Algorithm: as Construction of Cayley Graph which Embedded any Graph

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Abstract

C.Delorme gave a proposition of construction of vertex-transitive graph. For this there is a group G and a subgroup H, and a subset A of G. the graph [G,H,A]are constructed. The vertices of graph are the parts of G of the forms xH , their number is the index of H in G.

The adjacent of xH are xah where a ЄA when H is reduced to an neuter element of the group. Cayley graph is found and it is associated to group G and the part A .

If gЄG , then xH➔gxH is an automorphism Nihad M. [3] found that there exist an Extension which in (n-1) monomorphic , which contains any binary relation , then Cayley graph is vertex transitive so (n-1) – monomorphic. In this work it is found that an Algorithm as construction Cayley graph which embedded any binary relation, and this Extension perhaps is finite or infinite.

Keywords: Vertex-transitive, Cayley graph, automorphisme, (n-1) –monomorphic.

1. Introduction

Let G be a finite group, and S the set of generators of G . Cayley graph is denoted by Cay (G,S) with the vertices correspond to the set of elements of G , defined by: Let (V1,V2) is an are of Cay (G,S) if and only if ∃S_iЄ S such that x_2=x_1.$s_1$or x_1^{-1} . x_2ЄS .

Cayley graph in (n-1) monomorphic, when for all x,y of it is base E , the restrictions of cay (G,S) to E-x and E-y are isomorphic in [3], any binary relation reflexive or anti-reflexive is embedded in cay (G,S)

2. Definitions

*A Binary relation R on V in a Function:V^2[+ , -] ⟶
*The relation R of base V, the elements of V are called vertices of R
The pair (V , E) are called a graph , E represents the set of arcs (x , y) such that R (x ,y) = +
*G and it are two graphs , such that G = (V , E) , and H = (V' , E') , V , and V'are the same cardinal , and F in a objective from V to V' .
provided that f in an isomorphism from G on H if :
Ʉ (x, y) ϵV : (x ,y) Є E ⟶ [f(x) , f(y)] ϵ E'
*let R , S , be two relations of the same arity . R in embedded in S if and only if there exists a restriction of S isomorphic with R .

Notation:
(X) = { X + S : X Є S } set of successors λ^{+}
(X) = { X – S : X Є S } set of predecessors λ^{-}

Theorem1 N. Majed [3]: let x be a graph of cardinal n . then there exists a part S of Z_3^{n-1} such that x is embedded in cay (Z_3^{n-1}, S)
Proof:
(1)let X = (V,E) be a graph we shall be constructed by recurrence as a part S of Z_3^{n-1} and an injection f:V⟶ Z_3^{n-1} such that for all x , y Є V
(X,Y) ЄE iff f(Y) – f(X) Є S .
A) case1: n=2, there exist three graphs not isomorphic at two elements defined as follows .

1) For the graph without arcs, put S=ϕ , then cay (Z_3,S) is the empty graph .
2) For the graph with one arc, put S={1} , then cay (Z_3,S) is a cycle of three elements.
3)For the complete graph, put S={1,2} and cay (Z_3,S) is a symmetric complete graph .

(2) Suppose the theorem is true for n, let X=(V,E) be a graph of cardinal n+1 , let aЄ V, V' = V – {a} and X' is the restriction of X to V' , the set E' of arcs of X' is arcs set of E which in not in the from (a,x) or (x,a) x is any element of V by the hypothesis of recurrence there exist a part T of Z_3^{n-1} and an injective f from V' into Z_3^{n-1} such that for all x,y Є V' (X,Y) Є E' Iff f(y) – f(x) ЄT

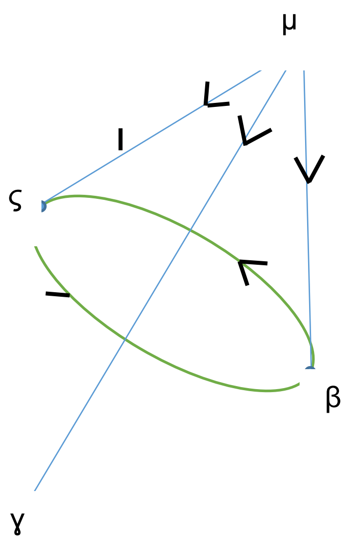
Let:
S1=T*{0}
S2 = { (f(x) , 1) ; X Єλ^{+} (a) }
S3 = { (-f(x) , -1) ; X Єλ^{-} (a) }
S = S1U S2 U S3
Let g be an application from V into Z_3^{n} . define by g(a)=(0,0,….,0) and g(x) = (f(x) , 1) Shall be proved that for all X,YЄ V
(X,Y) Є E iff g(y) – g(x) Є S
A) case1 (X,Y) Є V' = V – {a} , (X,Y) Є E' equivalent to f(y) – f(x) Є T equivalent g(y) – g(x) Є S1 , by def , to S1 .
B) case 2 , X= a and Y Є V' = V – {a} .
(a,y) ЄE , Y Є λ^{+} (a)⟷(f(y) , 1) Є S2
But (f(y) , 1) = g(y) = g(y) – g(a) .
C) case 3 , X Є V' and y = a , (x,a) Є E ⟷ X Є λ^{-} (a)⟷ (-f(x) , -1) Є S3
But (-f(x),-1) = -g(x) = g(a) – g(x) . in the three cases , if (x,y) Є E then g(y) – g(x) Є S .
The inverse , if the elements (respectively) belong to S1 , S2 , S3 then the last coordinate is (respectively) equal 0,1,2
Remark 1 :it can put in the proof Z_m (m ≥ 3) instead of Z3 .
Remark 2 : if X in symmetric graph , it can put Z2 instead of Z3 , and in this case S3=S2 .

Theorem 2 **[3]** : Each binary relation possesses a finite extension (n-1)–monomorphic .

For , if n is the cardinal of relation there exist for each integer m of the form K^{n-1}(K ≥ 3) , this extension is (m-1) – monomorphicand there exist an infinite extension (n-1) – monomorphic . of the relation .
Let A be a finite binary relation , and Ā one of these finite extensions
(n-1) – monomorphic , by the above theorem .
Let B by the relation by making the Z-sum of Ā (I.e. we replace each point of Z by copy of Ā) , it is clear that B is infinite of A , and then (n-1)-monomorphic .

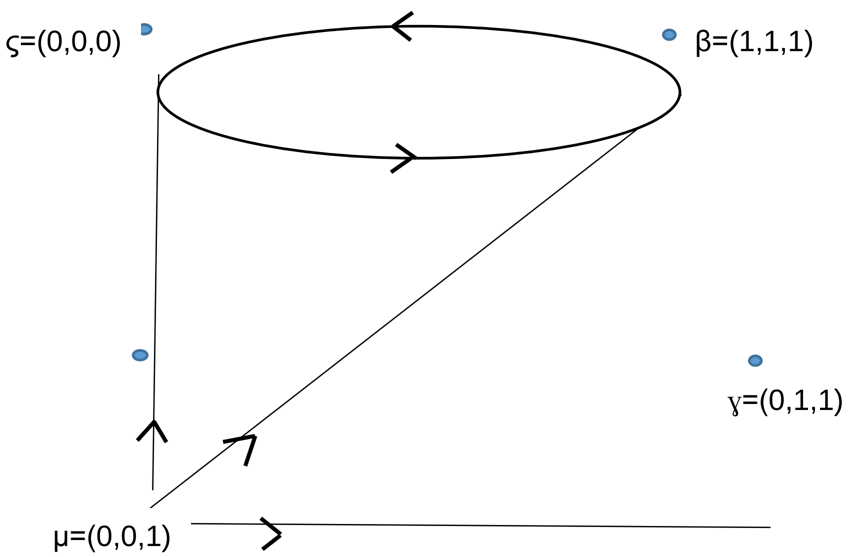
3. Algorithm

The following graph X is considered any two element S of X are chosen, for example.



ɣ and β .
In this case S=ϕ .
And X1 = cay (Z_3, ϕ)
f1 (ɣ) = 0
f1(β) = 1
= ϕ X {0} = ϕ.S_1'
= f_1(λ^+ (μ)) x {1} . thenS_2'
= f_1({ɣ,β}) x {1}S_2'
= {0,1} x{1}
= { (0,1) , (1,1) } .
= -[f(λ^- (μ) x {1}]S_3'
= -[ϕ]
= ϕ
S' = $S_1' U\ S_2'\ U\ S_3'$
S' = { (0,1) , (1,1) } .
(μ) = (0,0) f_2
(ɣ) = (f_1(ɣ),1) = (0,1) f_2
(β) = (f_1(β) ,1) = (1,1)f_2
= S' x {0} .S_1''
= { (0,1) , (1,1) } x {0} .S_1''
= { (0,1,0) , (1,1,0) } .
= f_2 (λ^+ (ς) x {1} S_2''
= { f_2(β) x {1} .
= { (1,1,1) } .
= -[f_2(λ^- (ς)) x {1}] .S_3''
= -[f_2(μ,β) x {1}] .
= -[(0,0,1) , (1,1,1)] .
= { (0,0,-1) , (-1,-1,-1) } .
= $S_1'' S_2'' S_3'' S''$
= { (0,1,0) , (1,1,0) , (1,1,1) , (0,0,-1) , (-1,-1,-1) } .
(ɣ)= (f_2(ɣ) , 1) = (0,1,1)f_3
(β)= (f_2(β) , 1) = (1,1,1)f_3
(μ)= (f_2(μ) , 1) = (0,0,1)f_3

(ς)= (0,0,0)f_3
It is found that the graph cay $\{Z_3^3$, [(0,1,0) , (1,1,0) , (1,1,1) , (0,0,-1) , (-1,-1,-1)] }
Which contains the following subgraph which is isomorphic to X



Conflict of Interests.
There are non-conflicts of interest .

4. References

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الخلاصة

في هذا البحث وجدت خوارزمية لبناء اشكال كيلي التي تكون (n−1) مونو مورفي وتحوي اشكال مهما كانت اصغر منها من حيث عدد العناصر.