Analysis of the Reliability of Complex Systems by Using Fuzzy Fault Tree

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Abstract

Reliability is the probability of performing the function of an item over a given period of time under the operating conditions encountered .
A Fault tree analysis (FTA) is exceedingly used reliability guess tool for complex engineering systems.
The classic fault tree analysis method, which contains (AND, OR) gates can efficacious build an analytical model to represent combinations of component failures that cause the failure of a system. In this paper, we analyze the fuzzy fault tree (FFT) to the qualitative analysis and quantitative analysis for the purpose of identifying the a minimal cut set and the statement of system failure through examples of complex systems (containing linking sequent and parallel). This paper includes reliability analysis as one of its basic tasks .

Keywords: Fuzzy Logic , Reliability , Membership Function , Fault Tree Analysis

الخلاصة

المعولية هي احتمال أداء وظيفة المكون على مدى فترة معينة من الزمن تحت ظروف محددة للتشغيل . ويستخدم تحليل شجرة الخطأ كأداة لتخمين معولية النظم الهندسية المعقدة . طريقة التحليل الكلاسيكية لشجرة الفشل تحتوي على ابواب منطقية مثل (و , او)ويمكن من خلالها تمثيل العناصر التي تسببت في فشل النظام , في هذا البحث نقوم بتحليل شجرة الخطأ الضبابية الى تحليل نوعي وتحليل كمي لغرض التعرف على اصغر المجموعات القاطعة والتي تسبب فشل النظام من خلال امثلة توضيحية لانظمة معقده (تحوي على ربط متوالي وربط متوازي).

الكلمات المفتاحية : المنطق الضبابي , المعولية , دالة الانتماء , تحليل شجرة الخطأ .

1. Introduction

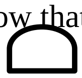
The reliability analysis comprises of Fault Tree Analysis (FTA) as one of the important tools or activities among the reliability tasks of an item design. A fault tree analysis is a graphical design technique that is for analyzing the top event, which causes the system failure. It is a top-down, deductive analysis structured as top-down to show that the down events lead to the occurrence of the top event. (FTA) is a useful tool for conducting a system safety analysis.

Though (FTA) was developed long back, it still cannot be performed practically without facing problems of imprecise failure input data and improper modeling problems. Towards this end, fuzzy sets can help to overcome this problem. Experts use fuzzy sets to subjectively describe the uncertainties of each given failure event, and then conduct mathematical operation to evaluate system reliability. These fuzzy sets are considered as the possibility of occurrence of the failure events. Therefore the problem is to calculate the probability of failure of the top event as a fuzzy set, given the possibilities of failure of the basic or major events .

2. Logic - Gates

They are two basic types of fault tree gates , the OR – gate and the AND – gate .

2.1The AND – Gate : [Nikolaos Limnios , 2007]

The AND- gate is used to show that the output fault happens only if all input fault happen . Which is symbolized by  as shown in Fault tree of the figure below :

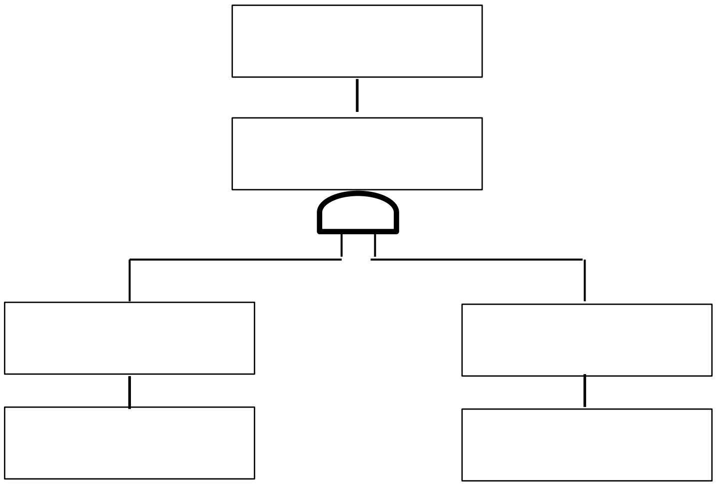
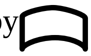


Fig . (1) AND gate

2.2The OR – Gate : [Sandler G. , 1963]

The OR – gate is used to show that the output event happens only if one or more of the input events happen , Which symbolized by  as shown in fault tree of the figure below :

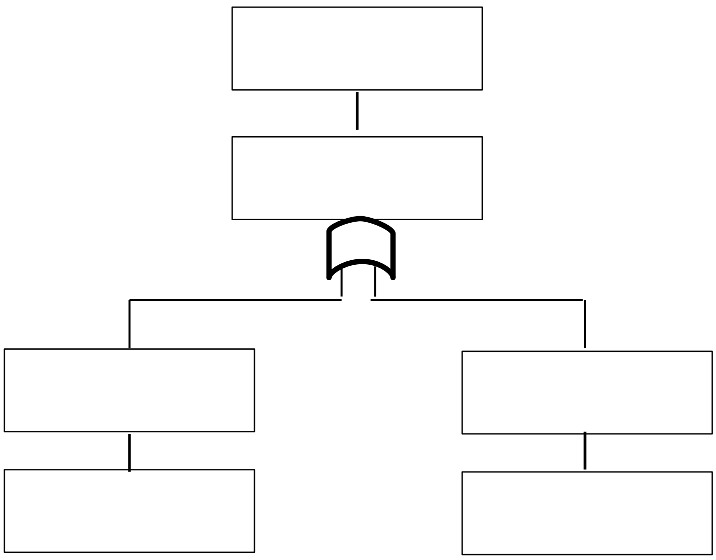


Fig . 2 OR–gate

3. Some Examples About Fault Tree Analysis

3.1 Fault Tree Analysis : [Yue – Lung , 2000]

A fault tree is a diagram representation of some relations which traces a system hazard underdeveloped to investigate for all it's possible causes. Such a system hazard is called the top event of the fault tree . Conventional , quantitative analysis , estimate the possibility of the occurrence of the top event in such case the probability of each major event is already known .

Example 1: Write the equation of the top event the following fault tree :

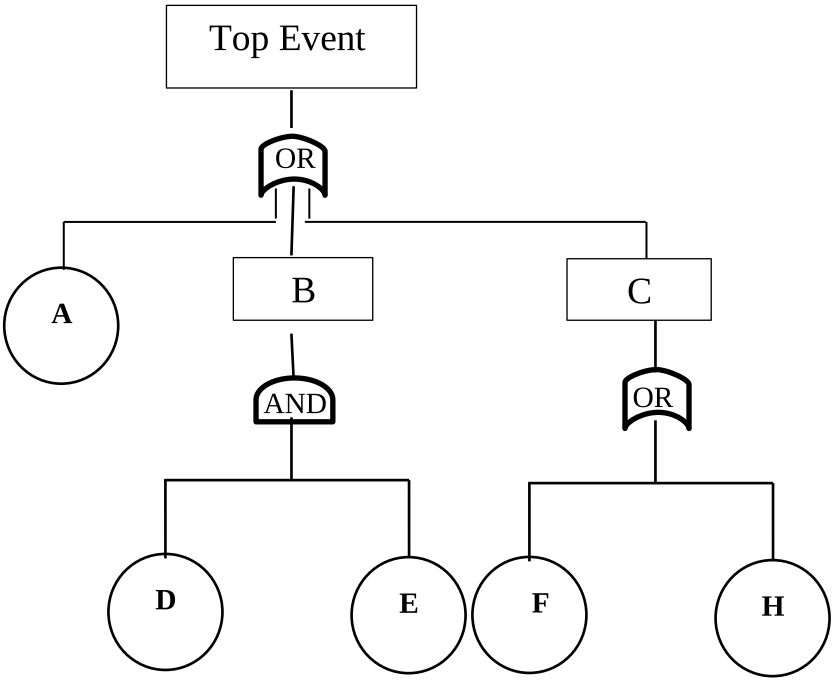


Fig . (3) A fault tree

By using the operations Boolean algebra we get :

$T = A \cup B \cup C$

$B = D \cap E$

$C = F \cup H$

$T = A \cup (D \cap E) \cup (F \cup H)$

$\quad = A + (D . E) + (F + H)$

3.2 Fuzzy Fault Tree Analysis : [Yue – Lung , 2000]

Taking into Considering conventional (crisp) sets first , Let H be a set and A a subset of H , in this case the relation between an element $x \in H$ and set A is either $x \in A$ or $x \notin A$. This fact can be shown or denoted by the following characteristic function :

$$\mu_A(x) = \begin{cases} 1 & if \quad x \in A \\ 0 & if \quad x \notin A \end{cases}$$

Where μ_A (x) is the characteristic function .

For a Fuzzy set sense the characteristic function may differ in the degree of value , say membership , between 0 and 1 to show the belief that such as x ∈ A . Hence , μ_A (x) is called the membership function .see[Yue-Lung Cheng,2000] .

If X is a collection of objects , denoted generally by x , then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs .

$$\tilde{A} = \{ (x , \mu_{\tilde{A}} (x) | x \in X \}$$

Where $\mu_{\tilde{A}} (x)$ is called the membership function of x in $\tilde{A}$ which maps X to the membership space [0 , 1]

3.3 A Fuzzy Probability : [Yue – Lung , 2000]

To say that an event A occurs with probability P the following membership function can be used :

$$\mu_P (x) = \begin{cases} one(1) & if \quad x = p \\ zero(0) & otherwise \end{cases}$$

The belief that the event A is said to happen with probability P ∈ [a ,b] , can be shown by the following membership function :

$$\mu_P (x) = \begin{cases} one(1) & if \quad x \in [a,b] \\ zero(0) & otherwise \end{cases}$$

3.4 Fuzzy Arithmetic

Arithmetic operations on fuzzy number based on resolution principle (α – cuts)
Let $\bar{A}$, $\bar{B}$ be two fuzzy numbers and

$\bar{A} [\alpha] = [a_1(\alpha) , a_2 (\alpha)] , \bar{B} [\alpha] = [b_1(\alpha), b_2(\alpha)]$

be α cuts of $\bar{A}$ *and* $\bar{B}$ respectively , α ∈ [0,1]

The operation * denoted any arithmetic operation : (+) , (-) , (.) , (÷) , ∧ , ∨) on $\bar{A} , \bar{B}$ denoted by $\bar{A} * \bar{B}$ gives a fuzzy number in R . for more detail see [George J.,1995] .

4.Some Definition and Concepts :

<u>Definition(4.1)</u> : [Nikolaos Limnios , 2007]

A fault tree is a diagram representation of some relations which traces a system hazard back words to each for all its possible causes .

<u>Definition(4.2)</u> : [Balagursumy, 1984]

A System is the overall structure being considered , which in turn consist of sub-ordinate structures called sub system or **"components"** .

<u>Definition(4.3)</u> : [Srinath, 1985]

Reliability is the likelihood that an article will do its meant role or function for a specified interval under certain conditions .

<u>Definition(4.4)</u> : [Keiser,1989]

The **network reliability** is the likelihood that a network element thereof , will do satisfactory for a given period of time.

Definition(4.5) : [Subrie , 1975]
A cut set is set of components such that when the components in the cut are removed the system there is no path from one terminal to the other .
Definition(4.6) : [Jain, 1986]
A cut set of a diagram G is a set of branches whose separation from G results in G **disconnected** (leaves the origin or source and sink in separate parts) .

Definition(4.7) : [Srinath, 2005]
A minimal cut set is a minimum group of fault tree initiators which if all occur , will make the top event to occur .

Definition (4.8) : [Sandler و 1963]
A **diagram** is a set of nodes together with a set of branches with the condition that each branch terminates in each end into a node .

Definition(4.9) : [Keiser, 1989]
A diagram G is said to be **connected** if every two of its vertices are connected. A top u is said to be a u – v path in G . i.e., A graph G is connected if it cannot expressed as the union of two graphs , otherwise it is disconnected
Example 2 :
For the following network for a complex system consists of nine components , determine the minimal cut sets .(which reprise a modification for a question taken from [NASA , 2002]) .

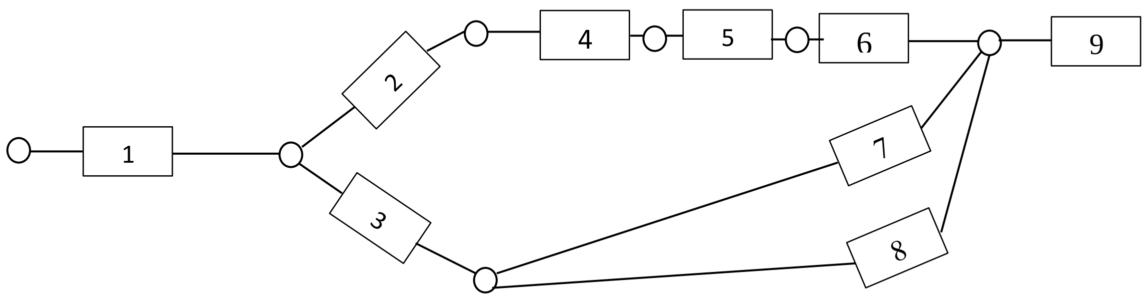


Fig . (4) Network representation for the system

The minimal cut set is one or more cuts of links whose failure causes the system to fail
Let X_i be represent the component i , the minimal cut sets (C_i) are :
$C_1 = \{ X_1 \}$, $C_2 = \{ X_2 , X_3 \}$, $C_3 = \{ X_2 , X_7 , X_8 \}$, $C_4 = \{ X_4 , X_3 \}$
$C_5 = \{ X_4 , X_7 , X_8 \}$, $C_6 = \{ X_5 , X_3 \}$, $C_7 = \{ X_5 , X_7 , X_8 \}$
$C_8 = \{X_6 , X_3 \}$, $C_9 = \{ X_6 , X_7 , X_8 \}$, $C_{10} = \{ X_9 \}$

The Figure (5) represents the minimal cut sets of the network in Figure (4) as a series – parallel system

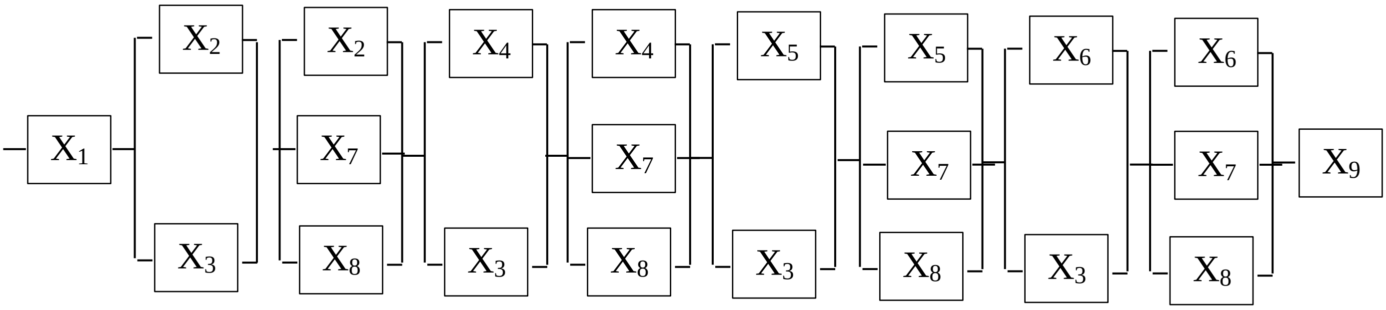


Fig .(5) Shows the minimal cut sets of the network in fig .(4)

The reliability (R_S) of the system of each cut set (C_i) are as follows :

$$Rs = \prod_{i=1}^{10} Ci$$

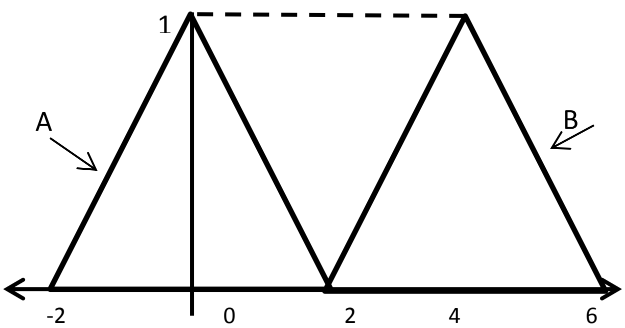
$Rc_1 = [\ 1 - (\ 1 - R_1\ )\ ]$
$Rc_2 = [\ 1 - (\ 1 - R_2\ )\ (\ 1 - R_3\ )\ ]$
$Rc_3 = [\ 1 - (\ 1 - R_2\ )\ (\ 1 - R_7\ )\ (\ 1 - R_8\ )\ ]$
$Rc_4 = [\ 1 - (\ 1 - R_4\ )\ (\ 1 - R_3\ )\ ]$
$Rc_5 = [\ 1 - (\ 1 - R_4\ )\ (\ 1 - R_7\ )\ (\ 1 - R_8\ )\ ]$
$Rc_6 = [\ 1 - (\ 1 - R_5\ )\ (\ 1 - R_3\ )\ ]$
$Rc_7 = [\ 1 - (\ 1 - R_5\ )\ (\ 1 - R_7\ )\ (\ 1 - R_8\ )\ ]$
$Rc_8 = [\ 1 - (\ 1 - R_6\ )\ (\ 1 - R_3\ )\ ]$
$Rc_9 = [\ 1 - (\ 1 - R_6\ )\ (\ 1 - R_7\ )\ (\ 1 - R_8\ )\ ]$
$Rc_{10} = [\ 1 - (\ 1 - R_9\ ]$
$Rs = [\ 1 - (\ 1 - P_1\ )\ ]\ [\ (\ 1 - (\ 1 - P_2\ )\ (\ 1 - P_3\ )\ ]\ [\ 1 - (\ 1 - P_{2)}\ (\ 1 - P_7\ )\ (\ 1 - P_8\ )\ ]\ [\ 1 - (\ 1 - P_4\ )\ (\ 1 - P_3\ )]\ [\ 1 - (\ 1\text{-}\ P_4\ )\ (\ 1 - P_7\ )\ (\ 1 - P_8\ )\ ]$
$[\ 1 - (\ 1 - P_5\ )\ (\ 1 - P_3\ )\ ]\ [\ 1 - (\ 1 - P_5)\ (\ 1 - P_7\ )\ (\ 1 - P_8\ )\ ]$
$[\ 1 - (\ 1 - P_6\ )\ (\ 1 - P_3\ )\ ]\ [\ 1 - (\ 1 - P_6\ )\ (\ 1 - P_7\ )\ (\ 1 - P_8\ )\ ]$
$[\ 1 - (\ 1 - P_9\ )\ ]$

If the components have the same probability
Pi = P then (Rs) we get :
$Rs = [\ 1 - (\ 1 - P\ )]^2\ [\ 1 - (\ 1 - P\ )^2]^4\ [\ 1 - (\ 1 - P\ )^3]^4$
Suppose P = 0.8 Then $Rs = [1\text{-}(1\text{-}0.8)]^2[1\text{-}(1\text{-}0.8)^2]^4[1\text{-}(1\text{-}0.8)^3]^4$
$R_s = 0.86$

Example 3 :(**Question take from [George and Boyuan , 1995])**
Let A , B be two fuzzy numbers whose membership functions are given by :

$$A\ (x) = \begin{cases} (x+2\ )/2 & for \quad -2 < x \le 0 \\ (2-x\ )/2 & for \quad 0 < x \le 2 \\ 0 & other\ wise \end{cases}$$

$$B(x) = \begin{cases} (\ x-2\ )/2 & for \quad 2 < x\ \le 4 \\ (\ 6-2\ )/2 & for \quad 4 < x\ \le 6 \\ 0 & other\ wise \end{cases}$$



Calculate the fuzzy numbers A + B , A – B

Solution :

Let (x+2) / 2 = α   Such that α ∈ [0 , 1]

2α = x + 2

x = 2α – 2

Let (2 – x) / 2 = α

2α = 2 – x

x = 2 - 2α

A (α) = [2α – 2 , 2 - 2α]

Let (x – 2) /2 = α

2α = x – 2

x = 2α + 2

Let (6 – x) / 2 = α

2α = 6 – x

x = 6 - 2α

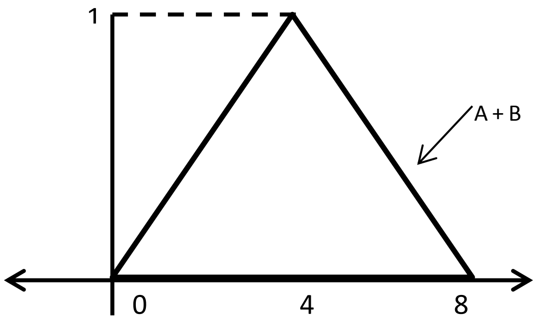
B(α) = [2α + 2 , 6 – 2α]

(A + B) (α) = [4α , 8 - 4α]

4α = x ,   α = $\frac{x}{4}$

if α = 0   then   x = 0

if α = 1   then   x = 4



8 - 4α = x

4α = 8 – x

α = $\frac{8-x}{4}$

if α = 0   then   x = 8

if α = 1   then   x = 4

$$(A+B)(x) = \begin{cases} 0 & if \quad x \le 0 \;,\; x \ge 8 \\ x/4 & if \quad 0 < x \le 4 \\ 8 - x/4 & if \quad 4 < x \le 8 \end{cases}$$

(A – B) (α) = [4α – 8 , -4 α]

4α – 8 = x ,   α = $\frac{x+8}{4}$

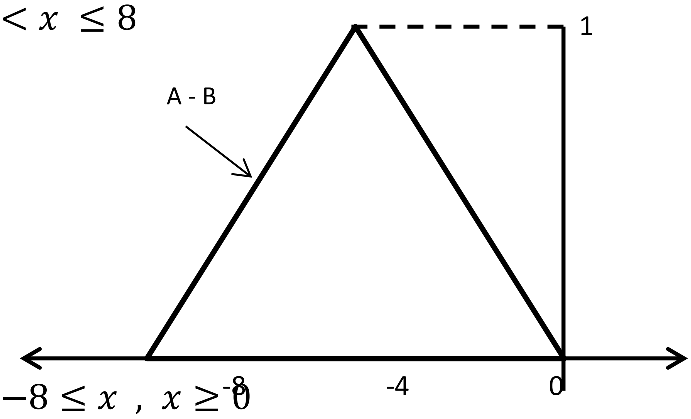
if α =0   then   x = -8

if α = 1   then   x = -4

-4α = x ,   α = $\frac{-x}{4}$

if α = 0   then   x = 0

if α = 1   then   x = -4

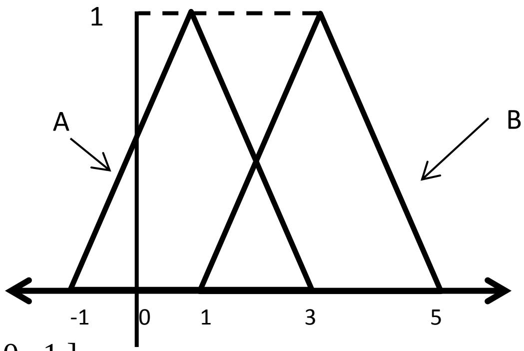


$$(A-B)(x) = \begin{cases} 0 & if \quad -8 \le x \;,\; x \ge 0 \\ (x+8)/4 & if \quad -8 \le x \le -4 \\ -x/4 & if \quad -4 < x \le 0 \end{cases}$$

Example 4 : Let A, B be two fuzzy numbers whose membership functions are given by:

$$A(x) = \begin{cases} 0 & x \le -1 \ , \ x \ge 3 \\ (x+1)/2 & -1 < x \le 1 \\ (3-x)/2 & 1 < x < 3 \end{cases}$$

$$B(x) = \begin{cases} 0 & x \le 1 \ , \quad x \ge 5 \\ (x-1)/2 & 1 < x \le 3 \\ (5-x)/2 & 3 < x < 5 \end{cases}$$

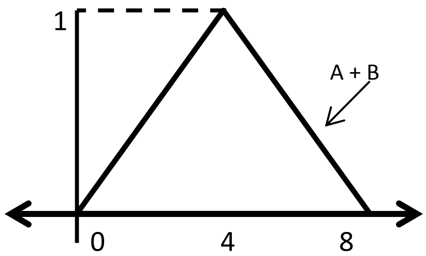


Find the fuzzy numbers A+ B , A-B

Solution :

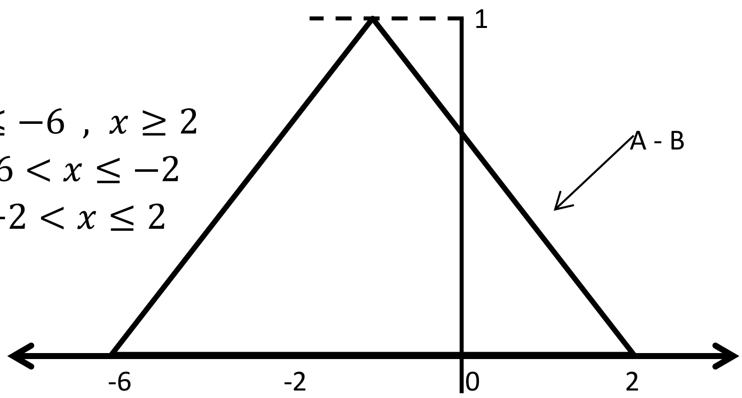
Let (x+1) /2 = α , 2α = x + 1 , x = 2α – 1 Such that α ∈ [0 , 1]
Let (3 – x) /2 = α , 2α = 3 – x , x = 3 - 2α
We get A (α) = [2α – 1 , 3- 2α]
Let (x – 1) / 2 = α , 2α = x – 1 , x = 2α + 1
Let (5 – x) /2 = α , 2α = 5 – x , x = 5 - 2 x = 5 - 2α
We get B (α) = [2α + 1 , 5 - 2α]
Now (A + B) (α) = [4α , 8 - 4α]

$$(A+B)(x) = \begin{cases} 0 & if \quad x \le 0 \ , \ x \ge 8 \\ x/4 & if \quad 0 < x \le 4 \\ 8 - x/4 & if \quad 4 < x < 8 \end{cases}$$

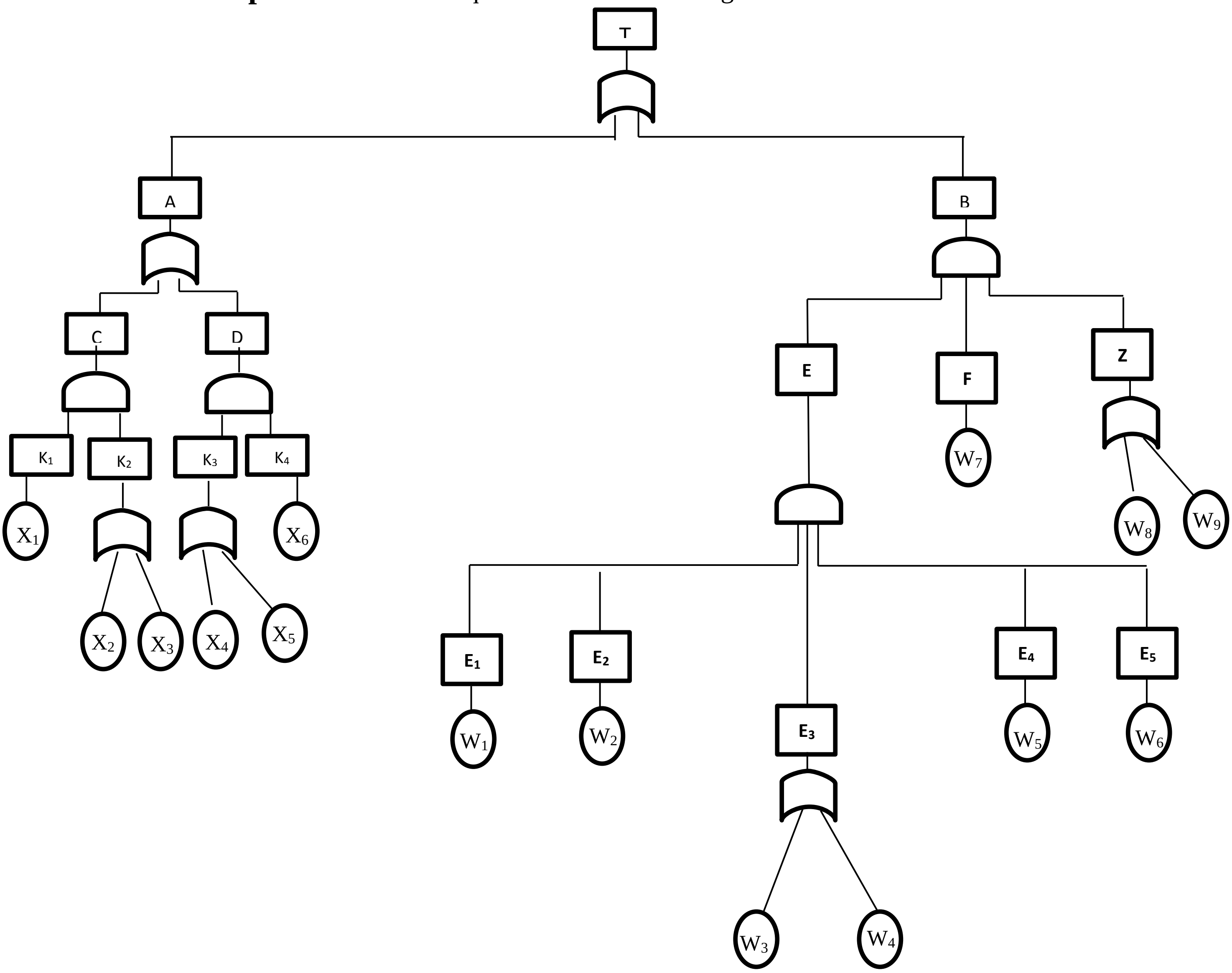


Also (A – B) (α) [4α – 6 , 2 - 4α)

$$\therefore (A-B)(x) = \begin{cases} 0 & if \quad x \le -6 \ , \ x \ge 2 \\ x + 6/4 & if \quad -6 < x \le -2 \\ 2 - x/4 & if \quad -2 < x \le 2 \end{cases}$$



Example 5 : Derive the equation for the following fault tree



Solution : Frist method

$T = A + B$

$A = C + D$

$C = K_1 . K_2$, $D = K_3 . K_4$

$K_1 = X_1$, $K_3 = X_4 + X_5$

$K_2 = X_2 + X_3$, $K_4 = X_6$

.: $A = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$ ………… (1)

$B = E . F . Z$

$E = E_1 + E_2 + E_3 + E_4 + E_5$

$E_1 = W_1$, $E_2 = W_2$, $E_3 = W_3 + W_4$

$E_4 = W_5$, $E_5 = W_6$

$F = W_7$, $Z = W_8 + W_9$

.: $B = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 + W_9$ … (2)

.: $T = X_1 . X_2 + X_3 + X_4 + X_5 + X_6 + W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 + W_9$

Second method :

$X_1 = K_1$, $X_2 + X_3 = K_2$

$K_1 . K_2 = C$, $X_4 + X_5 = K_3$, $X_6 = K_4$

$K_3 . K_4 = D$

$C + D = A$ ………… (1)

$W_1 = E_1$, $W_2 = E_2$, $W_3 + W_4 = E_3$, $W_5 = E_4$, $W_6 = E_5$

$E_1 + E_2 + E_3 + E_4 + E_5 = E$

$W_7 = F$, $W_8 + W_9 = Z$

$E . F . Z = B$ …………. (2)

$X_1 . X_2 + X_3 + X_4 + X_5 . X_6 + W_1 + W_2 + W_3 + W_4 + W_6 . W_7 . W_8 + W_9 = T$

Example 6 : Derive the equation for the following fault tree with analysis :

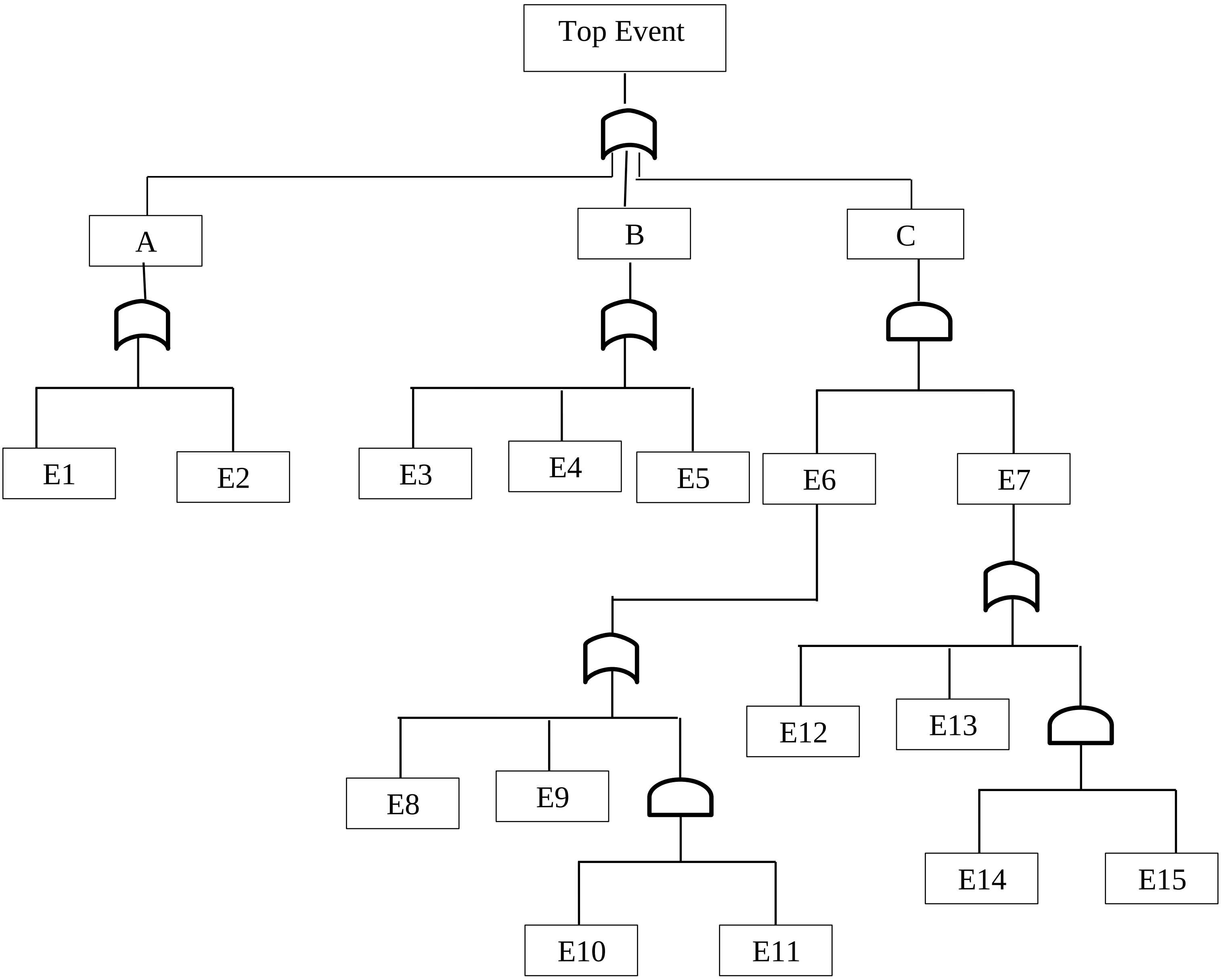


Fig .(6) Human Factors Fault tree

Solution :

$T = A + B + C$

$A = E_1 + E_2$

$B = E_3 + E_4 + E_5$

$C = E_6 \ . \ E_7$

$E_6 = E_8 + E_9 + E_{10} \ . \ E_{11}$

$E_7 = E_{12} + E_{13} + E_{14} + E_{15}$

.: $T = E_1 + E_2 + E_3 + E_4 + E_5 + E_6 \ . \ E_7$

.: $T = E_1 + E_2 + E_3 + E_4 + E_5 + E_8 +[E_9 + (E_{10} . E_{11})] .[E_{12} + E_{13} + (E_{14} . E_{15})]$

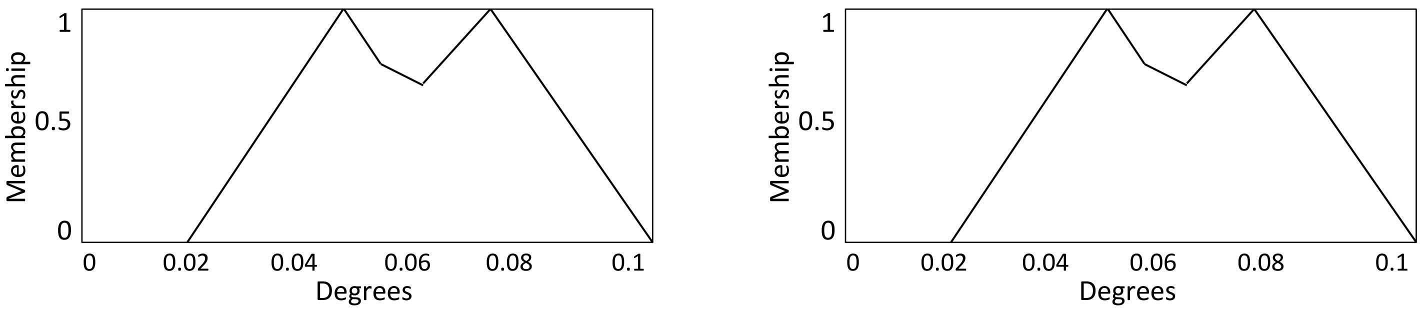
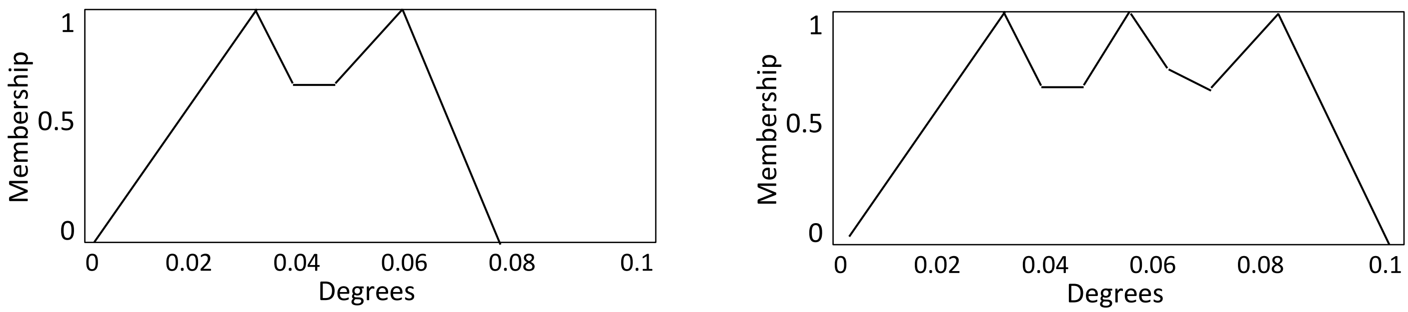


Fig .(7) Outcome of gathering of Intermediate Events and Top Event



Conclusion

Information of the major events are imperfect or mysterious. So, fault tree analysis is a helpful tool for safety performing a system by fault tree analysis. Also reliability analysis by fault tree demands perfect and accurate information about human faults . In many cases ,the thematic tree hazard estimate renders a false impression of accuracy and on fuzzificationg fault tree by representation major events it is possible fragile values such as short , medium , and high or hot ,warm ,cold to show the degree in which experts believe in major events happening . In lieu of fault tree AND , OR operations , fuzzy operators are used to confer conclusion about the top event in the form of fuzzy possibility score . This fuzzy possibility score value is transformed into fuzzy failure probability .

References :

Balagursumy E. (1984) " **Reliability Engineering** " , Tata Mc GRAW – Hill publishing company limited .

George J .Klir and Boyuan **,** (1995) **"Fuzzy Sets and Fuzzy Logic Theory and Applications "** New York .

Hany Sallam(2015) "**Human Factors Reliability Analysis Using Fuzzy Fault tree"** , (IJEIT) Volume 4 .

Jain S . P . , Gopal K. (1986) **" An Improved of Selecting , Network Topology for Optimal Terminal Reliability " .**

Keiser E . G. (1989) " **Local Area Networks** " , McGraw – Hill Inc . , New York .

NASA,*(2002)*"**Fault Tree and Book With Aerospace Applications" ,** office of Safety and Mission Assurance .

NarsinghDeo (2007) " **Graph Theory with Application to Engineering and Computers Science** " , since Hallof IndiaPrivate Limited New Delhi

Nikolaos Limnios(2007) **" Fault Tree ISTELTD " .**

Sandler G.(1963) " **System Reliability Engineering** " , prentice – hall Englewood Cliffs .

Srinath L.S . (1985) " **Concepts in Reliability Engineering** " , 2^{nd} edition , Affiliated East , West press Private Limited , New Delhi Madras – Hyclerabd .

Srinath L . S .(2005) " **Reliability Engineering** " , 4^{th} edtion Affiliated East , West press Private Limited – New – Delhi .

Subrie U.A .(1975) " **Some Reliability Models Subject to Corrective Maintenance**" M , Sc , thesis , university of Baghdad , Iraq .

Yue-Lung Cheng(2000) "**Uncertainties in Fault Tree Analysis***"* Department of Information Management, Husan Chuang College, 48, Husan-Chuange Rd.,HsinChu, Taiwan,R.O.C