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Studying $^{124\text{-}128}$Ba Transitional nuclei in the *IBM* Framework

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Abstract

Many properties of nuclear structure for even-even $^{124-128}_{56}Ba$ isotopes have been investigated with $IBM-1$ and $IBM-2$ .They examined carefully with benefit from continuously updating of nuclear decay schemes. Low-lying energy levels for even-even positive parity states, mixed symmetry states (MSS), reduced electric quadruple transitions probabilities B(E2), branching ratios, reduced magnetic dipole transitions probability B(M1), $\delta(E2/M1)$ ratio, reduced electric monopole transitions probability B(E0) and mixing ratio $X(E0/E2)$ and potential energy surface have been studied.

In the framework of $IBM-1$ noted the competition between the two parameters (a_0 and a_2) in $^{124\text{-}128}$Ba isotopes were the increases of a_0 associated with decreases of a_2 which mean that the γ-unstable features were continuous increases with opposite of rotational properties , in IBM-2 χ_π and χ_ν were (-1.3 and around 0.7) which signified similarity with IBM-1 expected. $^{124\text{-}128}$Ba isotopes described to be transitional nuclei , transitional between SU(3) and O(6) features.

Key word: Nuclear structure ,$^{124\text{-}128}$Ba isotopes, transitional nuclei, and Mixed Symmetry State.

1. Introduction

The interacting boson model (IBM) is based on the well-known shell model and on geometrical collective model of the atomic nucleus[1-3]. It is suitable for describing the structure of intermediate and heavy nuclei. In addition, it is of considerable theoretical interest since it shows the dynamical symmetries of the nuclei[4,5], which are made visible using lie algebra. The (IBM) was developed by Iachello and Arima [5-7]. According to (IBM) postulates, the nuclear pairs are represented by bosons with angular momentum $L = 0$ or $L = 2$, i.e. the number of bosons relies on the number of active nuclear particle number (or hole) of the pairs outside a closed shell. While the total boson number (N) is calculated by addition of the partial numbers i.e. $N = N_\pi + N_\nu$, where N_π and N_ν are the number of proton and neutron bosons respectively. There are many studies that attempted to explain the behavior of Barium nuclei by using different models[8-14]. In this research many properties of nuclear structure for even-even [124-128]Ba isotopes have been investigated with $IBM-1$ and $IBM-2$ they have been examined carefully with benefit from continuously updating of nuclear decay schemes.

2 .The Interacting Boson Model

In the $IBM1$, it is assumed that the Hamiltonian operator contains only one body and two body terms thus, introducing creation $(s^\dagger, d_m^\dagger)$ and annihilation (s, d_m) operators where the index $m = 0, \pm1, \pm2$. The most general Hamiltonian can be written as[15]:-

$$H = \varepsilon_s(s^\dagger s) + \varepsilon_d \sum_m d_m^\dagger d_m + \sum_{L=0,2,4} \frac{1}{2}(2L+1)^{\frac{1}{2}} C_L\left[(d^\dagger d^\dagger)^{(L)}.(dd)^L\right]^{(0)} + \frac{1}{\sqrt{2}}\upsilon_2\left[(d^\dagger d^\dagger)^{(2)}.(ds)^{(2)} + (d^\dagger s^\dagger)^{(2)}.(dd)^{(2)}\right]^{(0)} + \frac{1}{2}\upsilon_0\left[(d^\dagger d^\dagger)^{(0)}.(ss)^{(0)} + (s^\dagger s^\dagger)^{(0)}.(dd)^{(0)}\right]^{(0)} + \mathrm{u}_2\left[(d^\dagger s^\dagger)^{(2)}.(ds)^2\right]^{(0)} + \frac{1}{2}u_0\left[(s^\dagger s^\dagger)^{(0)}.(ss)^{(2)}\right]^{(0)} \quad (1)$$

Where $C_L(L = 0,2,4), \upsilon_L(L = 0,2), u_L(L = 0,2)$ describe the boson interaction.
The most commonly used form of the IBM-1 Hamiltonian is[16]:-

$$H = \varepsilon n_d + a_0 P^\dagger P + a_1 L.L + a_2 Q.Q + a_3 T_3 T_3 + a_4 T_4 T_4 \quad (2)$$

Where $\varepsilon = \varepsilon_d - \varepsilon_s$ is the boson energy, for simplicity $\varepsilon_s$was set equal to zero only $\varepsilon = \varepsilon_d$ appears , and a_0, a_1, a_2, a_3, and a_4 designate the strengths of the quadrupole, angular momentum, pairing, octupole and hexadecapole interacting between bosons respectively. The five component of d boson and the single component of the s boson extend across a six dimentional space and for a fixed number of boson N, the group structure of the problem is that of $U(6)$. Considering the different reductions of $U(6)$, three dynamical symmetries emerge, namely $U(5)$, $SU(3)$ and $U(6)$; these symmetries are related to the geometrical idea of the spherical vibrator, deformed rotor and a symmetric (γ −soft) deformed rotor, respectively[17-19]. In order to calculate transition rates the simplest form of $IBM-1$ the one body transition operator has been given as follows[16-19]:-.

$$T_m^l = \alpha_2 \delta_{l2}[d^\dagger s + s^\dagger d]_m^{(2)} + \beta_l [d^\dagger d]_m^{(l)} + \gamma_0 \delta_{l0} \delta_{m0} [s^\dagger s]_0^{(0)} \quad (3)$$

where $\alpha_2, \beta_l, \gamma_0$are the coefficients of the various terms in the operators. The three, corresponding dynamical symmetries group chains of $U(6)$can be written as [16-19]:-

$$I - U(6) \supset SU(5) \supset O(5) \supset O(3) \supset O(2)$$
$$II - U(6) \supset SU\,(3) \supset O(3) \supset O(2)$$
$$III - U(6) \supset O(6) \supset O(5) \supset O(3) \supset O(2) \quad (4)$$

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The branching ratio for three, corresponding dynamical symmetries have been written as[15]:-

$$R=\frac{B(E2;4_1^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)};R'=\frac{B(E2;2_2^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)};R''=\frac{B(E2;0_2^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)};R=R'=R''=\frac{2(N-1)}{N}<2\ in\ U(5)$$

$$R=\frac{B(E2;4_1^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)}=\frac{10(N-1)(2N+5)}{7N(2N+3)}\approx\frac{10}{7};R'=\frac{B(E2;2_2^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)};R''=\frac{B(E2;0_2^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)}\Rightarrow R'=R''=0\ in\ SU(3)$$

$$R=\frac{B(E2;4_1^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)};R'=\frac{B(E2;2_2^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)};R''=\frac{B(E2;0_2^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)},R=R'=\frac{10}{7}\frac{(N-1)(N+5)}{N(N+4)}<\frac{10}{7};R''=0\ \ in\ O(6) \quad (5)$$

The quadrupole moments, in $U(5)$, $SU(3)$ and $O(6)$ limits defined as

$$Q_{2_1}^+=\beta_2\sqrt{16\pi/5}\sqrt{2/7},Q_{2_1}^+=-\alpha_2\sqrt{2\pi/5}\,2/7(4N+3)\ \text{and}\ Q_{2_1}^+=0 \quad (6)$$

$IBM-2$ assumes proton and neutron bosons to have an intrinsic quantity, called $F-$spin, of value $F=1/2$ [16-20] which can be introduced in a formalism similar to that used for isotopic spin. Then proton bosons will have a $Z-$projection $F_Z=+1/2$ while neutron bosons have$F_Z=-1/2$, thus one can write

$$|\pi\rangle=\left|\frac{1}{2},+\frac{1}{2}\right\rangle,\ |\nu\rangle=\left|\frac{1}{2},-\frac{1}{2}\right\rangle \quad (7)$$

The underlying group structure to $F-$spin is $SU(2)$. The three generators can be written explicitly as $F_+=d_\pi^\dagger d_\nu+s_\pi^\dagger s_\nu$, $F_-=d_\nu^\dagger d_\pi+s_\nu^\dagger s_\pi$ and its zero-component

$$F_0=\frac{1}{2}(d_\pi^\dagger d_\pi+s_\pi^\dagger s_\pi-d_\nu^\dagger d_\nu+s_\nu^\dagger s_\nu) \quad (8)$$

From the last equation it follows that F_Z is always a good quantum number for a given nucleus .A state with $N=N_\pi+N_\nu$ bosons is fully symmetric under the interchange of neutron and proton boson if it has maximal $F-$spin

$$F_{max}=\left|\frac{N_\pi+N_\nu}{2}\right|\rightarrow\left|\frac{N}{2}\right|,F=F_{max}-1,F_{max}-2,\dots,F_{min}=\left|\frac{N_\pi-N_\nu}{2}\right| \quad (9)$$

While the mixed symmetry states characterized by decreasing $F-$spin value. A state with only s- boson is naturally fully symmetric and has $F=N/2$ it should be noted that the state of maximum $F-$spin are in one to one correspondence with the states of $IBM-1$ while the states with $F-$spin less than the maximum value of $N/2$ have no counterparts in $IBM-1$. They have mixed proton-neutron symmetry character, thus they are called mixed symmetry states. These states are considered as the main interest in studying the $IBM-2$.

A simple schematic Hamiltonian guided by microscopic consideration is given by [15,16, 21]

$$H=\varepsilon(n_{d\pi}+n_{d\nu})+\kappa Q_\pi.Q_\nu+V_{\pi\pi}+V_{\nu\nu}+M_{\pi\nu} \quad (10)$$

$\varepsilon_\pi, \varepsilon_\nu$ are proton and neutron energy respectively, they are assumed equal $\varepsilon_\pi = \varepsilon_\nu = \varepsilon$. The last term in eq. (10) contains the Majorana operator $M_{\pi\nu}$ and it is usually added in order to remove states of mixed proton neutron symmetry. this term can be written as [15,22,23]

$$M_{\pi\nu} = \zeta_2 (s_\nu^\dagger d_\pi^\dagger - d_\nu^\dagger s_\pi^\dagger)^{(2)}.(s_\nu d_\pi - d_\nu s_\pi)^{(2)} + \sum_{k=1,3} \zeta_k (d_\nu^\dagger d_\pi^\dagger)^{(k)} - (d_\nu d_\pi)^{(k)} \quad (11)$$

If there is experimental evidence for so called mixed symmetry state then the Majorana parameter are varied to fix the location of these states in the spectrum. The general single boson transition operator of angular momentum ℓ has the same form as in eq.(3) in $IBM-1$ except the fact that in each term one has to consider π, ν degree of freedom and this can be written as[24-27]:-

$$T^{(l)} = \alpha_{2\rho}\delta_{\ell 2}[d^\dagger s + s^\dagger d]_\rho^{(2)} + \beta_{\ell\rho}[d^\dagger d]_\rho^{(\ell)} + \gamma_{0\rho}\delta_{\ell 0}[s^\dagger s]_\rho^{(0)} \ldots \rho = \pi \text{ or } \nu \quad (12)$$

For $E2$ operator[15,26]

$$T^{E2} = e_\pi Q_\pi + e_\nu Q_\nu \quad (13)$$

Where Q_ρ is the same as in eq.(10) and e_π, e_ν are boson effective charges depending on the boson number N and they can take any value to fit the experimental result. The quadruple moment of 2_1^+ can be in $IBM2$ defined as:-

$$Q_{2_1^+} = \langle L, M=2 | \hat{Q} | L, M=2 \rangle \quad (14)$$

$$Q_{2_1^+} = \sqrt{16\pi/5}\sqrt{L(2L+1)B(E2)\uparrow/(2L+1)(L+1)(2L+3)} \quad (15)$$

The $M1$ operator obtained by letting $\ell = 1$ in eq. (12) is written [15,25] as

$$T^{(M1)} = [\tfrac{3}{4\pi}]^{\frac{1}{2}}(g_\pi L_\pi^{(1)} + g_\nu L_\nu^{(1)}) \quad (16)$$

Where g_π , g_ν are the boson g −factor in unit of $\mu_N$and $L_\rho^{(1)} = \sqrt{10}(d^\dagger d)_\rho^{(1)}$, this operator can be written alternatively as:-

$$T^{(M1)} = [\tfrac{3}{4\pi}]^{\frac{1}{2}}[\tfrac{1}{2}(g_\pi + g_\nu)(L_\pi^{(1)} + L_\nu^{(1)}) + \tfrac{1}{2}(g_\pi - g_\nu)(L_\pi^{(1)} - L_\nu^{(1)})] \quad (17)$$

The first term on the right hand side of this equation is diagonal; therefore, for $M1$ transition the previous equation may be written as:-

$$T^{(M1)} = 0.77[(d^\dagger d)_\pi^{(1)} - (d^\dagger d)_\nu^{(1)}](g_\pi - g_\nu) \quad (18)$$

The M1 strength many be expressed in terms of the multipole mixing

$$\delta\left(\frac{E2}{M1}\right) = 0.835 E_\gamma(MeV).\Delta \quad \text{where} \quad \Delta = \frac{\langle J_f \| T^{(E2)} \| J_i \rangle}{\langle J_f \| T^{(M1)} \| J_i \rangle} \quad (19)$$

The $E0$ reduced transitions probability is expressed as [28].

$$B(E0; I_i \to I_f) = e^2 R^2 \rho^2(E0) \quad (20)$$

Where e indicates the electronic charge, R stands for the nuclear radius and ρ (E0) is the matrix element transition which is gotten by [28]:-

$$\rho(E0) = Z/R^2 \sum \tilde{\beta}_{0\rho} \langle f | d_\rho^\dagger \times d_\rho | i \rangle, \rho = \pi, \nu \quad (21)$$

where $\beta_{o\pi}$, $\beta_{o\nu}$ represent the deformation parameters for (protons and neutrons)

The X(E0/E2) ratio can be calculated as follows [28]:

$$X(E0/E2) = \frac{B(E0; I_i \to I_f)}{B(E2; I_i \to I_{f'})} \quad (22)$$

Where $I_f = I_{f'}$ for $I_i = I_f \neq 0$, and $I_{f'} = 2$ for $I_i = I_f = 0$.

This ratio is so essential as it mirrors to what extent the transition between $B(E2)$ and $B(E0)$ is strong.

The general formula for the potential energy surface as a function of geometrical variables (β) and γ is given by [15,16] :-

$$V(\beta,\gamma) = \frac{N(\varepsilon_s+\varepsilon_d\beta^2)}{1+\beta^2} + \frac{N(N+1)}{(1+\beta^2)^2}(\alpha_1\beta^4 + \alpha_2\beta^3 Cos3\gamma + \alpha_3\beta^2 + \alpha_4) \quad (23)$$

$$\text{with } \alpha_1 = \frac{C_0}{10} + \frac{C_2}{7} + \frac{9C_4}{35}, \alpha_2 = -\sqrt{\frac{8}{35}}\upsilon_2,\ \alpha_3 = \frac{(\upsilon_0+u_2)}{\sqrt{5}}, \alpha_4 = u_0 \quad (24)$$

where N is the total boson number β is the quadrupole deformation parameter operator from $\beta = 0 - 2.4$. γ is the distortion parameter operator or (asymmetry angle) for $0^\circ \leq \gamma \leq 60^\circ$. The variables $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are related to the parameters in eq.(1). The energy surfaces has minima only at $\gamma = 0^\circ$ and 60° these expressions give for large N, $\beta_{min} = 0, \sqrt{2}, and$ 1 for $U(5), SU(3)$ and $O(6)$ respectively.

3. Results and Discussion

The $^{124\text{-}128}$Ba isotopes, have $Z = 56$, then 3 particle bosons the number of protons and neutrons lying between 50 , 82 and 82, 126 magic shells, respectively, $^{124\text{-}128}$Ba have 68-72 neutrons which mean (7-5) hole neutron bosons respectively, The parameters estimated for the calculations of the levels of low- lying excited energy for Barium isotopes have been given the table (1).

Table 1: The parameters have been used in the $IBM-1$ and $IBM-2$ Hamiltonian for even-even $^{124\text{-}128}$Ba isotopes (in MeV) except χ , χ_ν and χ_π were unit less.

Isotopes	$IBM-1$ parameters in MeV unless χ							
	N_ρ	ε	a_0	a_1	a_2	a_3	a_4	χ
^{124}Ba	10	0.0	0.018	0.016	-0.025	0.04	0.0	-0.4
^{126}Ba	9	0.0	0.019	0.016	-0.028	0.04	0.0	-0.4
^{128}Ba	8	0.0	0.02	0.0172	-0.031	0.041	0.0	-0.42

Isotopes	IBM-2 parameters in MeV unless χ_ν , χ_π= -1.3 , N_π =3						
	N_ν	ε_d	κ	χ_ν	ζ_2	$\zeta_{1,3}$	C_ν^L	C_π^L
^{124}Ba	7	0.59	-0.14	0.8	0.03	0.02	-0.57, 0.1, 0.0	-0.74, 0.1 ,0.0
^{126}Ba	6	0.61	-0.18	0.7	0.03	0.03	-0.68, 0.15, -0.08	-0.6,0.15, 0.0
^{128}Ba	5	0.61	-0.17	0.7	0.04	0.03	-0.8,0.15,-0.09	-0.6,0.15,0.0

Experimental and theoretical energy ratios ($E4_1^+/E2_1^+$), ($E6_1^+/E2_1^+$) and ($E8_1^+/E2_1^+$) [15] compared to the standard ratios for $U(5)$, $SU(3)$, and $O(6)$ limits have been calculated for $^{124\text{-}128}$Ba isotopes shown in figure (1) as a function of mass numbers.

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The calculated energy levels comparison with experimental data [29-36] for $^{124-128}$Ba isotopes illustrated in figures from (2) to (4) .

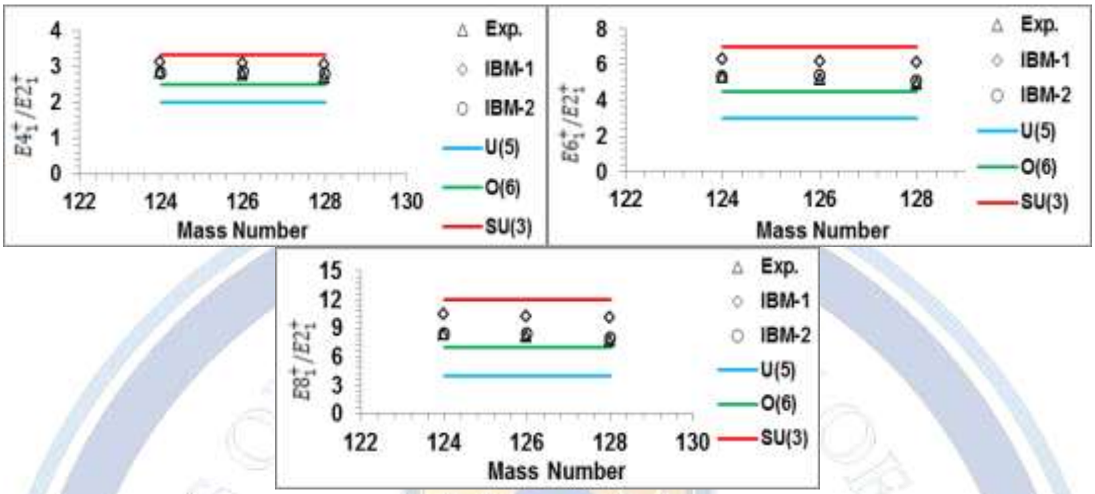

Figure 1:The experimental [29-36], theoretical and standard [15] energy ratios ($E4_1^+/E2_1^+$,$E6_1^+/E2_1^+$, and $E8_1^+/E2_1^+$) respectively as a function of mass numbers for even –even $^{124-128}$Ba isotopes.

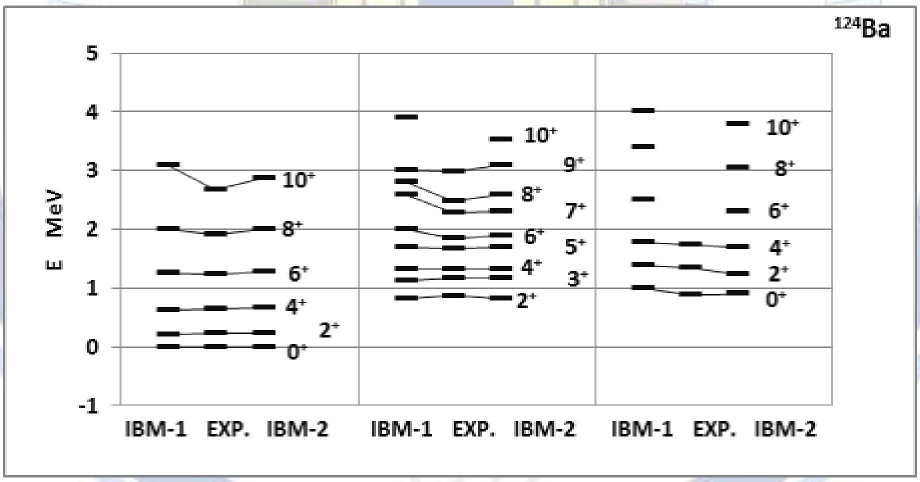

Figure 2: Comparison between experimental [34] and calculated energy levels for ^{124}Ba isotope.

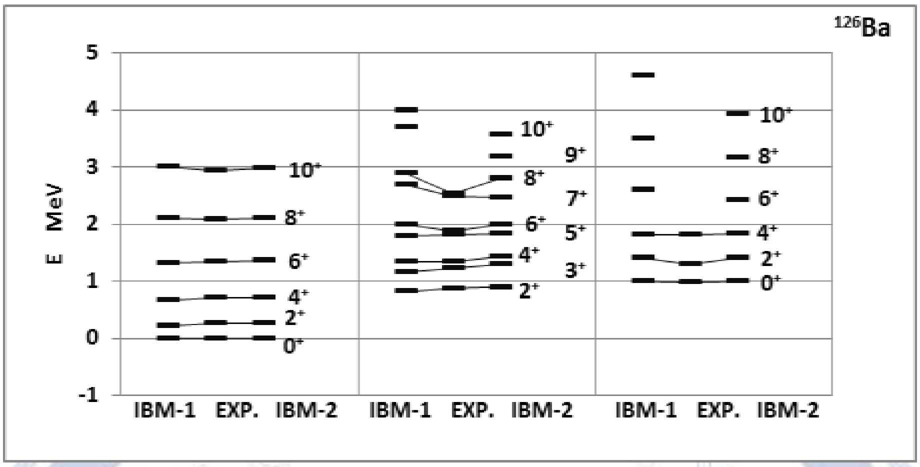

Figure 3: Comparison between experimental [35] and calculated energy levels for ^{126}Ba isotope.

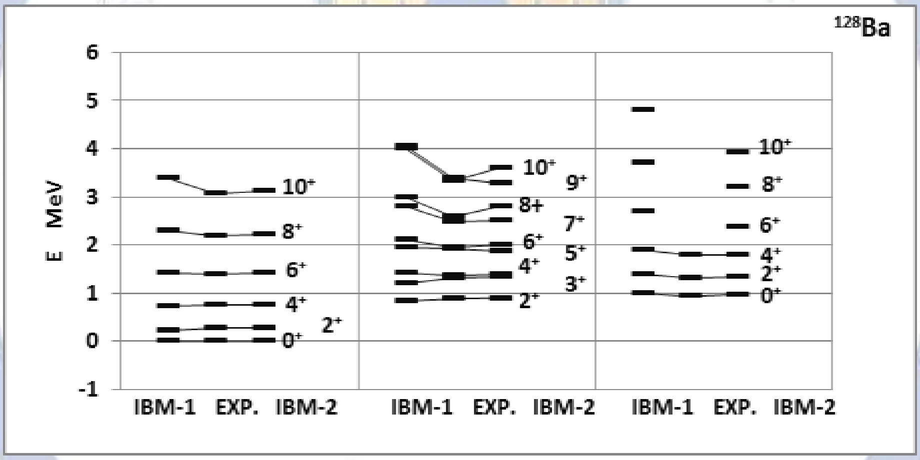

Figure 4: Comparison between experimental [36] and calculated energy levels for ^{128}Ba isotope.

The effect of the Majorana parameters ($\zeta_{1,3,}\zeta_2$) on the level of the calculated excitation energy for $^{124-128}$Ba isotopes have been conducted for all isotopes; this effect varies the ζ_2 around the best-fitted with experimental data for the states $(2_3^+, 2_4^+, 2_5^+, 3_1^+, 5_1^+, 1_1^+)$. The energy variation of these states is stated in figure (5) as a function of the Majorana parameter ζ_2.

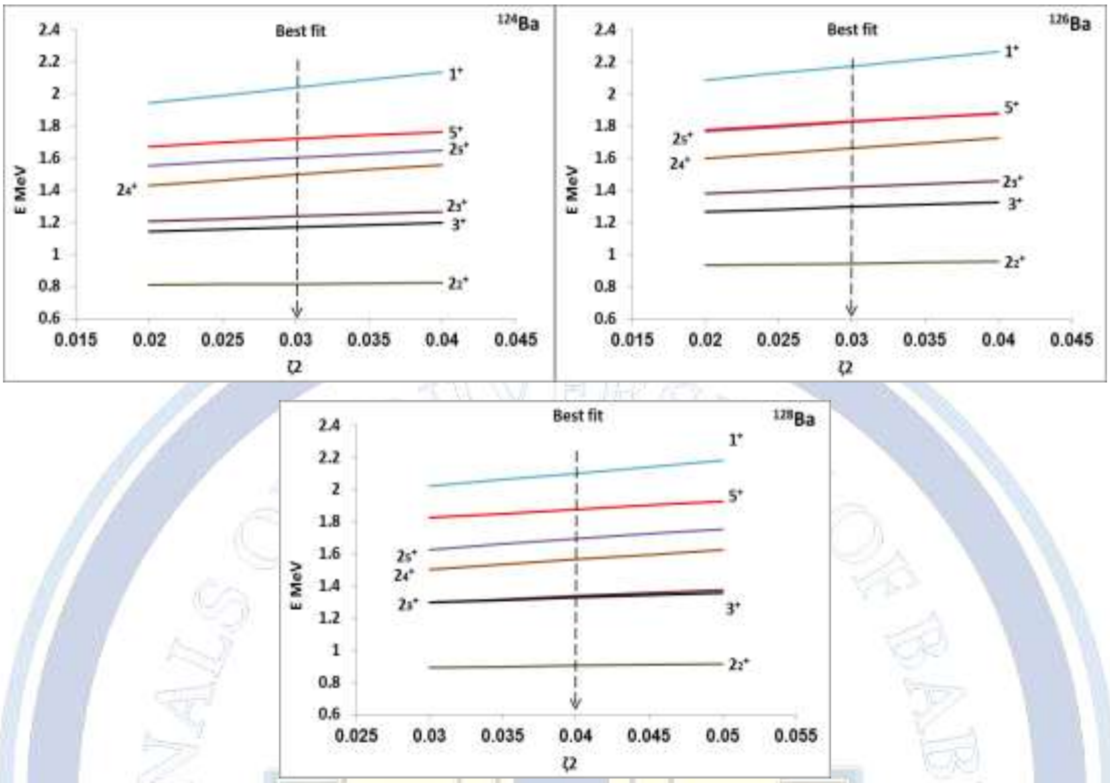

Figure (5) Mixed symmetry states (MSS) in even- even $^{124\text{-}128}$Ba isotopes.

Effective boson charges used in $IBM-1$ and $IBM-2$ for electric quadruple transitions and calculated branching ratios listed in table (2) have been compared with available experimental data[29-36]. Therefore calculated reduced electric quadruple transitions probability $B(E2)$, and electric quadrupole moment of 2_1^+ state comparison with the experimental values [29-36] for $^{124\text{-}128}$Ba isotopes listed in tables (3).

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Table 2: The effective boson charges used in $IBM-1$ and $IBM-2$ to calculate B(E2) transition and comparisons between calculated branching ratios with experimental data[29-36] for even-even $^{124\text{-}128}$Ba.

Isotopes	The effective boson charges (eb)			
	$IBM-1$		$IBM-2$	
	$E2SD$	$E2DD$	e_ν	e_π
^{124}Ba	0.1028	-0.127	0.037	0.27
^{126}Ba	0.1046	-0.126	0.035	0.25
^{128}Ba	0.1032	-0.123	0.029	0.23

Isotopes	R			R'			R''		
	Exp.	IBM-1	IBM-2	Exp.	IBM-1	IBM-2	Exp.	IBM-1	IBM-2
^{124}Ba	--	1.416	1.366	--	0.0861	0.01	--	0.004	0.0038
^{126}Ba	1.335	1.392	1.36	--	0.1122	0.133	--	0.0044	0.005
^{128}Ba	1.28	1.4	1.362	--	0.134	0.147	--	0.0046	0.007

Table 3:Calculated reduced electric quadruple transitions probability $B(E2)$ and electric quadrupole moment of 2_1^+ state comparison with the experimental values [29-36] for $^{124\text{-}128}$Ba isotopes.

Isotopes	$B(E2)$ values in (e^2b^2) ^{124}Ba			$B(E2)$ values in (e^2b^2) ^{126}Ba		
$J_i^+ \rightarrow J_f^+$	$Exp.$	$IBM-1$	$IBM-2$	$Exp.$	$IBM-1$	$IBM-2$
$2_1 \rightarrow 0_1$	0.418	0.418	0.416	0.352	0.352	0.351
$4_1 \rightarrow 2_1$	--	0.592	0.569	0.472	0.49	0.479
$6_1 \rightarrow 4_1$	--	0.63	0.62	0.55	0.52	0.56
$8_1 \rightarrow 6_1$	--	0.62	0.66	0.63	0.509	0.58
$10_1 \rightarrow 8_1$	--	0.58	0.56	0.606	0.466	0.603
$0_2 \rightarrow 2_1$	--	0.00168	0.0022	--	0.00156	0.002
$2_2 \rightarrow 0_2$	--	0.039	0.016	--	0.03	0.0185
$2_2 \rightarrow 2_1$	--	0.36	0.53	--	0.039	0.04
$4_2 \rightarrow 2_2$	--	0.195	0.2	--	0.164	0.198
$3_1 \rightarrow 2_2$	--	0.458	0.5	--	0.3699	0.49
$3_1 \rightarrow 4_3$	--	0.13	0.12	--	0.13	0.138
$5_1 \rightarrow 3_1$	--	0.27	0.35	--	0.227	0.29
$7_1 \rightarrow 5_1$	--	0.35	0.437	--	0.284	0.3
$9_1 \rightarrow 7_1$	--	0.364	0.434	--	0.279	0.34
$4_2 \rightarrow 4_1$	--	0.032	0.04	--	0.033	0.055
$3_1 \rightarrow 1_1$	--	--	0.00404	--	0.008	--

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$1_1 \rightarrow 2_1$	--	--	0.053	--	0.042	--
$1_1 \rightarrow 2_2$	--	--	0.00025	--	0.00056	--
$Q_{2_1^+}(eb)$	--	-1.69	-1.53	--	-1.5	-1.449

Isotopes	B(E2) values in (e^2b^2) ^{128}Ba						
$J_i^+ \rightarrow J_f^+$	Exp.	IBM-1	IBM-2	$J_i^+ \rightarrow J_f^+$	Exp	IBM-1	IBM-2
$2_1 \rightarrow 0_1$	0.275	0.275	0.71	$3_1 \rightarrow 4_3$	--	0.109	0.104
$4_1 \rightarrow 2_1$	0.352	0.385	0.369	$5_1 \rightarrow 3_1$	--	0.169	0.22
$6_1 \rightarrow 4_1$	0.38	0.402	0.372	$7_1 \rightarrow 5_1$	--	0.204	0.27
$8_1 \rightarrow 6_1$	0.309	0.38	0.34	$9_1 \rightarrow 7_1$	--	0.189	0.24
$10_1 \rightarrow 8_1$	0.32	0.33	0.314	$4_2 \rightarrow 4_1$	--	0.029	0.036
$0_2 \rightarrow 2_1$	--	0.0012	0.002	$3_1 \rightarrow 1_1$	--	0.0083	--
$2_2 \rightarrow 0_2$	--	0.03	0.164	$1_1 \rightarrow 2_1$	--	0.035	--
$2_2 \rightarrow 2_1$	--	0.037	0.054	$1_1 \rightarrow 2_2$	--	0.0004	--
$4_2 \rightarrow 2_2$	--	0.1267	0.15	$Q_{2_1}^+$ (eb)	--	-1.35	-1.22
$3_1 \rightarrow 2_2$	--	0.275	0.3				

To calculate $B(M1)$ transition probability the effective g −factors for proton g_π and neutron g_ν calculated for for even -even $^{124\text{-}128}$Ba isotopes were $g_\pi = (0.4\mu_N)$ and $g_\nu = (0.34\mu_N)$ equations (18) have been used in $IBM-2$ to calculate the $B(M1)$ transition probabilities as it is shown in table (4). The calculation values for $B(M1)$ and mixing ratio $\delta(E2/M1)$ were compared with the available experiments data [29-36].

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Table 4: Comparison between calculated magnetic dipole transitions $B(M1)$ in(μ_N^2) and mixing ratio $\delta(E2/M1)$ with the available experimental data [29-36] for even - even $^{124-128}$Ba isotopes.

Isotopes	^{124}Ba				^{126}Ba			
$J_i^+ \rightarrow J_f^+$	$B(M1)\ \mu_N^2$		$\delta(E2/M1)$		$B(M1)\ \mu_N^2$		$\delta(E2/M1)$	
	$Exp.$	$IBM-2$	$Exp.$	IBM	$Exp.$	$IBM-2$	$Exp.$	$IBM-2$
$1_1 \rightarrow 2_1$	--	0.000462	--	--	--	0.0005	--	--
$1_1 \rightarrow 2_2$	--	0.000218	--	--	--	0.00027	--	--
$1_1 \rightarrow 2_3$	--	4.21×10^{-6}	--	--	--	3.23×10^{-6}	--	--
$2_2 \rightarrow 2_1$	--	0.000135	--	10.61	--	0.00017	$+5^{+2}_{-1}$	+8.33
$2_2 \rightarrow 2_3$	--	4.77×10^{-6}	--	--	--	6.73×10^{-6}	--	--
$2_1 \rightarrow 2_3$	--	2.93×10^{-7}	--	--	--	9.14×10^{-6}	$+1.9^{+11}_{-9}$	+6.16
$3_1 \rightarrow 2_1$	--	9.83×10^{-5}	--	--	--	0.00015	$+5.5^{+11}_{-9}$	-1.53
$3_1 \rightarrow 2_2$	--	0.0002	--	--	--	0.000217	-1.7	-2.34
$4_1 \rightarrow 3_1$	--	0.000173	--	--	--	0.00022	--	+6.97
$4_2 \rightarrow 4_1$	--	0.000419	-0.15^{+25}_{-20}	5.68	--	0.0006	$+1.4^{+80}_{-3}$	+5.06
$5_1 \rightarrow 4_1$	--	0.000199	--	-6.48	--	0.000313	--	--
$6_2 \rightarrow 6_1$	--	0.00066	--	5.35	--	0.0009	$+2.8^{+24}_{-9}$	+3.48

Isotopes	^{128}Ba								
$J_i^+ \rightarrow J_f^+$	$B(M1)\ \mu_N^2$		$\delta(E2/M1)$		$J_i^+ \rightarrow J_f^+$	$B(M1)\ \mu_N^2$		$\delta(E2/M1)$	
	$Exp.$	$IBM-2$	$Exp.$	IBM		$Exp.$	$IBM-2$	$Exp.$	IBM
$1_1 \rightarrow 2_1$	--	0.000125	--	--	$3_1 \rightarrow 2_1$	--	3.93×10^{-5}	$+4^{+2}_{-1}$	+7.27
$1_1 \rightarrow 2_2$	--	6.95×10^{-5}	--	--	$3_1 \rightarrow 2_2$	--	5.437×10^{-5}	--	-8.21
$1_1 \rightarrow 2_3$	--	8.07×10^{-7}	--	--	$4_1 \rightarrow 3_1$	--	5.726×10^{-5}	$+3.7^{+25}_{-12}$	+11.53
$2_2 \rightarrow 2_1$	5.58×10^{-5}	4.47×10^{-5}	$+13^{+16}_{-4}$	+17.3	$4_2 \rightarrow 4_1$	0.00027	0.00015	$+14^{+8}_{-16}$	+9.87
$2_2 \rightarrow 2_3$	--	1.68×10^{-5}	--	--	$5_1 \rightarrow 4_1$	--	7.83×10^{-5}	--	--
$2_1 \rightarrow 2_3$	--	2.28×10^{-6}	--	--					

The deformation parameters for protons and neutrons have been used to calculate monopole transition matrix elements $B(E0)$ for $^{124-128}$Ba isotopes are ($\beta_{o\pi} = 0.053 fm^2, \beta_{ov} = -0.052 fm^2$). The monopole transition matrix element $B(E0)$and

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mixing ratio $X(E0/E2)$ have been calculated using eq.(21,22). Calculation values of monopole transition matrix element $B(E0)\ e^2b^2$ and $X(E0/E2)$ listed in table (5).

Table 5: Compression between calculated monopole transition matrix element $B(E0)$ e^2b^2 and $X(E0/E2)$ with the evaluable experimental data [34,37] for even -even $^{124-128}$Ba isotopes.

Isotopes	^{124}Ba			^{126}Ba		
$J_i^+ \to J_f^+$	$B(E0)e^2b^2$	$X(E0/E2)$		$B(E0)e^2b^2$	$X(E0/E2)$	
	$IBM-2$	$IBM-2$	$Exp.$	$IBM-2$	$IBM-2$	$Exp.$
$0_2 \to 0_1$	0.038	0.0913	0.09	0.039	0.112	--
$0_3 \to 0_1$	0.0031	0.00766	--	0.00075	0.00213	--
$0_3 \to 0_2$	0.00206	0.0049	--	0.000117	0.00033	--
$2_2 \to 2_1$	0.0803	1.5	--	0.0918	1.94	--
$2_3 \to 2_1$	0.000465	0.12	--	0.0023	5.15	--
$2_2 \to 2_3$	0.0145	1.35	--	0.00209	0.37	--
$4_2 \to 4_1$	0.0113	0.26	--	0.164	2.95	--

Isotopes	^{128}Ba						
$J_i^+ \to J_f^+$	$B(E0)e^2b^2$	$X(E0/E2)$		$J_i^+ \to J_f^+$	$B(E0)e^2b^2$	$X(E0/E2)$	
	$IBM-2$	$IBM-2$	$Exp.$		$IBM-2$	$IBM-2$	$Exp.$
$0_2 \to 0_1$	0.0349	0.128	--	$2_3 \to 2_1$	0.51	41.2	--
$0_3 \to 0_1$	0.0027	0.01	--	$2_2 \to 2_3$	0.133	27.04	--
$0_3 \to 0_2$	9.93×10^{-6}	3.6×10^{-5}	--	$4_2 \to 4_1$	1.28	35.2	--
$2_2 \to 2_1$	0.14	2.68	--				

The potential energy surface as a function to β with contour diagrams for $^{124-128}$Ba isotopes which calculated from equation (23) with $IBMP$ computer code represented in figure (6).

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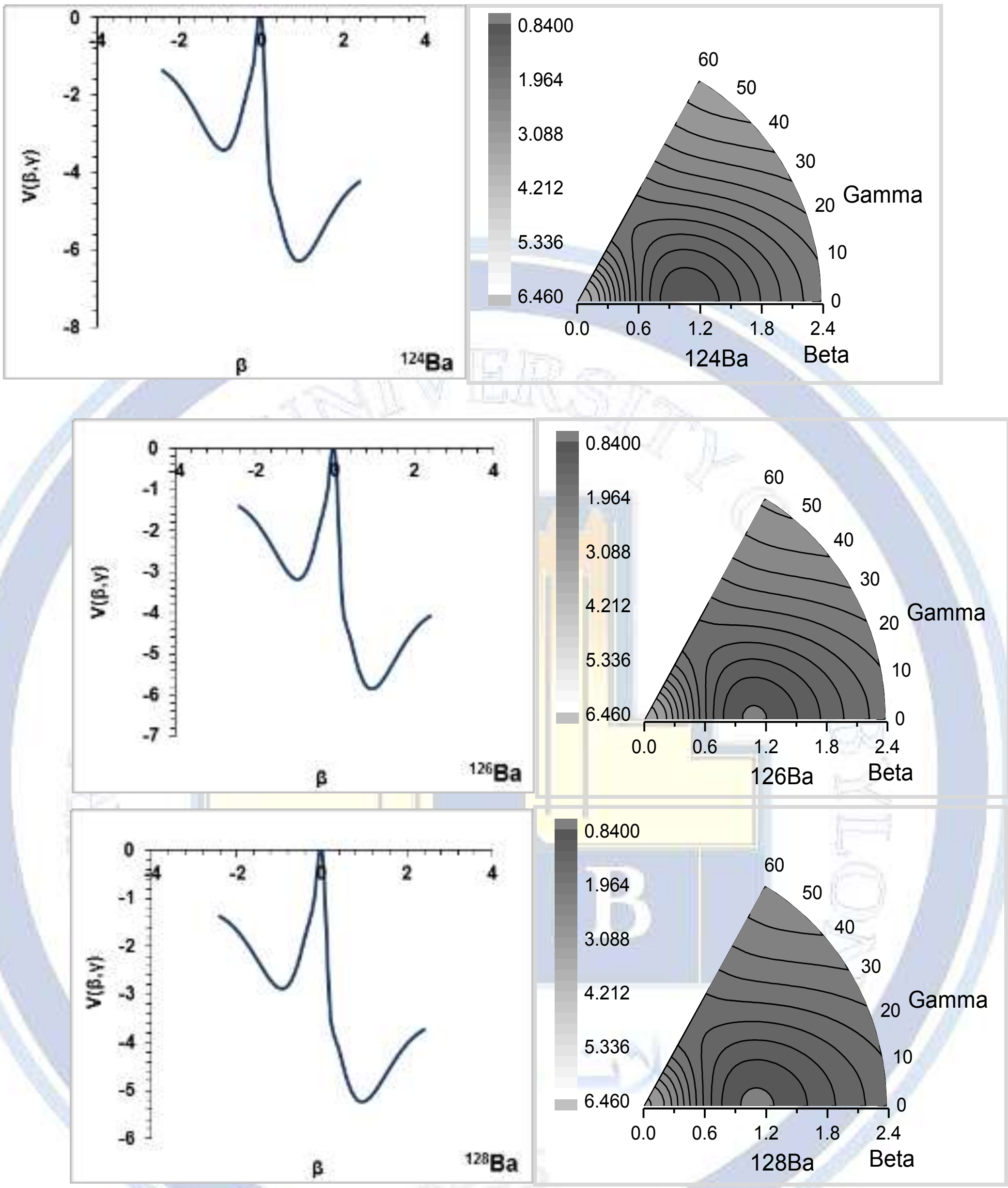

Figure 6: Potential energy surface as a function to β with contour diagrams for even -even $^{124-128}$Ba isotopes

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4. Conclusion

Barium nuclei always have been a great interest the evidence of structure evolutions from weakly to well deformation or γ soft shapes as a function of mass number. The interacting boson model has been applied to calculated barium isotopes due to flexibility of the model to describe not only the exact symmetries corresponding to different geometrical models but also transitional cases within the same framework. The continuous updating of the decay schemes and the experimental information related to the barium nuclei have been presented in restudying these isotopes and rearranging them according to the dynamic symmetry, $^{124-128}$Ba isotopes where the researchers disagreed about their characteristics, some of them describe them as transitional characteristics between vibrational $U(5)$ and γ −unstable $O(6)$, and the others described as pure γ −unstable nuclei. In the framework of $IBM-1$ the competition is noted between the two parameters (a_0 and a_2) in $^{124-128}$Ba isotopes where the increases of a_0 associated with decreases of a_2. This mean that the γ −unstable features are continuous increases with opposite of rotational properties, in $IBM-2$, χ_π and $\chi_\nu$are (-1.3 and around 0.7), signifying the similarity with $IBM-1$ expected as tabled in table (1).The experimental energy ratios ($E4_1^+/E2_1^+$),($E6_1^+/E2_1^+$ and $E8_1^+/E2_1^+$) have been sloped downward from higher values in ^{124}Ba (2.83,5.33 and 8.36) which seemed as transitions between rotational and γ soft nuclei to the lowest values in ^{128}Ba (2.68, 4.95 and 7.70) as diagrammed in figure (1). The Majorana parameter effect (ζ_2) on the calculated excitation energy level for $^{124-128}$Ba isotopes, have been investigated by vary the ζ_2 around the best-fitted. The lowest mixed state was J=2^+, 2_2^+ state staying conservative to its value in all isotopes denoted that this states may have a full symmetry. We cannot compare the calculated value of 1^+ state with experimental value due to data lack. The calculated energy of scissor state 1^+ of $^{124-128}$Ba is (2.039, 2.173 and 2.102) MeV respectively which are close to observed energy of 1^+ state of the other neighbored nuclei .The another states (2_3^+,2_4^+,3_1^+,5_1^+ and 1_1^+) in $^{124-128}$Ba isotopes were slowly increased with ζ_2 as shown in figure (5). In tables (2)–(5) listed the calculated branching ratios, $B(E2)$, $B(M1)$ and $B(E0)$ depended on comparisons with fewer experimental data; however, the results are in accept agreement. The electric quadrupole moment $Q_{2_1}^+$ in $IBM-1$ and $IBM-2$ ranged from (-1.69 and -1.53) (eb) for ^{124}Ba to (-1.35and -1.22)(eb) for ^{128}Ba indicated a reduced rotational index .In some places of the nuclear chart, the shape is very sensitive to structural effects; this is also able to alter from one specific nucleus to its neighbor. Moreover, the shape can also take different form with angular momentum or excitation energy within the same nucleus shape besides changes with proton or neutron number. These changes are happened by the orbital configuration rearrangement of the nucleons or by the nuclear system dynamic response to rotation. In some cases, configurations conforming to the various shapes coexist at similar energies as illustrated in the potential energy $V(\beta,\gamma)$, where it is calculated as a function of β and γ deformations parameters for $^{124-128}$Ba isotopes, diagrammed in figure (6) that shows the transition behavior in 124-

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^{128}Ba isotopes where $V(\beta,\gamma)$ with minimum β values approaching from (1) in $\gamma = 0^o$ side equal to (-6.267,-5.833, and -5.224).Then $^{124\text{-}128}$Ba isotopes described to be transitional nuclei, transitional from $SU(3)$ features toured $O(6)$.

Conflict of Interests.

There are non-conflicts of interest .

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الخلاصة

تمت دراسة العديد من خصائص التركيب النووي باستعمال $IBM-1$ و $IBM-2$,حيث فحصت بدقة بالاستفادة من التحديث المستمر لمخططات الانحلال النووية. تم دراسة مستويات الطاقة الواطئة الزوجية –الزوجية موجبة التماثل والمستويات مختلطة التماثل MSSو احتمالية الانتقال رباعي القطب المختزل $B(E2)$ ونسب التفرع حساب احتمالية الانتقال ثنائي القطب المغناطيسي المختزلة $B(M1)$ونسبة الخلط $\delta(E2/M1)$ وحساب احتمالية الانتقال الكهربائي احادي القطب المختزل $B(E0)$ ونسبة الخلط $X(E0/E2)$ و سطوح تساوي الجهد.

لتحديد شكل النوى ضمن .اطار نموذج IBM−1 لوحظ ان هناك تنافس بين المعاملين a_0 و a_2 في نوى $^{124-128}Ba$ حيث ان زيادة المعلم a_0 مترافقة بنقصان a_2 والذي يعني استمرار زيادة صفات كاما غير المستقرة بالضد من الصفات الدورانية ,في نموذج IBM−2 كانت قيم χ_π و χ_ν (1.3 وتقريبا 0.7) والذي يعطي تشابه مع توقع نموذج IBM−1 . ان نظائر $^{124-128}Ba$ اعتبرت كنوى انتقالية , تنتقل بين صفات التحديد SU(3) و O(6).