A Special kind of Non AssociativeSeminear-Ring With BCK Algebra

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Abstract

In this paper, we discuss an algebraic system introduced in 2015 by Mohammed and Abdul Wahab called it a Special kind of non associative seminear-ring with BCK Algebra (*SNAK-seminear ring*) where we prove some properties and give some examples . We define three types of it we, call the first is an ideal of type one, the second is an ideal of type two and the third is an ideal of type three. We prove some of properties and give some example .

Keywords: BCK-algebra, BCI-algebras,Semigroup , Seminear-Ring ,non associative seminear-ring

الخلاصة

ناقشنا في هذا البحث النظام الجبري الذي يدعى نوع خاص من شبه الحلقة القريبة غير التجميعية مع جبر BCK (شبه الحلقة القريبة – *SNAK*) حيث برهنا عدد من القضايا وأعطينا عدد من الأمثلة .ثم عرفنا ثلاث أنواع من المثاليات وهم مثالية من النوع الأول و مثالية من النوع الثاني ومثالية من النوع الثالث حيث برهنا عدد من القضايا وأعطينا عدد من الأمثلة .

الكلمات المفتاحية: جبر –BCI, جبر –BCK , شبه الزمرة, شبه الحلقة القريبة, شبه الحلقة القريبة غير التجميعية.

1. Introduction

The notion of BCK- algebras was introduced first in 1966 by Imai and Iseki, [Imai and Iseki, 1966]. In the same year, Iseki [Iseki, 1966] introduced two classes of abstract algebras: BCK-algebras and BCI-algebras where the class of BCK algebras is a proper subclass of the class of BCI-algebras." In 1967 ,Van Hoorn and Van Root Selaar introduced the concept of seminear-rings and discussed a general theory of seminear-rings", [Hollings,2009]. Seminear-ring,or near- semiring in another term . Is generalization of near-ring and semiring , and distributive seminear-ring a common are semirings. In 2015, S.K.Mohammed and Abdul Wahab introduced the notion of a special kind of non associative seminear-ring with BCK algebra (SNAK-seminear ring). The main goal of this work is to study properties of a special kind of non associative seminear-ring with BCK algebra where a non empty set $(X, \bullet, *, 0)$ with two binary operations '$*$' and '$\bullet$' with constant 0 is called a Special kind of non associative seminear-ring with BCK algebra if satisfying the following conditions:

1) $(X , \bullet)$ is a semigroup
2) $(X , * , 0)$ is a BCK algebra
3) $(x \bullet y) * z = (x \bullet z) * (y \bullet z) \quad \forall\ x , y , z \in X$
4) $0 \bullet x = x \bullet 0 = x \quad \forall\ x \in X$

and introduce three types of ideals of it . We call that the first is the ideal of type one, the second is the ideal of type two and the third is the ideal of type three where we study some properties and give some examples.

1.Basic Concepts and Notations

This section contains some basic ordinary concepts about semigroups, seminear-ring, a non-associative seminear-ring,BCK-algebra with some examples and some propositions.

Definition1.1 **[Petrich, 1973]**

Let Sbe a non-empty set. $(S , \bullet)$ is said to be a ***semigroup*** if on S is defined a binary operation '$\bullet$' such that for all $a, b \in S, a \bullet b \in S$ and
$(a \bullet b) \bullet c = a \bullet (b \bullet c)$ *for all* $a, b, c \in S$.

Definition 1.2 **[Harju1996]**

The **direct product** $S \times T$ of two semigroups $(S, \bullet)$ and $(T, \bullet)$ is defined by
$(x_1, y_1) \bullet (x_2, y_2) = (x_1 \bullet x_2 , y_1 \bullet y_2)$ *where* $x_1,x_2 \in S, y_1,y_2 \in T$.
It is easy to show that the direct product is a semigroup.

Definition 1.3 **[Harju1996]**

Let $(S, \bullet)$ *and* $(P, *)$ be two semigroups. A mapping f: S → P is a ***homomorphism*** if
$\forall$ x, y $\in$ S : $f(x \bullet y) = f(x) * f(y)$.

Definition 1.4 **[Vasantha ,2002]**

A semigroup $(S, \bullet)$ is said to be ***commutative*** if $a \bullet b = b \bullet a$ for all $a, b \in S$.

Definition 1.5 **[Vasantha ,2002]**

Let $(S , \bullet)$ be a semigroup. P a non-empty proper subset of S is said
to be a ***subsemigroup*** if $(P,\bullet)$ is a semigroup .

Definition 1. 6 **[Petrich , ,1973]**

Let $(X, \bullet)$ be a semigroup and x an element of X . An element e of X is a ***left identity*** of x if $e \bullet x = x$, a ***right identity*** of x if $x \bullet e = x$, an ***identity*** of x *if*
$x \bullet e = e \bullet x = x$.

Definition 1.7 **[Mordeson,2003]**

A semigroup $(S, \bullet)$ is called ***normal*** if $a \bullet S = S \bullet a$ for all $a \in S$.

Definition 1.8 **[Zulfiqar,2009]**

A non empty set R with two binary operations + (addition) and $\bullet$ (multiplication) is called a ***seminear-ring*** , if it satisfies the following axioms:
(1) $(R, +)$ *and* $(R, \bullet)$ are semigroups,
(2) $(x + y) \bullet z = x \bullet z + y \bullet z$ *for all* $x, y, z \in R$.
Precisely speaking, it is a right seminear-ring because it satisfies the right distributive law.

Definition 1.9 **[Vasantha ,2002]**

Let $(N, + , \bullet)$ be a non-empty set with two binary operation '+' *and* '$\bullet$' satisfying the following conditions :
a. $(N, +)$ is a semigroup.
b. $(N, \bullet)$ is a groupoid.
c. $(a + b) \bullet c = a \bullet c + b \bullet c$ *for all* $a, b, c \in N$; $(N, +, \bullet)$ is called the ***right seminear-ring which is non-associative.***

If we replace (c) by $a \bullet (b + c) = a \bullet b + a \bullet c$ *for all* $a, b, c \in N$; then
$(N, +, \bullet)$ is a non-associative left seminear-ring .
In this text we denote by $(X, +, \bullet)$ a non-associative right seminear-ring and by default of notation call X just a non-associative seminear-ring.

Definition 1.10 **[Vasantha ,2002]**

Let $(N, +, \bullet)$ be a seminear-ring which is not associative. A subset P of N is said to be a ***subseminear-ring*** if $(P, +, \bullet)$ is a seminear-ring.

Definition 1.11 **[Vasantha ,2002]**

A mapping g between two seminear- rings is called ***seminear ring homomorphism*** if g is a homomorphism.

Definition 1.12 **[Vasantha ,2002]**

Let $(N, +, \bullet)$ be a non associative seminear-ring . Then a non-empty subset I of N is called ***left ideal*** in N if

1) $(I, +)$ is a normal subsemigroup of $(N, +)$.
2) $n \bullet (n_1 + i) + n \bullet n_1 \in I$ *for each* $i \in I, n, n_1 \in N$.

Definition 1.13 **[Vasantha ,2002]**

Let $(N, +, \bullet)$ be a non-associative seminear-ring. A nonempty subset I of N is called an ***ideal*** in N if

1) I is a left ideal.
2) $I \bullet N \subseteq I$.

Definition 1.14 **[Samaei & Azadani, 2011]**

Let X be a non-empty set with binary operation, *, and 0 is a constant. An algebraic system $(X, *, 0)$ is called a ***BCK algebra*** if it satisfies the following conditions:

1) $((x * y) * (x * z)) * (z * y) = 0$,
2) $(x * (x * y)) * y = 0$,
3) $x * x = 0$,
4) *if* $x * y = 0$ *and* $y * x = 0$ *then* $x = y, \ \forall x, y, z \in X$
5) $0 * x = 0$.

Remarks 1.15 **[Nisari,2009]**

Let X be a BCK algebra then :

a) A partial ordering " $\leq$ " on X can be defined by $x \leq y$ if and only if $x * y = 0$.

b) A BCK-algebra X has the following properties:

1) $x * 0 = x$.
2) $(x*y)*z=(x*z)*y$.

Definition 1.16 **[Iseki,1974]**

Let $(X,*, 0)$ *and* $(X', *', 0')$ be two BCK-algebras. A mapping $f : X \to Y$ is called a homomorphism from X to X' if for any $x, y \in X$, $f(x * y) = f(x) *' f(y)$.

Definition 1.17 **[Jun, 2012]**

A BCK-algebra is said to be ***commutative*** if $x * (x * y) = y * (y * x)$ *for any* $x, y \in X$.

Definition 1.18 **[Kuroki, 1977]**

A subsemigroup A of a semigroup S is called normal if $x \bullet A = A \bullet x$ for all elements x of S.

2 . Some Properties of A Special kind of Non Associative Seminear-Ring With BCK Algebra.

In this section , we discuss a special kind of non associative seminear-ring with BCK algebra and study some of properties.

Definition 2.1.1 **[Mohammed and Abdul Wahab,2015]**

Let $(X, \bullet, *, 0)$ be a non-empty set with two binary operations '$*$' and '$\bullet$' and 0 is constant satisfying the following conditions :

a) $(X, \bullet)$ is a semigroup .
b) $(X, *, 0)$ is a BCK algebra.
c) $(x \bullet y) * z = (x * z) \bullet (y * z)$, *for all* $x, y, z \in X$ which is called the distributive law
e) $0 \bullet x = x \bullet 0 = x$, for all $x \in X$

Then; $(X, \bullet, *, 0)$ is called ***A Special kind of Non Associative Seminear-Ring With BCK Algebra***, we refer to by ***SNAK-Seminear Ring.***

Example 2.1.2

Let $X=\{0,1,2,3\}$with two binary operations $'*'$ and $'\bullet'$ are defined by the following tables :

$\bullet$	**0**	**1**	**2**	**3**
0	0	1	2	3
1	1	1	2	3
2	2	2	2	2
3	3	3	2	2

$*$	**0**	**1**	**2**	**3**
0	0	0	0	0
1	1	0	0	0
2	2	2	0	0
3	3	3	0	0

Then usual calculation we have $(X ,\bullet, *,0)$ is SNAK-seminear ring

Proposition 2.1.3 **[Mohammed and Abdul Wahab,2015]**

Let S , T be a SNAK-seminear ring . Then $S\times T = \{(s , t) : s\in S , t\in T\}$ is a SNAK-seminear ring, where the binary operations '$\bullet$' and '*' are defined by the following :

$(a_1 , b_1) \bullet (a_2 , b_2) = (a_1\bullet a_2 , b_1\bullet b_2)$

$(a_1 , b_1) * (a_2 , b_2) = (a_1 * a_2 , b_1 * b_2)$, *for all* $(a_1,b_1) , (a_2,b_2) \in S\times T$

Definition 2.1.4 **[Mohammed and AbdulWahab,2015]**

Let $(X , \bullet, * ,0)$ is a SNAK-seminear ring a non empty subset P of X is said to be ***a Special Kind of Non Associative Sub Seminear-Ring With BCK Algebra*** if $(P, \bullet, *,0)$ is a SNAK-seminear ring , we denoted by sub SNAK-seminear ring. Note that every sub SNAK-seminear ring is SNAK-seminear ring .

Remark 2.1.5 **[Mohammed and AbdulWahab,2015]**

If B is a sub SNAK-seminear ring of a SNAK-seminear ring X then it is clear that $0\in B$ since B is a BCK algebra.

Remark 2.1.6 **[Mohammed and AbdulWahab,2015]**

Let $(X_1, \bullet , * , 0)$, $(X_2 , \bullet , * , 0)$ be a Sub SNAK-seminear ring of X. Then the following are Sub SNAK-seminear ring.

1) $(X_1\cap X_2 , \bullet , * , 0)$

2) $(X_1\cup X_2 , \bullet , * , 0)$ *such that* $X_1\subseteq X_2$ *or* $X_2\subseteq X_1$

Remark 2.1.7

The converse of above remark is not true in general

Proof

To show that the converse of (1) in remark above is not true in general , Let $X=\{0,1,2,3\}$with two binary operations $'\bullet'$ and $'*'$ be defined by the following tables :

$\bullet$	**0**	**1**	**2**	**3**
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

$*$	**0**	**1**	**2**	**3**
0	0	0	0	0
1	1	0	0	0
2	2	1	0	0
3	3	1	0	0

Then $(X , \bullet , *, 0)$ is a SNAK-seminear ring.Let $X_1 = \{0 , 1,3\}$ *and* $X_2 = \{0 , 1 , 2\}\subseteq X$ .Then $X_1\cap X_2 = \{0 , 1\}$ is a SNAK-seminear ring.

But X_1 and X_2 is not SNAK-seminear ring since X_1 and X_2 is not semigroup.Now to show that the converse of (2) in above remark is not true since if we take $(X,\bullet , *, 0)$ as in the following example : Let $X=\{0,1,2\}$with two binary operations $'\bullet'$ and $'*'$ be defined by the following tables :

•	0	1	2
0	0	1	2
1	1	1	1
2	2	1	2

*	0	1	2
0	0	0	0
1	1	0	2
2	2	0	0

$X_1=\{0, 1\}$ and $X_2=\{0, 2\}$then it is clear that X_1 and X_2 Sub SNAK-seminear ring . and $X_1 \cup X_2=\{0,1,2\}$ is a Sub SNAK-seminear ring ,but $X_1 \not\subseteq X_2$ and $X_2 \not\subseteq X_1$.

Definition 2.1.8

If (X , .) is abelian semigroup in a SNAK seminear-ring we say that (X , • , ∗ , 0) is ***Abelian Special Kind Of Non Associative Seminear-Ring With BCK Algebra*** (abelian SNAK Seminear-Ring)

Definition 2.1.9

If (X , ∗) is a commutative BCK algebra in a SNAK seminear-ring we say that (X , • , ∗ , 0) is ***Commutative Special Kind Of Non Associative Seminear-Ring With BCK Algebra*** (Commutative SNAK Seminear-Ring)

Proposition2.1.10

1- If X_1 , X_2 is commutative SNAK-seminear ring then $(X_1 \times X_2 , \bullet , * , 0)$ is commutative SNAK-seminear ring .
2- If X_1 , X_2 is a commutative Sub SNAK-seminear ring , then
3- 1) $(X_1 \cap X_2 , \bullet , * , 0)$
4- 2) $(X_1 \cup X_2 , \bullet , * , 0)$ s.t. $X_1 \subseteq X_2$ or $X_2 \subseteq X_1$ are commutative Sub SNAK-seminear ring) .

Proof: clear

Proposition 2.1.11

Let X be a SNAK-seminear ring. If X_1 and X_2 are Sub SNAK-seminear ring of X such that $x * y = x \ \forall x \in X_1 , y \in X_2$. Then $X_1 \cap X_2 = \{ 0 \}$.

Proof

Let X_1 and X_2 are Sub SNAK-seminear ring of X ,since $0 \in X_1$ and $0 \in X_1$ so $X_1 \cap X_2 \neq \emptyset$. Now, suppose $X_1 \cap X_2 \neq \{ 0 \}$
$\Rightarrow \exists a \in X_1 \cap X_2$ and $a \neq 0 \Rightarrow a * a = a$ by hypotheses
but $a * a = 0$ so $a = 0$ contradiction $\Rightarrow X_1 \cap X_2 = \{ 0\}$

2.2 On Homomorphism of SNAK-seminear ring

In this section we study homomorphism on a SNAK-seminear ring and prove some results .

Definition 2.2.1 :

Let X and X' be SNAK-seminear ring and $f : X \rightarrow X'$ is mapping, then:
f is called a ***homomorphism*** if $f(x \bullet y) = f(x) \bullet f(y)$ and $f(x * y) = f(x) * f(y)$ *for all* $x, y \in X$. f is called a ***monomorphism*** if f is a one-to-one homomorphism. f is called an ***epimorphism*** if f is an onto homomorphism. f is called an ***isomorphism*** if f is a bijective homomorphism. The set $ker\, f = \{ x \in X : f(x) = 0\}$ is called the ***kernel of f***

Lemma 2.2.2

Let $f : X \rightarrow X'$ be a SNAK-seminear ring homomorphism . Then
(1) $f(0) = 0$,
(2) if $x \leq y$ then $f(x) \leq f(y)$.
(3) if $x \wedge y = x * (x * y)$ then $f(x \wedge y) = f(x) \wedge f(y)$.

Proof

Let $f : X \to X'$ be a SNAK-seminear ring

(1) Let $x \in X$ Then $f(0) = f(x * x) = f(x) * f(x) = 0$

(2) Let $x \le y$. Then we have $x * y = 0$ [by remarks 1.15]

Thus [by 1]we have $f(x * y) = f(0) = 0 \Rightarrow f(x) * f(y) = 0 \Rightarrow f(x) \le f(y)$

(3) Let $x, y \in X$ and $x \wedge y = x * (x * y)$

$\Rightarrow f(x \wedge y) = f(x * (x * y)) = f(x) * (f(x) * f(y)) = f(x) \wedge f(y)$ [since f is a SNAK-seminear ring homomorphism]

Proposition 2.2.3

Let X, Y, Z be SNAK-seminear ring and let $f : X \to Y$ and $g : Y \to Z$ are SNAK-seminear ring homomorphisms. Then $g \circ f : X \to Z$ is also a SNAK-seminear ring homomorphism.

Proof:clear

Proposition 2.2.4

Let $f : X \to Y$ is a SNAK-seminear ring homomorphism and $A \subseteq X$ be a Sub SNAK-seminear ring . Then $f(A)$ also

Proof:Let X be a SNAK-seminear ring Then $(X, \bullet)$ is a semigroup since $A \subseteq X$. Then $f(A) \subseteq Y$ so it is clear that $(f(A), \bullet)$ is a semigroup.

Now, to prove that $(f(A), *, 0)$ is a BCK algebra since $0 \in A$ by [2.1. 5] so $0 = f(0) \in f(A)$ .Now , let $x', y', z' \in f(A) \Rightarrow \exists x, y, z \in A \subseteq X$ s.t $f(x) = x'$, $f(y) = y'$, $f(z) = z'$ then

$$((x' * y') * (x' * z')) * (z' * y') = ((f(x) * f(y)) * (f(x) * f(z))) * (f(z) * f(y))$$
$$= (f(x * y) * f(x * z)) * f(z * y) = f(((x * y) * (x * z)) * (z * y))$$
$$= f(0) = 0 \text{ [Since } x, y, z \in A \subseteq X \text{ and } X \text{ is a SNAK-seminear ring]}$$

.And $(f(x) * (f(x) * f(y)) * f(y) = f((x * (x * y)) * y) = f(0) = 0$

Also, $f(x) * f(x) = f(x * x) = f(0) = 0$ [by 3 of definition 1.14]

$f(x) * f(0) = f(x * 0) = f(x)$ [by 1 of remark 1.15]

and $f(0) * f(x) = f(0 * x) = f(0)$

$\Rightarrow (f(A), *, 0))$ is a BCK algebra .Now to prove the distributive law

Let $x', y', z' \in f(A) \Rightarrow \exists x, y, z \in A$ s.t $f(x) = x'$, $f(y) = y'$, $f(z) = z'$

$$(x' \bullet y') * z' = [f(x) \bullet f(y)] * f(z) = f(x \bullet y) * f(z) = f((x \bullet y) * z)$$
$$= f(x * z \bullet y * z) \text{ [by c of definition 2.1.1]}$$
$$= f(x * z) \bullet f(y * z) = f(x) * f(z) \bullet f(y) * f(z).$$

Let $x' \in f(A) \Rightarrow \exists x \in A$ s.t $f(x) = x'$ then

$f(0) \bullet f(x) = f(0 \bullet x) = f(x)$ and $f(x) \bullet f(0) = f(x \bullet 0) = f(x)$ then $(f(A), \bullet, *, 0)$ is Sub SNAK-seminear ring..

Proposition 2.2.5

Let $f : X \to X'$ be a SNAK-seminear ring homomorphism. If X is a commutative , then $f(X)$ is also .

Proof: *Let* $f : X \to X'$ be SNAK-seminear ring homomorphism.So by 2.2.4 $f(X)$ is a sub SNAK-seminear ring since $X \subseteq X$

Now, *let* $f(x), f(y), f(z) \in f(X)$ *for some* $x, y, z \in X$.Suppose that X is a commutative .Then $f(x) * (f(x) * f(y)) = f(x) * f(x * y) = f(x * (x * y)) = f(y * (y * x))$

$= f(y) * f(y * x) = f(y) * (f(y) * f(x))$ then $f(X)$ is a commutative sub SNAK-seminear ring

Proposition 2.2.6

Let $f : X \to X'$ be SNAK-seminear ring monomorphism . If $f(X)$ is a commutative .Then X is also.

Proof: Let $f : X \to X'$ be SNAK-seminear ring monomorphism and let $x, y, z \in X$. Then $f(x), f(y), f(z) \in f(X)$. Suppose that $f(X)$ is a commutative .

so $f(x) * (f(x) * f(y)) = f(y) * (f(y) * f(x))$.

$f(x * (x * y)) = f(y * (y * x)$

since f is a monomorphism so $x * (x * y) = y * (y * x)$.Then, X is a commutative SNAK-seminear ring

Proposition 2.2.7

Let $f : X \to Y$ be SNAK-seminear ring epimorphism such that X is a commutative . Then Y is also

Proof:clear

Proposition 2.2.8

Let $f : X \to X'$ be a SNAK-seminear ring. homomorphism. Then $kerf$ is a Sub SNAK-seminear ring of X.

Proof:Since $f(0)= 0$ so $0 \in kerf$ so $kerf \neq \emptyset$ then let $x,y \in kerf$ so $f(x)= 0$ and $f(y)=0$ $\Rightarrow f(x \bullet y)=0 \Rightarrow x \bullet y \in kerf$

and if $x, y,z \in kerf \subseteq X$ we have $(x \bullet y) \bullet z= x \bullet (y \bullet z)$ since X is a SNAK-seminear ring . so $(kerf , \bullet)$ is a semigroup.Now, since $kerf \subseteq X$ so, it is easy to prove that all conditions of BCK algebra satisfy so $(kerf, *, 0)$ is a BCK algebra**.**

If $x, y, z \in kerf \subseteq X$ then it is easy to prove that $(x \bullet y)*z = x*z \bullet y*z$ since X is a SNAK-seminear ring.Let $x \in kerf \subseteq X$ *then* $x \bullet 0 = 0 \bullet x = x$ *so* $(kerf, \bullet , *, 0)$ *is a* Sub SNAK-seminear ring.

3. Ideal of Type One on SNAK-seminear ring

In this section ,we introduce the notions of ideal of type one in SNAK-seminear ring and discuss some problems and give some examples.

Definition 3.1.1

Let X be a SNAK-seminear ring. A non empty subset I of X is called an ***ideal of type one*** in X if satisfies the following conditions :

1) $(I , \bullet)$ is a normal subsemigroup of $(X , \bullet)$

2) $(n * (n_1 \bullet i)) \bullet (n * n_1) \in I$ for each $i \in I , n , n_1 \in X$.

3) $I * X \subseteq I$

Example 3.1.2:

Let $X=\{0,1,2,3\}$ be SNAK-seminear ring with two binary operations '$\bullet$' *and* '$*$' be defined by the following tables :

•	0	1	2	3
0	0	1	2	3
1	1	1	2	2
2	2	2	1	1
3	3	2	1	0

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	3	0	0
3	3	3	0	0

Then by usual calculation, we can prove that $I = \{ 0 , 1\}$ is an ideal of type one .

Example 3.1. 3:

Let $X=\{0,1,2,3\}$with two binary operations '$\bullet$' *and* '$*$' be defined by the following tables :

$\bullet$	0	1	2	3
0	0	1	2	3
1	1	1	1	1
2	2	1	1	2
3	3	1	2	0

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	0	0	1
3	3	0	0	0

Then $I = \{0, 1\}$ is not ideal of type one since $0, 2 \in X$ and $1 \in I$ but $(2 * (0 \bullet 1)) \bullet (2 * 0) = 0 \bullet 2 = 2 \notin I$.

Remark 3.1.4

Let X be a SNAK-seminear ring and $I \subseteq X$ be an ideal of type one, then $0 \in I$.

proof: *clear*

Proposition 3.1.5

Let X be a SNAK-seminear ring. If $I \subseteq X$ be an ideal of type one of SNAK-seminear ring, then $(I, \bullet, *, 0)$ is a Sub SNAK-seminear ring.

proof: Let X be a SNAK-seminear ring and $I \subseteq X$ be an ideal of type one, Since $(I, \bullet)$ is a normal sub semigroup so $(I, \bullet)$ is a semigroup and since $0 \in I$ [by remark 3.1.4] so if $x, y, z \in I \subseteq X$, so it is easy to prove that all conditions of BCK algebra satisfy so $(I, *, 0)$ is a BCK algebra. Now , if $x, y, z \in I \subseteq X$,then $(x \bullet y) * z = (x * z) \bullet (y * z)$ since X is a SNAK-seminear ring let $x \in I \subseteq X$ then it is clear that $0 \bullet x = x = x \bullet 0$, therefore $(I, \bullet, *, 0)$ is a Sub SNAK-seminear ring.

Remark 3.1.6

The converse of proposition (3.1.5) in general is not true where we explain this by the following example : Let $X=\{0,1,2, 3\}$with two binary operations '$\bullet$' and '$*$' are defined by the following tables :

$\bullet$	0	1	2	3
0	0	1	2	3
1	1	1	1	1
2	2	2	2	2
3	3	2	2	3

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	3
2	2	0	0	3
3	3	0	0	0

It is clear that $I = \{0, 2\}$ is a sub SNAK-seminear ring but I is not ideal of type one since $2*3 = 3 \notin I$.

Proposition 3.1.7

Let $f : X \rightarrow Y$ be a SNAK-seminear ring homomorphism such that X is an abelian and $x^2 = 0 \ \forall x \in X$, then $ker\, f$ is an ideal of type one .

Proof: Let X be an abelian SNAK-seminear ring such that $x^2 = 0 \ \forall x \in X$

1) Let $a, b \in ker\, f$. Then $f(a) = f(b) = 0$

So, $f(a \bullet b) = f(a) \bullet f(b) \Rightarrow f(a \bullet b) = 0 \bullet 0 = 0 \Rightarrow a \bullet b \in ker\, f$. Now,

let $x, y, z \in ker\, f \subseteq X$ then $(x \bullet y) \bullet z = x \bullet (y \bullet z)$ .So, $(ker\, f, \bullet)$ is a semigroup

Since X is an abelian so $(ker\, f, \bullet)$ is a normal subsemigroup

2) Let $n, n_1 \in X$ and $i \in ker\, f$ so $f(i) = 0$

$\Rightarrow f(((n * (n_1 \bullet i)) \bullet (n * n_1)) = ((f(n) * (f(n_1) \bullet f(i))) \bullet (f(n) * f(n_1))$

$= (f(n) * f(n_1)) \bullet (f(n) * f(n_1)) = f((n * n_1) \bullet (n * n_1))$

$= f(n^2 * n_1)$ [by c of definition 2.1.1]

$= f(0 * n_1) = f(0)$ [by hypothesis]

$= 0$ [by 1 of Lemma 2.2.2]

3) let $i \in ker\, f$ and $n \in X$,$f(i * n) = f(i) * f(n) = 0 * f(n) = 0$
$\Rightarrow i * n \in ker\, f \Rightarrow ker\, f * X \subseteq ker\, f$ .Then $ker\, f$ is an ideal of type one

Proposition 3.1.8

Let X be an abelian SNAK-seminear ring and let A , B be ideals of type one of X . Then $A \cap B$ is an ideal of type one of X .

Proof clear

Proposition 3.1.9

Let X be a SNAK-seminear ring and let A , B be an ideal of type one of X. Then $A \cup B$ is an ideal of type one of X if $A \subseteq B$ or $B \subseteq A$ and the converse is not true in general

Proof: proof is clear. Now ,we show that the converse is not true in general since if we take A,B and $A \cup B$ are ideals of type one of X then A,B and $A \cup B$ are Sub SNAK-seminear ring of X [by proposition 3.1.5] so in general $A \not\subset B$ and $B \not\subset A$ by [2.1.7].

Proposition 3.1.10

Let $f : X \rightarrow Y$ be a SNAK-seminear ring epimorphism if A is an ideal of type one of X . Then $f(A)$ is an ideal of type one of X.

Proof:Let X is a SNAK-seminear ring then

1) a. Since A is an ideal of type one . Then $(A , \bullet)$ is a subsemigroup implies that A is a semigroup

b. Let $x' , y' \in f(A)$ since f is an epimorphism $\Rightarrow \exists\, x , y \in A \ni f(x) = x' , f(y) = y'$
$\Rightarrow x \bullet y \in A$ then $f(x) \bullet f(y) = f(x \bullet y) \in f(A) \Rightarrow x' \bullet y' \in f(A)$
Let $x' , y' , z' \in f(A) \subseteq Y \Rightarrow (x' \bullet y') \bullet z' = x' \bullet (y' \bullet z')$ [since Y is a SNAK-seminear ring] hence $(f(A) , \bullet)$ is a semigroup. Now, let $a' \in Y$ then $\exists\, a \in X \ni f(a) = a' \Rightarrow a' \bullet f(A) = f(a) \bullet f(A) = f(a \bullet A) = f(A \bullet a)$ [Since A is a normal]
$= f(A) \bullet f(a) = f(A) \bullet a'$ so $((f(A) , \bullet)$ is a normal subsemigroup

2) To prove that $((n * (n_1 \bullet r_1)) \bullet (n * n_1) \in f(A)$ for each $r_1 \in f(A)$, $n , n_1 \in Y$. Since f is an epimorphism $\Rightarrow \exists\, x , x_1 \in X$ and $r \in A \ni f(x) = n , f(x_1) = n_1 , f(r) = r_1$ since A is an ideal of type one then $((x * (x_1 \bullet r)) \bullet (x * x_1) \in A$
$\Rightarrow f((x * (x_1 \bullet r)) \bullet (x * x_1)) \in f(A)$
$\Rightarrow ((f(x) * (f(x_1) \bullet f(r))) \bullet (f(x) * f(x_1)) \in f(A)$
$\Rightarrow ((n * (n_1 \bullet r_1)) \bullet (n * n_1)) \in f(A)$

3) To prove that $f(A) * Y \subseteq f(A)$ .Let $a' \in f(A)$ and $m' \in Y$ so $a' * m' \in f(A) * Y$ where $m' \in Y \Rightarrow \exists\, x \in A$ and $m \in X \ni f(x) = a'$, $f(m) = m'$
$\Rightarrow a' * m' = f(x) * f(m)$
$= f(x * m) \in f(A)$ [since $x * m \in A$ because that A is an ideal of type one]
$\Rightarrow a' * m' \in f(A)$ so $f(A) * Y \subseteq f(A)$
Therefore, f (A) is an ideal of type one of X .

3.2 Ideal of Type Two of SNAK-seminear ring

In this section, we introduce the notion of ideal of type two on a SNAK-seminear ring and study its properties and give some examples .

Definition 3.2.1

Let X be a SNAK-seminear ring A non empty subset I of X is said ***ideal of type two*** on X if satisfies the following conditions :

1) if $a \bullet b \in I$ or $b \bullet a \in I$ and $a \in I$ then $b \in I \, \forall\, a , b \in X$

2) $I * X \subseteq I$

Example 3.2.2:

Let $X=\{0,1,2\}$ with two binary operations '$\bullet$' and '$*$' be defined by the following tables

•	0	1	2
0	0	1	2
1	1	1	1
2	2	1	0

*	0	1	2
0	0	0	0
1	1	0	1
2	2	0	0

Then by usual calculation we can prove that $I = \{0 , 2\} \subseteq X$ is an ideal of type two .

Example 3.2.3:

Let $X=\{0,1,2,3\}$with two binary operations '$\bullet$' and '$*$' be defined by the following tables :

•	0	1	2	3
0	0	1	2	3
1	1	1	1	1
2	2	1	1	1
3	3	1	2	3

*	0	1	2	3
0	0	0	0	0
1	1	0	0	3
2	2	0	0	3
3	3	0	0	0

Then $I = \{0 , 2\} \subseteq X$ is not ideal of type two , since $3 \bullet 2 = 2 \in I$ *and* $2 \in I$ *but* $3 \notin I$, also $2 * 3 = 3 \notin I$ *so* $I * x \not\subset I \forall a \in I, x \in X$. Then I is not an ideal of type two.

Remark 3.2.4

Lel I be an ideal of type two of SNAK-seminear ring then it is clear that $,0 \in I$

Proposition 3.2.5

Let X be a SNAK-seminear ring and let A , B be ideals of type two of X . Then $A \cap B$ is an ideal of type two of X .

Proof:Let X be a SNAK-seminear ring, since $A \cap B \neq \emptyset$

1) *Let* $a \bullet b \in A \cap B$ *or* $b \bullet a \in A \cap B$ and $a \in A \cap B$ since A , B be ideals of type two$\Rightarrow a \bullet b \in A$ *or* $b \bullet a \in A$ *and* $a \in A$ and $a \bullet b \in B$ *or* $b \bullet a \in B$ *and* $a \in B \Rightarrow b \in B$ and $b \in A$ then $b \in A \cap B$

2) *Let* $a \in A \cap B$ *then* $a * X \subseteq A$ *and* $a * X \subseteq B$ *since* A , B *is an ideal of type two* $\Rightarrow a * X \subseteq A \cap B \ \forall a \in A \cap B \Rightarrow A \cap B * X \subseteq A \cap B$ then $A \cap B$ is an ideal of type two .

Remark 3.2.6

Let X be a SNAK-seminear ring and let A , B be ideals of type two of X then $A \cup B$ is an ideal of type two of X if $A \subseteq B$ *or* $B \subseteq A$

Proof: clear .

***Example* 3.2.7**

Let $X=\{0,1,2,3\}$with two binary operations '$\bullet$' and '$*$' be defined by the following tables :

•	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	2	0	0
3	3	2	0	0

Let $A = \{0 , 1\}$ and $B = \{0 , 2\}$ then A and B is ideal of type two but $A \cup B = \{0 , 1 , 2\}$ is not ideal of type two since $2 \bullet 3 \in A$ and $2 \in A$ but $3 \notin A$ so the converse of remark 3.2.6 is not true in general.

Proposition 3.2.8

Let $f : X \to Y$ be a SNAK-seminear ring homomorphism . Then $ker\, f$ is an ideal of type two of X

Proof: Let X be SNAK-seminear ring and $f : X \to Y$ be a SNAK-seminear ring homomorphism

1) let $a \bullet b \in ker\, f$ or $b \bullet a \in ker\, f$ and $a \in ker\, f$

$\Rightarrow 0 = f(a \bullet b) = f(a) \bullet f(b) = 0 \bullet f(b) = f(b)$ [by e of definition 2.1.1]

$\Rightarrow f(b) = 0,$ in a similar way if $f(b \bullet a) = 0$ then $f(b) = 0 \Rightarrow b \in ker\, f$

2) in a simaller way of 3.1.7 we have $ker\, f * X \subseteq ker\, f$ for each $a \in ker\, f$ and $x \in X$ then, $ker\, f$ is an ideal of type two .

Proposition 3.2.9

Let $f : X \to Y$ be an SNAK-seminear ring epimorphism if A is an ideal of type two of X. Then $f(A)$ is an ideal of type two of Y.

Proof: Let X be a SNAK-seminear ring and A be an ideal of type two of X

1) Let $a' \in f(A)$ and $b' \in Y$, since f is an epimorphism $\Rightarrow \exists\ a \in A$ and $b \in X$ such that $f(a) = a'$, $f(b) = b' \Rightarrow$ let $a' \bullet b' \in f(A)$ or $b' \bullet a' \in f(A)$ and $a' \in f(A)$

$\Rightarrow f(a) \bullet f(b) \in f(A)$ or $f(b) \bullet f(a) \in f(A)$ and $f(a) \in f(A)$

$\Rightarrow f(a \bullet b) \in f(A)$ or $f(b \bullet a) \in f(A)$ and $f(a) \in f(A)$

$\Rightarrow a \bullet b \in A$ or $b \bullet a \in A$ and $a \in A \Rightarrow b \in A$ [Since A is an ideal of type two]

$\Rightarrow f(b) \in f(A)$ then $b' \in f(A)$

2) in a simaller way of *[proposition 3.2.9]* we have $f(A) * Y \subseteq f(A)$.Then, f(A) is an ideal of type two .

Proposition 3.2.10

Let X and X' be SNAK-seminear rings and let $f : X \to X'$ be a homomorphism of X if B is an ideal of type two of X' . Then $f^{-1}(B) = \{ a \in X / f(a) \in B \}$ is an ideal of type two of X .

Proof:

Let X and X' be SNAK-seminear rings

let $f : X \to X'$ be a SNAK-seminear ring homomorphism and let B be an ideal of type two of X'

1) Let $a \bullet b \in f^{-1}(B)$ or $b \bullet a \in f^{-1}(B)$ and $a \in f^{-1}(B)$

$\Rightarrow f(a \bullet b) \in B$ or $f(b \bullet a) \in B$ and $f(a) \in B \Rightarrow f(a) \bullet f(b) \in B$ or $f(b) \bullet f(a) \in B$ and $f(a) \in B$ since B is an ideal of type two we have $f(b) \in B \Rightarrow b \in f^{-1}(B)$

2) Let $a \in f^{-1}(B)$ and $x \in X$ so $f(x) \in X' \Rightarrow f(a) \in B$ and $f(x) \in X'$

$\Rightarrow f(a) * f(x) = f(a * x) \in B$ [since B is an ideal of type two]

$\Rightarrow a * x \in f^{-1}(B)\ \forall a \in f^{-1}(B)$ and $x \in X \Rightarrow f^{-1}(B) * X \subseteq f^{-1}(B) \Rightarrow f^{-1}(B)$ is an ideal of type two .

Proposition3.2.11

Let X be a SNAK-seminear ring and let I and J be ideals of type two of X . Then $I \times J$ is an ideal of type two of $X \times X$

Proof

Let X be a SNAK-seminear ring and let I , J be ideals of type two of X.

1) Let $(a , a') \bullet (b , b') \in I \times J$ or $(b , b') \bullet (a , a') \in I \times J$

and $(a , a') \in I \times J \Rightarrow (a \bullet b , a' \bullet b') \in I \times J$ or $(b \bullet a , b' \bullet a') \in I \times J$

and $(a , a') \in I\times J$

$\Rightarrow a \bullet b \in I$ *or* $b \bullet a \in I$ *and* $a \in I$ *also* $a' \bullet b' \in J$ *or* $b' \bullet a' \in J$

and $a' \in J$

then $b \in I$, $b' \in J$ [since I, J are ideals of type two of $X] \Rightarrow (b , b') \in I\times J$

2) Let $(x_1 , x_2) \in X \times X$ *and* $(a_1 , a_2) \in I\times J$

$\Rightarrow a_1 * x_1 \in I , a_2 * x_2 \in J$ because I and J are ideals of type two .Then

$(a_1 , a_2) * (x_1 , x_2) = (a_1 * x_1 , a_2 * x_2) \in I\times J$.Then $I\times J$ is an ideal of type two

<u>***3.3 Ideal Of Type Three on SNAK-seminear ring***</u>

In this section, we introduce the notion of ideal of type three on a SNAK-seminear ring and study its properties and give some examples .

<u>***Definition 3.3.1***</u>

Let X be a SNAK-seminear ring. A non empty subset I of X is said ***ideal of type three*** on X if satisfies the following conditions :

1) $a \bullet b \in I \; \forall a , b \in I$

2) $I * X \subseteq I$

<u>***Example 3.3.2:***</u>

Let $X=\{0,1,2\}$ with two binary operations '$\bullet$' and '$*$' be defined by the following tables :

•	0	1	2
0	0	1	2
1	1	1	1
2	2	1	0

*	0	1	2
0	0	0	0
1	1	0	1
2	2	0	0

Then by usual calculation we can prove that $I = \{0 , 1\} \subseteq X$ is an ideal of type three

<u>***Example 3.3.3:***</u>

Let $X=\{0,1,2,3\}$ with two binary operations '$\bullet$' and '$*$' be defined by the following tables :

•	0	1	2	3
0	0	1	2	3
1	1	1	1	1
2	2	1	1	1
3	3	1	2	3

*	0	1	2	3
0	0	0	0	0
1	1	0	0	3
2	2	0	0	3
3	3	0	0	0

Then $I = \{0 , 2\} \subseteq X$ is not ideal of type three since $2 \in I$ but $2 \bullet 2 = 1 \notin I$

<u>***Remark 3.3.4***</u>

If I is an ideal of type three of SNAK-seminear ring, then it is clear that , $0 \in I$

<u>***Remark 3.3.5***</u>

If I is an ideal of type three of SNAK-seminear ring , then I is sub SNAK-seminear ring.

<u>***Proof:***</u> Let I be an ideal of type three of SNAK-seminear ring and let $a , b \in I$ $\Rightarrow a \bullet b \in I$.Let $x , y , z \in I \subseteq X \Rightarrow (x \bullet y) \bullet z = x \bullet (y \bullet z)$ since $(X , \bullet)$ is a semigroup so $(I , \bullet)$ is a semigroup .Now, since $I * X \subseteq I$ $\forall x \in I$ so $i * j \in I$ $\forall i , j \in I$ so I is closed under the operation ($*$).Now, since $0 \in I$ [by remark 3.3.4] then in easy way we can show that all conditions of BCK algebra are satisfies so *(I , **

, *0)* is a BCK algebra .Also, the distribution law satisfies for all $x , y , z \in I \subseteq X$ where X is a SNAK-seminear ring and let $x \in I \subseteq X$ then it is clear that $0 \bullet x = x = x \bullet 0$ so $(I , \bullet , * , 0)$ is a sub SNAK-seminear ring .

Remark 3.3.6

The converse of above remark in general is not true .

Proof

We will prove it by using the example (3.2.3),
take $I = \{0 , 1\} \subseteq X$ it is clear that is a sub SNAK-seminear ring but I is not ideal of type three since $I * x \not\subset I$ where $1 \in I$ and $3 \in X$ but $1 * 3 = 3 \notin I$.

Proposition 3.3.7

Let X be a SNAK-seminear ring and let A , B be ideals of type three of X .Then $A \cap B$ is an ideal of type three of X and the converse is not true in general.

Proof: prove is clear .Now to prove the converse take $A = \{0 , 1\}$ and $B = \{0 , 1 , 2\}$ in (example 3.2.7) then $A \cap B= \{0 , 1\}$ ideal of type three but $B = \{0 , 1 , 2\}$ is not ideal of type three since $2 \bullet 1 = 3 \notin B$

Remark 3.3.9

Let X be a SNAK-seminear ring and let A , B be ideals of type three of X. Then $A \cup B$ is an ideal of type three of X if $A \subseteq B$ or $B \subseteq A$ and the converse is not true in general.

Proof: proof is clear and we can prove the converse of this remark in a similar way of remark 3.1.9 .

Proposition 3.3.10

Let $f : X \rightarrow Y$ be a SNAK-seminear ring homomorphism . Then $ker\, f$ is an ideal of type three of X .

Proof:.Let $f : X \rightarrow Y$ be a SNAK-seminear ring homomorphism. Then

1) $a , b \in ker\, f \Rightarrow f(a) = 0$ and $f(b) = 0 \Rightarrow f(a \bullet b) = f(a) \bullet f(b) = 0 \bullet 0 = 0$
$\Rightarrow f(a \bullet b) = 0 \Rightarrow a \bullet b \in ker\, f$

2) We can prove that $ker\, f * X \subseteq ker\, f$ in the same manner used in (3.1.7) .Then $ker\, f$ is an ideal of type three

we can easily prove the proposition 3.3.11-3.316

Proposition 3.3.11

Let $f : X \rightarrow Y$ be a SNAK-seminear ring epimorphism if A is an ideal of type three of X, then $f(A)$ is an ideal of type three of Y.

Proposition 3.3.12

Let X be a SNAK-seminear ring and let $f : X \rightarrow X'$ be SNAK-seminear ring homomorphism of X *if* B is an ideal of type three of X'. Then $f^{-1}(B) = \{ a \in X : f(a) \in B \}$ is an ideal of type three of X .

Proposition 3.3.13

Let X be a SNAK-seminear ring and let I, J be ideals of type three of X . Then $I \times J$ is an ideal of type three of $X \times X$.

Proposition 3.3.14

Let X be a SNAK-seminear ring and let $I' = \{(a , 0) / a \in X\}$ and $J' = \{(0 , b) / b \in X\}$. Then I' and J' are ideals of type three of $X \times X$.

Remark 3.3.15

Let X be a SNAK-seminear ring and let I' and J' be defined as in the above proposition. Then $I' \cap J' = (0 , 0)$.

Proposition 3.3.16

Let X be a SNAK-seminear ring . If I and J be ideals of type three of X and $x * y = x\ \forall x \in I, y \in J$, then $I \cap J = \{ 0 \}$.

Proposition 3.3.17

Let X be an abelian SNAK-seminear ring and let $I = \{a \in X : a^2 = a\}$. Then I an ideal of type three .

Proof: Let X be a commutative

1) Let $a, b \in I \Rightarrow a = a^2, b = b^2$

$\Rightarrow a \bullet b = a \bullet a \bullet b \bullet b = (a \bullet b)^2$ [since X is an abelian] so $a \bullet b \in I$

2) Let $a \in I$ and $x \in X \Rightarrow a = a^2$ and $x \in X$

$a * x = a^2 * x = (a \bullet a) * x = a * x \bullet a * x = (a * x)^2$ so $a * x \in I$.Then I is an ideal of type three .

Proposition 3.3.18

Let X be an abelian SNAK-seminear ring and let A be an ideal of type three. Then $\bar{A} = \{a \in X : a \bullet x \in A$ for some $x \in A\}$ is an ideal of type three of X and $A \subseteq \bar{A}$.

Proof : Let X be SNAK-seminear ring

let $a, b \in \bar{A} \Rightarrow a \bullet x, b \bullet y \in A$ for some $x, y \in A$

$\Rightarrow (a \bullet b) \bullet (x \bullet y) = (a \bullet x) \bullet (b \bullet y)$ [since X is an abelian]

Since $a \bullet x \in A$ and $b \bullet y \in A$ then $(a \bullet x) \bullet (b \bullet y) \in A$ [since A is an ideal of type three] but $x \bullet y \in A$ so $a \bullet b \in \bar{A}$

let $r \in X$ and $a \in \bar{A} \Rightarrow \exists x \in A \ni a \bullet x \in A \Rightarrow a * r \bullet x * r = (a \bullet x) * r \in A$ [since A is an ideal of type three] but $A * X \subseteq A$ [since A is an ideal of type three]

so $x * r \in A \Rightarrow a * r \in \bar{A}$ [by definition of $\bar{A}$].Now , to prove that $A \subseteq \bar{A}$

Let $x \in A \Rightarrow x \bullet 0 \in A$ where $0 \in A$ [since A an ideal of type three]

so $x \in \bar{A}$ then $A \subseteq \bar{A}$.

Proposition 3.3.19

Let X be a SNAK-seminear ring and $B_a = \{x \in X, (x * a) * a = 0\}$ where $a \in X$. Then B_a is an ideal of type three.

Proof : Let X be a SNAK-seminear ring and Let $x, y \in B_a$

$\Rightarrow (x * a) * a = 0$ and $(y * a) * a = 0$

1) $(x * a) * a \bullet (y * a) * a = [(x * a) \bullet (y * a)] * a = ((x \bullet y) * a) * a = 0 \Rightarrow x \bullet y \in B_a$

Let $r \in B_a$ and $x \in X$ to prove that $r * x \in B_a$

Since $r \in B_a \Rightarrow (r * a) * a = 0$,

$[(r * x) * a] * a = [(r * a) * x] * a$ [by 3 of remark 1.15]

$= [(r * a) * a] * x = 0 * x = 0$ [by 4 definition 1.14]

$\Rightarrow r * x \in B_a \Rightarrow B_a$ ideal of type three

Corollary 3.3.20

Let X be an abelian SNAK-seminear ring and A be an ideal of type three of X. Then $\bar{A} = A$ if and only if A is an ideal of type two .

Proof

Let X be an abelian and A be an ideal of type three. Suppose that $\bar{A} = A$,

if $a \bullet b \in A$ or $b \bullet a \in A$ and $a \in A$

$\Rightarrow a \bullet b = b \bullet a \in A$ and $a \in A$ [since X is an abelian] $\Rightarrow b \in \bar{A}$

but $\bar{A} = A$ so $b \in A \Rightarrow A$ is an ideal of type two .Conversely , suppose that A is an ideal of type two so $A \subseteq \bar{A}$ [by 3.3.18].Let $a \in \bar{A} \Rightarrow a \bullet x \in A$ for some $x \in A$

$\Rightarrow a \bullet x = x \bullet a \in A$ and $x \in A$ [since X is an abelian] .But A is an ideal of type two $\Rightarrow a \in A$, so $A \subseteq \bar{A}$ therefore $\bar{A} = A$

Proposition 3.3.21

Let X be an abelian SNAK-seminear ring and Let A , B be ideals of type three of X such that $A \subseteq B$. Then $\overline{A} \subseteq \overline{B}$

Proof: Let X be a SNAK-seminear ring and Let A , B be ideals of type three of X such that $A \subseteq B$

Let $a \in \overline{A}$ then $a \bullet x \in A$ for some $x \in A \Rightarrow a \bullet x \in B$ *where* $x \in B$ [since $A \subseteq B$] .
Hence, $a \in \overline{B} \Rightarrow \overline{A} \subseteq \overline{B}$

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