it is sufficient to have only one element whose image is not the null soft set . in such case the soft set will not be null soft set and we will explain that in an example.

One of the agreed upon facts which concern the origin of the soft set is the dynamicity by which these types of sets have been invented in the following way :

$$F_A = \{(a, F(a)), a \in A, F(a) \in IP(\chi)\} \text{ and } \forall a \in A, \text{ then } (a, F(a)) \in F_A .$$

Many of the researches studies the family points which is the result of the cross between the two coordinates of the two soft sets

(I . e $\{a\} \times F(a) = \{(a, x), x \in F(a)\} \subseteq A \times \chi$.

From above we can conclude that each of the ordered pairs may be dealt whit individually as (soft set) if we considered the set $F(a) = \{x\}$when the study is on the point $a \in A$, so :

(x, a) is an $a -$ soft point iff $\{(a, x)\} \subseteq \{a\} \times F(a)$ is a soft set .

All these notes be explained more in the following papers .

In [6] the example (2.6) is incorrect in that we believe that the soft set G_B , does not necessary represent a soft set according to the definition introduced by molodtsov [1] , because of the fact 0that the set $\neg A$ is a subset of the set of all Parameters E_χ , if it is so , it is possible to be a domain of the relation F , and F_A will be a soft set . in fact the set $\neg A$, is an ambiguous set (whose feature are not clear) from the set E_χ; because E_χ is also ambiguous . As we mentioned that they might be signs , features , or symbol , and the signs and the feature are facts that cannot be restricted . and excluding element of a subset from E_χ does not mean that the remaining is always excluding (A) elements. In other word we can’t from which the rest of the unknown number parameters that represent $\neg A$, consequently , this expression will not be accurate at all.

As an example , if the universal set χ contains a set of pictures as ($A \subseteq E_\chi$) and it contains of several parameters as (sad , angry , proud) , the negation of the adjective (sadness) will be (happiness) and the negation of the adjective of (anger) will be (quietness) and the same with the adjective of (pride) it will be (humbleness) .

From the above example , we want to say that each parameter should have a direct antonym and it is accurately the opposite . This note has lost the focus of several researches in spite of the fact that it represents a basic factors in drawing a soft set . There are problems appeared as we have indicated in example (17) from [6] , the resource [8] and example (6) , the research falls in the same problem and in the resource [7] as well. On this statue , we opine to make E_χ by the following expression :

$E_\chi = E \cup \neg E$, where $\neg E$ is the negation set of E , this means that E and , $\neg E$ are disjoint which implies that :

$E_\chi \backslash E = \neg E$ and $E_\chi \backslash \neg E = E$.

By taking in to consideration the last period , the care in this field has gained many development , and the voice of this field spread in rapid range, because of the light nature that it stands on a soft sets .In [9] [4] maji et al had studied [soft set theory] and used this theory in several problems that had a relation in making decision. in addition to that he presented a concepts [fuzzy soft set] and generalized several concepts in [fuzzy soft set] , and he studied its properties as well .In [10] Ali etal defined some of new operations in soft set , in [6] Aktas and Naim wrote in [soft sets and soft groups], in [11] [12] also applied the soft set theory in problems relation making decision .

We will study the basic concepts in soft set theory and what follows it in soft topological spaces just like (soft a- closure ,a- soft interior , a-soft closed set , a-soft denes set and its properties that related to it at the point $a \in A$.

Preliminary

In this section, we will mention the most important concepts, definitions and results which have been reached to in previous studies , and we present new concepts depend on the same way that is depended by the Greece researches [D.A . GEORGION and A .C . MEGAITIS] in [13] .

We will denoted by χ to the initial universe , E_χ the set of possible parameters under consideration with respect to χ , and $A \subseteq E_\chi$.

Difinition1.1 [1**]**

A pair (F, A) is called soft set over χ , where F is a mapping defined as : $F: A \longrightarrow IP(\chi)$, we are simply to the soft set by F_A .

The soft set has the property $F(a) = \chi$ for all $a \in A$ is called the absolute soft set and denoted denoted by $\tilde{\chi}_A$.

The soft set has the property $F(a) = \varphi$ for all $a \in A$ is called the null soft set and denoted ` by $\widetilde{\Phi}_A$. null

Remark1.2

$S(\chi)$ refers to the set of all soft sets over the universe χ with respect to $A \subseteq E_\chi$.

Difinition1.3

If F_A be any soft set over the universe χ . $\forall\ a \in A$ and $x \in \chi$,we say that x_a is an a −soft point of F_A, and defined as:

$$x_a = \{(x, a)\ , x \in F(a)\} \subseteq \{a\} \times F(a)$$

Also we can say that $x \in_a F_A$ iff $x \in F(a)$,

And we say that $x_a \tilde{\in} F_A$ if $\{(a, x)\} \subseteq \{(a, F(a)\}$.

Difinition1.4

If F_A be any soft set over the universe χ , then F_A is called an $a-$ absolute soft if $F(a) = \{\chi\}$, it is dented by $\tilde{\chi}_a$. for each $a \in A$

(I .e $\tilde{\chi}_a = \{(a, \{\chi\})\}$) .

also F_A is called an $a-$null soft set if $F(a) = \{\varphi\}$, it is denoted by $\tilde{\Phi}_a$.

(I .e $\tilde{\Phi}_a = \{(a, \{\varphi\})\}$.

the $a-$soft complement is an $a-$soft set over χ defined as :

$\tilde{\chi}_A -_a F_A = H_a$ such that $H_a = \{(a, \chi - F(a))\}$.

Note1.5

We will denoted to the soft set F_A at the point $a \in A$ by F_a .

Difinition1.6

If F_A , G_A be any two soft sets over the universe χ , then:

(1) F_A is an $a-$soft subset of G_A iff $F(a) \subseteq G(a)$ and we write the form $F_A \tilde{\subseteq}_a G_A$. for each $a \in A$.

(2) the $a-$soft intersection of F_A and G_A , is an $a-$soft set is defined as follows :

$F_A \tilde{\cap}_a G_A = H_a$ such that $H_a = \{(a, F(a) \cap G(a))\}$.

(3) the $a-$soft union of F_A and G_A , is an $a-$soft set defined as follows :

$F_A \tilde{\cup}_a G_A = H_a$ such that $H_a = \{(a, F(a) \cup G(a))\}$.

(4) F_A and G_A are called $a-$soft equal iff $F(a) = G(a)$, and denoted by $F_A =_a G_A$.

Proposition1.7

If F_A , G_A be any two soft sets over the universe χ, . for each $a \in A$, we have :

(1) $\tilde{\chi}_a -_a [F_A \tilde{\cup}_a G_A] = [\tilde{\chi}_a -_a F_A] \tilde{\cap}_a [\tilde{\chi}_a -_a G_A]$

(2) $\tilde{\chi}_a -_a [F_A \tilde{\cap}_a G_A] = [\tilde{\chi}_a -_a F_A] \tilde{\cap}_a [\tilde{\chi}_a -_a G_A]$

Proof //These properties can be considered as a special case of proposition (2.2) [6] and we can proof them by [definition 1.6]

We can generalize this proposition to the following proposition :

Proposition1.8

If $I = \{F_{i_A}, i \in I\}$, be a family of a soft sets over the universe χ

Then for $a \in A$:

(1) $\tilde{\chi}_A -_a \{\tilde{\cup}_a (F_{i_A})\} = \tilde{\cap}_a (\tilde{\chi}_A -_a F_{i_A}))$

(2) $\tilde{\chi}_A -_a \{\tilde{\cap}_a (F_{i_A})\} = \tilde{\cup}_a (\tilde{\chi}_A -_a F_{i_A}))$

Proposition1.9

If F_A , G_A be any two soft sets over the universe χ , for each a ∈ A the following are true :

(1) $F_A \tilde{\cap}_a \tilde{\Phi}_A =_a \tilde{\Phi}_a$

(2) $F_A \tilde{\cap}_a \tilde{\chi}_A =_a F_a$

(3) $F_A \tilde{\cup}_a \tilde{\Phi}_A =_a F_a$

(4) $F_A \tilde{\cup}_a \tilde{\chi}_A =_a \tilde{\chi}_a$

We can conclude these facts directly from [definition1.8] it can be considered as a special case of [proposition 2.3] in [6]

Proposition1.10

If F_A , G_A be any two soft sets over the universe χ , . for each a ∈ A the following are true :

(1) $F_A \tilde{\subseteq}_a G_A$ iff $F_A \tilde{\cap}_a G_A =_a F_a$

(2) $F_A \tilde{\subseteq}_a G_A$ iff $F_A \tilde{\cup}_a G_A =_a G_a$

Proof//directly from definition [1.7 ,1.8].

Remark1.11

Note that $\tilde{\Phi}_a \tilde{\subseteq}_a F_a \tilde{\subseteq}_a \tilde{\chi}_a$

Remark1.12

All of theorems and properties that are true in [4] [5] are also be true in this research (I .e when we considered the point a ∈ A as a base of this work) .

Proposition1.13

If F_A , G_A and H_A be a soft sets over the universe χ , then for a ∈ A , the following are true :

(1) if $F_A \tilde{\cap}_a G_A =_a \tilde{\Phi}_A$, then $F_A \tilde{\subseteq}_a (\tilde{\chi}_A -_a G_A)$

(2) $F_A \tilde{\subseteq}_a G_A$ iff $(\tilde{\chi}_A -_a G_A) \tilde{\subseteq}_a (\tilde{\chi}_A -_a F_A)$

the proof of them is directly from the definitions and the properties.

(1.2)soft topology

Definition 2.1

Let χ be an initial universal set , and $A \subseteq E$ be a set of parameters, Let $\tilde{\tau}$ be a subfamily of a the family of all soft sets over χwe say that the family $\tilde{\tau}$ is a soft topology on χ if the following axioms are holds :

(1) $\widetilde{\Phi}_A$, $\tilde{\chi}_A \widetilde{\in} \tilde{\tau}$

(2) if F_A ,$G_A \widetilde{\in} \tilde{\tau}$, then then $F_A \widetilde{\cap} G_A \widetilde{\in} \tilde{\tau}$

(3) $G_{i_A} \widetilde{\in} \tilde{\tau}$, for any $i \in I$, then $\widetilde{\cup} \{G_{i_A},\ i \in I\} \widetilde{\in} \tilde{\tau}$.

The triple $(\tilde{\chi}_A, \tilde{\tau}, A)$ is called soft topological space or (soft space) .

The members of $\tilde{\tau}$,are called soft open sets

A soft set F_A is called soft closed set iff its complement is soft open

The family of all soft closed set is denoted by :

$$C(\tilde{\chi}) = \{\tilde{\chi}_A - F_A , F_A \widetilde{\in} \tilde{\tau}\}$$

Definition 2.2

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space , for each $a \in A$ and $x \in \chi$,a soft open set $F_A \widetilde{\in} \tilde{\tau}$, F_A is called $a -$ soft open neighborhood of x iff $x \in F(a)$.

Also we say that F_A is an $a -$ soft open set at the point x if $x \in F(a)$ and denoted by $F_{(a,x)}$.

We will denoted the (neighborhood) as simply by (nhd) .

Definition 2.3

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space , for each $a \in A$, a soft set F_A is called $a -$ soft nhd of a point $x \in \chi$ iff there exists an $a -$ soft open nhd $G_{(a,x)}$ such that $G_{(a,x)} \widetilde{\subseteq}_a F_A$.

Also we say that a soft closed set G_A is a s $a -$ soft closed nhd of $x \in \chi$ iff $x \in_a G_A$.

Remark 2.4

The set of all $a -$ soft nhd of a point $x \in \chi$ is called the $a -$ soft nhd system of x and denoted by $N_{\tilde{\tau}(x_a)}$.

Proposition2.5

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space , The $a -$soft nhd system

of a point x has the following properties :

(1) if $G_{(a,x)} \widetilde{\in} N_{\tilde{\tau}(x_a)}$ then $x_a \widetilde{\in} G_{(a,x)}$.

(2) if $G_{(a,x)} \widetilde{\in} N_{\tilde{\tau}(x_a)}$ and $G_{(a,x)} \widetilde{\subseteq}_a H_A$, then $H_A \widetilde{\in} N_{\tilde{\tau}(x_a)}$.

(3) if $G_{(a,x)}$ and $H_{(a,x)} \widetilde{\in} N_{\tilde{\tau}(x_a)}$, then $G_{(a,x)} \widetilde{\cap}_a H_{(a,x)} \widetilde{\in} N_{\tilde{\tau}(x_a)}$.

The proof of them is directly from definition [2.3 , 1.8,1.7].

Definition 2.6

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space , for each $a \in A$ and for any soft set F_A over the universe χ , then :

(1) The $a-$ soft closure of F_A is denoted by $a - Cl(F_A)$ defined as follow :

$a - Cl(F_A) = \tilde{\cap}_a \{S_A, S_A$ is soft closed set such that $F_A \tilde{\subseteq}_a S_A\}$.

(2) The $a-$ soft interior of F_A is denoted by $a - Int(F_A)$ defined as follow :

$a - Int(F_A) =_a \tilde{\cup}_a \{S_A, S_A$ is soft open set such that $S_A \tilde{\subseteq}_a F_A\}$.

Note that $a - Cl(F_A)$, need not be necessary soft closed set .

And $a - Int(F_A)$, need not be necessary soft open set .also Note that in general we have $a - Cl(F_A) \tilde{\subseteq} Cl(F_A)$, for each $a \in A$.

the following example shows this fact .

Example2.7

Let χ be a universal set such that $\chi = \{x_1, x_2, x_3\}$,and $A \subseteq E$, be a subset of parameters , such that : $A = \{a_1, a_2, a_3\}$,

Let $\tilde{\tau} = \{\tilde{\Phi}_A, \tilde{\chi}_A, F_{1_A}, F_{2_A}\}$, be a soft topological space ,where :

$F_{1_A} = \{(a_1, \{x_1, x_2\}), (a_2, \{x_2\}), (a_3, \{\varphi\})\}$, $F_{2_A} = \{(a_1, \{x_2\}), (a_2, \{\varphi\}), (a_3, \{\varphi\})\}$

If we consider a soft set $F_A = \{(a_1, \{x_1, x_2\})(a_2, \{x_1, x_2, x_3\}), (a_3, \{\varphi\})\}$, then :

For a point $a_1 \in A$, we get that :

$a_1 - Cl(F_A) = \tilde{\chi}_A -_a F_{1_A}$ and $a_1 - Int(F_A) = \{(a_1, \{x_1, x_2\})\}$

$a_2 - Cl(F_A) =_a \{(a_2, \{x_1, x_2, x_3\})\}$ and $a_2 - Int(F_A) =_a \{(a_1, \{x_2\})\}$

Now, since $\tilde{\chi}_A$ is the only soft closed set containing F_A , thus $Cl(F_A) = \tilde{\chi}_A$, and $Int(F_A) = F_{1_A}$

Note 2.8

From the definition (2.6 part -2-), we say that a point $x \in \chi$ is an $a-$ soft interior point of a soft set F_A iff there is a soft open set $G_{(a,x)}$ of x such that $G(a) \subseteq F(a)$.

Proposition2.9

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space , for each $a \in A$ and for any two soft sets F_A and G_A over the universe χ , then :

(1) $\tilde{\chi}_A -_a Cl(F_A) =_a a - Int(\tilde{\chi}_A -_a F_A)$

(2) $F_A \tilde{\subseteq}_a G_A$ then $(a - Cl(F_A)) \tilde{\subseteq}_a (a - Cl(G_A))$

(3) $F_A \tilde{\subseteq}_a G_A$ then $(a - Int(F_A)) \tilde{\subseteq}_a (a - Int(G_A))$

(4) $\tilde{\chi}_A -_a \mathrm{Int}(F_A) =_a a - \mathrm{Cl}(\tilde{\chi}_A -_a F_A)$

The proof is directly from the definitions 2.6 , 1.6 .

Corollary2.10

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space , for each $a \in A$ and for any soft sets F_A over the universe χ , then the following properties are holds :

(1) $a - \mathrm{Int}(F_A) =_a \tilde{\chi}_A - a - \mathrm{Cl}(\tilde{\chi}_A -_a F_A)$

(2) $a - \mathrm{Cl}(F_A) =_a \tilde{\chi}_A -_a \mathrm{Int}(\tilde{\chi}_A -_a F_A)$

The proof of them is directly from above theorem .

Remark2.11

All theorems and properties of a soft closure and soft interior of a soft set F_A in [1, 4, 5] are true on a point $a \in A$.

Theorem2.12

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space, for each $a \in A$, and $x \in \chi$ a soft point $x_a \tilde{\in} a - \mathrm{Cl}(F_A)$ iff

$$F_A \tilde{\cap}_a G_{(a,x)} \neq_a \tilde{\Phi}_a$$

for any soft open set $G_{(a,x)}$ of x .

Proof// it can proved easily by a contradiction .

Definition 2.13

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space ,let $a \in A$, for any soft sets F_A over the universe χ , F_A is called soft dense set iff $\mathrm{Cl}(F_A) = \tilde{\chi}_A$, and for some point $a \in A$ we called F_A is an $a -$ soft dense iff

$a - \mathrm{Cl}(F_A) =_a \tilde{\chi}_a$.

Note that if F_A is an $a -$soft dense , then it is not necessary be a soft dense set , the following example shows this fact :

Example 2.14

If we consider the following universal set :

$\chi = \{x_1, x_2, x_3, x_4\}$, and the set of parameters $A \subseteq E$ as :

$A = \{a_1, a_2, a_3\}$, then we have ,$\tilde{\tau} = \{\tilde{\Phi}_A, \tilde{\chi}_A, F_{1_A}, F_{2_A}\}$. be a soft topological space,where $F_{1_A} = \{ (a_1, \{x_1, x_2, x_3\}), (a_2, \{x_2, x_4\}), (a_3, \{\varphi\}), (a_4, \{\varphi\})\}$

$F_{2_A} = \{(a_1, \{x_1, x_2\}), (a_2, \{x_2, x_4\}), (a_3, \{\varphi\}), (a_4, \{\varphi\})\}$.

Suppose that $F_A = \{(a_1, \{x_1, x_2, x_3\}), (a_2, \{x_2, x_3, x_4\}), (a_3, \{\varphi\}), (a_4, \{\varphi\})\}$

Now for a point $a_2 \in A$ $a_2 - \mathrm{Cl}(F_A) =_{a_2} \tilde{\chi}_a$,

F_A is soft dense at a_2 , but it is not soft dense .

Definition 2.15

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space , for each $a \in A$ and for any soft sets F_A over the universe χ , F_A is called soft codense set iff the soft complement of F_A is soft dense set .

For some point $a \in A$ we called F_A is an a −soft codense set iff $\tilde{\chi}_A -_a F_A =_a \tilde{\chi}_a$.

Example 2.16

If we consider example [2.14] , for $a_1 \in A$, for the null soft set $\tilde{\Phi}_A$, since

$\tilde{\chi}_A -_{a_1} \tilde{\Phi}_A =_{a_1} \tilde{\chi}_a$, is an $a_1 -$ soft dense set , then by the above definition

$\tilde{\Phi}_A$ is a soft a_1 −softcodense set .

Definition 2.17

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space , a soft set F_A over the universe χ , is called soft nowhere dense if $\mathrm{Int}\big(\mathrm{Cl}(F_A)\big) = \tilde{\Phi}_A$, for some point $a \in A$ F_A is called a −soft nowhere dense if $(a - \mathrm{Int}(a - \mathrm{Cl}(F_A) =_a \tilde{\Phi}_a$.

Theorem 2.18

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space, let $a \in A$,and let F_A be a soft set over the universe χ , then , following statements are holds :

(1)$a - \mathrm{Int}(F_A)$ is ana −soft dense iff $\tilde{\chi}_A -_a F_A$ is ana −soft nowhere dense .
(2)$a - \mathrm{Int}(\tilde{\chi}_A -_a F_A)$ is an a −soft dense iff F_A is an a −soft nowhere dense .
(3)F_A is an a −soft nowhere dense iff $-\mathrm{Cl}(\tilde{\chi}_A -_a F_A)$ is an $a -$ soft dense .
(4)if F_A is an a −soft nowhere dense , then $\tilde{\chi}_A -_a F_A$ is an $a -$ soft dense .

proof//(1)

$a - \mathrm{Int}(F_A)$ is ana −soft dense iff $a - \mathrm{Cl}(F_A)\big(a - \mathrm{Int}(F_A)\big) =_a \tilde{\chi}_a$ [definition 2.17] iff $(\tilde{\chi}_A - a - \mathrm{Cl}(F_A)\big(a - \mathrm{Int}(F_A)\big) =_a \tilde{\Phi}_A$ iff $a - \mathrm{Int}(a - \mathrm{Cl}(\tilde{\chi}_A -_a F_A) =_a \tilde{\Phi}_A$ iff $\tilde{\chi}_A -_a F_A$ is an a −soft nowhere dense .

proof//(2)

$a - \mathrm{Int}(\tilde{\chi}_A -_a F_A)$ is an a −soft dense iff F_A is an a −soft nowhere dense (1) , and $a - \mathrm{Int}(\tilde{\chi}_A -_a F_A)$ is an a −soft soft dense iff $a - \mathrm{Cl}(F_A)\big(a - \mathrm{Int}(F_A)\big) =_a \tilde{\chi}_A$ iff $\tilde{\chi}_A -_a (a - \mathrm{Int}(a - \mathrm{Cl}(\tilde{\chi}_A -_a F_A)) =_a \tilde{\chi}_A$ iff $a - \mathrm{Int}(a - \mathrm{Cl}(F_A) =_a \tilde{\Phi}_A$

Iff F_A is an a −soft nowhere dense .

proof//(3)

F_A is an a −soft nowhere dense iff $\tilde{\chi}_A -_a F_A$ is an a −soft soft dense ,that is F_A is an a −soft nowhere dense , also

$a \; -\mathrm{Int}(a - \mathrm{Cl}(F_A) =_a \tilde{\Phi}_a$ iff $\tilde{\chi}_A -_a \Big(a - \mathrm{Int}\big(a - \mathrm{Cl}(F_A)\big)\Big) =_a \tilde{\chi}_a$ iff $a - \mathrm{Cl}(\tilde{\chi}_A -_a \big(a - \mathrm{Cl}(F_A)\big)) =_a \tilde{\chi}_a$iff $\tilde{\chi}_A -_a \big(a - \mathrm{Cl}(F_A)\big)$ is an asoft dense.

proof//(4)

F_A is an a −soft nowhere dense iff $\tilde{\chi}_A -_a F_A$ is an a −soft soft dense , (I . e $a - Int(a - Cl(F_A) =_a \tilde{\Phi}_a)$, then

$a - Cl(a - Int(F_A) =_a \tilde{\chi}_A$ and $a - Int(\tilde{\chi}_A -_a F_A) \tilde{\subseteq}_a \tilde{\chi}_A -_a F_A$,so $\tilde{\chi}_A =_a a - Int(a - Cl(\tilde{\chi}_A -_a F_A) =_a a - Cl(F_A)$.

Theorem 2.19

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space ,let $a \in A$,and let F_A be a soft set over the universe χ , F_A is ana −soft dense , then for any soft open set G_A , we have :

$$G_A \tilde{\subseteq}_a \left(a - Cl(F_A \tilde{\cap}_a G_A)\right)$$

Proof//

Assume G_A be a soft open set such that :

Case (1) \if $G_A =_a \tilde{\Phi}_A$, the result is done .

Case (2)\ if x_a is an a −soft point of G_A . I .e. $G_A \neq_a \tilde{\Phi}_A$.

And if possible that $x_a \tilde{\notin} \left(a - Cl(F_A \tilde{\cap}_a G_A)\right)$, then by [theorem2.12] there is a soft open set $H_{(a,x)}$ such that :

$(F_A \tilde{\cap}_a G_A) \tilde{\cap}_a H_{(a,x)} =_a \tilde{\Phi}_A$, but $G_A \tilde{\cap}_a H_{(a,x)} =_a K_{(a,x)}$ is a soft open set of x_a [2.5part 3] but F_A is am a −soft dense ; a contradiction , hence $x_a \tilde{\notin} \left(a - Cl(F_A \tilde{\cap}_a G_A)\right)$ and

$G_A \tilde{\subseteq}_a \left(a - Cl(F_A \tilde{\cap}_a G_A)\right)$.

Theorem 2.20

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space , for each $a \in A$,and for any soft set F_A over the universe χ , then the following statements are equivalent :

(1) F_A is ana −soft dense .
(2)For any soft open set G_A with $G(a) \neq \varphi$, $F_A \tilde{\cap}_a G_A \neq_a \tilde{\Phi}_a$
(3)$\tilde{\chi}_A -_a \left(a - Cl(F_A)\right)) =_a \tilde{\Phi}_a$
(4)$a - Int(\tilde{\chi}_A -_a F_A) =_a \tilde{\Phi}_a$
(5)$\tilde{\chi}_A -_a F_A$ has no non-empty soft open set

Proof//
$1 \Longrightarrow 2$: suppose that F_A is an a −soft dense set , and let G_A be a soft open set with $G(a) \neq \varphi$, For any a −soft point $x_a \tilde{\in} G_A$, then $x_a \tilde{\in} \tilde{\chi}_A$, but F_A is an a-soft dense set , $a - Cl(F_A) =_a \tilde{\chi}_A$, implies that
$x_a \tilde{\in}\ a - Cl(F_A)$, which means that $F_A \tilde{\cap}_a G_A \neq_a \tilde{\Phi}_a$ [2.12]
$2 \Longrightarrow 1$: let x_a be an a −soft point of $\tilde{\chi}_A$, and G_A be a soft open set containing x_a ,by [2] we get that $F_A \tilde{\cap}_a G_A \neq_a \tilde{\Phi}_a$ and by [theorem 28]
$x_a \tilde{\in}\ a - Cl(F_A)$, imply that $\tilde{\chi}_A \tilde{\subseteq}_a\ a - Cl(F_A)$, thus

$\tilde{\chi}_A =_a a - Cl(F_A)$.

$1 \Longrightarrow 3$:since F_A is ana $-$soft dense set , $\tilde{\chi}_A =_a a - Cl(F_A)$, so

$\tilde{\chi}_A -_a \left(a - Cl(F_A)\right) =_a \tilde{\Phi}_a$.

$3 \Leftrightarrow 4$: since $\tilde{\chi}_A -_a \left(a - Cl(F_A)\right) =_a \tilde{\Phi}_a$ iff

$a - Int(\tilde{\chi}_A -_a F_A) =_a \tilde{\Phi}_a$ [corollary 2.10].

$4 \Longrightarrow 5$: if possible that $\tilde{\chi}_A -_a F_A$ has an $a -$softempty open set G_A with $G(a) \neq \varphi$, that is $G_A \tilde{\subseteq}_a \tilde{\chi}_A -_a F_A$, implies that $G_A \tilde{\subseteq}_a (a - Int(\tilde{\chi}_A -_a F_A) \neq_a \tilde{\Phi}_a$, which a contradiction with[4] , there fore $\tilde{\chi}_A -_a F_A$ has no non a $-$empty soft open set .

$5 \Longrightarrow 4$: since $a - Int(\tilde{\chi}_A -_a F_A) =_a \tilde{\cup}_a \{G_A \tilde{\in} \tilde{\tau}, G_A \tilde{\subseteq}_a \tilde{\chi}_A -_a F_A\}$ but by [5] $\tilde{\chi}_A -_a F_A$ has no non a $-$empty soft open set . such that $\tilde{\Phi}_A$ is the only soft open set containing in $\tilde{\chi}_A -_a F_A$, hence $a - Int(\tilde{\chi}_A -_a F_A) =_a \tilde{\Phi}_a$.

$4 \Longrightarrow 1$: by [4] we have $a - Int(\tilde{\chi}_A -_a F_A) =_a \tilde{\Phi}_a$ so by[corollary25 part 1] we get that $\tilde{\chi}_A -_a\left(a - Cl(F_A)\right) =_a \tilde{\Phi}_a$, so $a - Cl(F_A) =_a \tilde{\chi}_a$, hence F_A is an $a -$soft dense .

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