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Equilibrium (Fixed) Vector of a Markov and Absorbing Markov Chains

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Abstract

Owing to evolve the technological and scientific requirements, reliability plays an essential part in mathematical and Statistical disciplines. The method of independent trials has been the focus of most contemporary probability assessments. These operations are the cornerstone of classical probability and a large part of statistics. The "Law of Large Numbers" and the "Central Limit Theorem" were two of the primary theorems for such processes that were studied. We demonstrated that the general outcome occurs when a series of chance tests create an independent testing procedure.

Keywords: Markov, vector, probability, absorbing.

الخلاصة

نظرًا لتطور المتطلبات التكنولوجية والعلمية، تلعب الموثوقية دورًا أساسيًا في التخصصات الرياضية والإحصائية. كانت طريقة التجارب المستقلة هي محور معظم تقييمات الاحتمالات المعاصرة. هذه العمليات هي حجر الزاوية في الاحتمال الكلاسيكي وجزء كبير من الإحصائيات. كان "قانون الأعداد الكبيرة" و "نظرية الحدود المركزية" اثنين من النظريات الأولية لمثل هذه العمليات التي تمت دراستها. لقد أظهرنا أن النتيجة العامة تحدث عندما تؤدي سلسلة من اختبارات الصدفة إلى إنشاء إجراء اختبار مستقل.

الكلمات المفتاحية: ماركوف، احتمالية، متجه، ممتص

Introduction

Much of our probability analysis has dealt with the method of independent trials. Such processes are the basis of the concept of classical probability and a major part of statistics[1]. Two of the major theorems for these procedures were discussed: The Law of Large Numbers and the Central Limit Theorem. We have shown that the general result is where a sequence of chance tests forms an independent testing procedure. The general result is the same for any experiment and happens with the same probability [2, 3]. In addition, our estimates for the results of the next experiment are not affected by the experience of the results of the previous experiments. The distribution of the results of a single experiment is sufficientto create a tree measure for a set of n experiments, and by using this tree measure, we can answer some probability questions regarding these experiments[4-9]. Modern probability theory examines potential causes for which the

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experience informs forecasts for future trials of previous findings. All the past results may affect our forecasts for the next experiment while viewinga seriesof chance experiments. The outcomesmay be the case, for example, isprojectinga student's grades on a seriesof examinations in a course. But it would find it very difficult to prove general outcomes to have so much generality [10, 11].

Some important preliminaries

***Definition (2.1)* [6, 12]**

Although reliability theory considers the influence of mean time to fix on total system failure rates, such estimates are not necessary for critical systems since a fundamental performance criterion is operational failures.In other words, Reliability is if the "probability"of a significantperiod, the system wouldsurvive is formed.In the perspective of random parameter $\mathbb{T}$, this probability could be written as the time to system failure.

***Definition (2.2)* [13, 14]**

A "stochastic process" is characterized as a set of random parameters $\mathbb{X} = \{xt: t \in \mathbb{T}\}$characterized on a typical"probability space", taking values in a typicalset $\mathcal{S}$ (the state space), and integratedby a set $\mathbb{T}$, frequently $\mathbb{N}$in[0,∞) and conceived of as a time series ("discrete or continuous", respectively),frequentlyimplies$\mathbb{T} = \{0, 1, 2, 3, \ldots.\}$. A "discrete-time process" is which$\{x(0), x(1), x(2), x(3),\ldots\}$: a random number associatedwith 0,1,2… per unit time."

***Definition (2.3)* [14]**

The "Markov process" is a stochastic procedure that operates in both "continuous-time" and "discrete-state space". The "discrete-state continuous-time" model would be discussed in this paper.

***Definition (2.4)* [3, 11]**

Every Markov model represented by the collection of probabilities $\mathbb{P}_{ij}$ has the possibility of changing from entry-stage "i" to another stage "j." One of the "Markov model's" distinctive features is that the transformation probability $\mathbb{P}_{ij}$ is based exclusively on stages "i" and "j," with all earlier stages, including the ultimate, being ignored.

Definition (2.5) Matrix of Minors

Creating a "Matrix of Minors"is the initial stage. Most computations are in this stage. For each matrix element, consider writing:

- disregard the existing row and column entries.
- Compute the determinant of the leftover entries.
- Generate a matrix from those determinants (the "Matrix of Minors").

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Equilibrium (Fixed) Vector of a Markov Chain:

Whenever a “Markov chain” with a typical“transition matrix”$\mathbb{P}$ exists, then theirexistenceprobability vector V such that[13]

$$V\mathbb{P} = V$$

The “equilibrium vector” is also known as the “fixed vector”.

1.1.Example (1): Determine the regular stochastic matrix’s unique fixed probability vector.

$$\mathbb{P} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Solution: We find $V = (v_1 v_2)$, where $v_1 + v_2 = 1$

$$(v_1 v_2)\begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = (v_1 v_2)$$

$$\frac{3}{4}v_1 + \frac{1}{2}v_2 = v_1 \qquad \dots\dots(1)$$

$$\frac{1}{4}v_1 + \frac{1}{2}v_2 = v_2 \qquad \dots\dots(2)$$

Since $v_1 + v_2 = 1 \quad \Rightarrow v_2 = 1 - v_1 \qquad \dots\dots(3)$

By substituting Eq.(3) into Eq. (1), we get$\frac{3}{4}v_1 + \frac{1-v_1}{2} = v_1$

$$\Rightarrow v_1 = {}^2/_3$$

$$v_2 = 1 - v_1$$

$$= 1 - {}^2/_3 = {}^1/_3$$

Hence $V = \left(\frac{2}{3}\frac{1}{3}\right)$.

*1.2.Example (2):*In a computing system instance with a “transition matrix”, find the “long-range forecasting” for the “Markov chain”.

$$\mathbb{P} = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.3 & 0.5 \\ 0.4 & 0.5 & 0.1 \end{bmatrix}$$

Solution: We find $V\mathbb{P} = V$, in which,$V = (v_1 v_2\ v_3)$ and $v_1 + v_2 + v_3 = 1$

$$(v_1\ v_2 v_3)\begin{bmatrix}0.5 & 0.3 & 0.2\\0.2 & 0.3 & 0.5\\0.4 & 0.5 & 0.1\end{bmatrix} = (v_1 v_2 v_3)$$

$$0.5v_1 + 0.2v_2 + 0.4v_3 = v_1 \qquad \dots\dots(1)$$

$$0.3v_1 + 0.3v_2 + 0.5v_3 = v_2 \qquad \dots\dots(2)$$

$$0.2v_1 + 0.5v_2 + 0.1v_3 = v_3 \qquad \dots\dots(3)$$

$$-0.5v_1 + 0.2v_2 + 0.4v_3 = 0$$

$$0.3v_1 - 0.7v_2 + 0.5v_3 = 0$$

Since $v_1 + v_2 + v_3 = 1$

We find v_1 , v_2 and$v_3$by Cramer's Rule

$$v_1 = \frac{\begin{vmatrix}0 & 0.2 & 0.4\\0 & -0.7 & 0.5\\1 & 1 & 1\end{vmatrix}}{\begin{vmatrix}-0.5 & 0.2 & 0.4\\0.3 & -0.7 & 0.5\\1 & 1 & 1\end{vmatrix}} = \frac{0.38}{1.04} = 0.36538$$

$$v_2 = \frac{\begin{vmatrix}-0.5 & 0 & 0.4\\0.3 & 0 & 0.5\\1 & 1 & 1\end{vmatrix}}{1.04} = \frac{0.37}{1.04} = 0.35576$$

$$v_3 = \frac{\begin{vmatrix}-0.5 & 0.2 & 0\\0.3 & -0.7 & 0\\1 & 1 & 1\end{vmatrix}}{1.04} = \frac{0.29}{1.04} = 0.27884$$

Now, by taking some powers to transition $\mathbb{P}$, we note

$$\mathbb{P} = \begin{bmatrix}0.5 & 0.3 & 0.2\\0.2 & 0.3 & 0.5\\0.4 & 0.5 & 0.1\end{bmatrix}$$

$$\mathbb{P}^5 = \begin{bmatrix}0.3656 & 0.3557 & 0.2786\\0.3648 & 0.3550 & 0.2803\\0.3659 & 0.3568 & 0.2773\end{bmatrix}$$

$$\mathbb{P}^{12} = \begin{bmatrix}0.3654 & 0.3558 & 0.2788\\0.3654 & 0.3558 & 0.2788\\0.3654 & 0.3558 & 0.2788\end{bmatrix}$$

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The process would eventually arrive at a fixed vector V.

Absorbing Markov Chains:

The "Absorbing Markov Chains"is a common type of "Markov chain" utilized in the various fields of science. The absorbing process creates an "absorbing state".

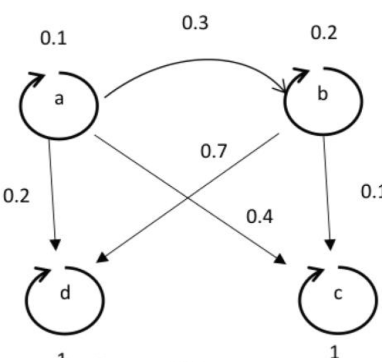
In an absorbing state,$p_{ii} = 1$.

Only if the aforementioned two criteria are met, a "Markov Chain" is an "Absorbing Chain":

- A minimum single "absorbing state" exists in the chain.
- Every "non-absorbing state" can be converted to an "absorbing state".

1.3.Example (3):

$$P = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{matrix} a & b & c & d \end{matrix} \\ \begin{bmatrix} 0.1 & 0.3 & 0.4 & 0.2 \\ 0 & 0.2 & 0.1 & 0.7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$



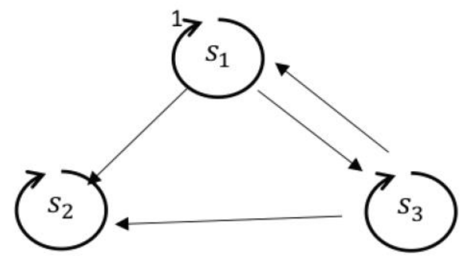
$\mathbb{P}$ is an absorbing matrix.

Fig(1)

1.4.Example (4):

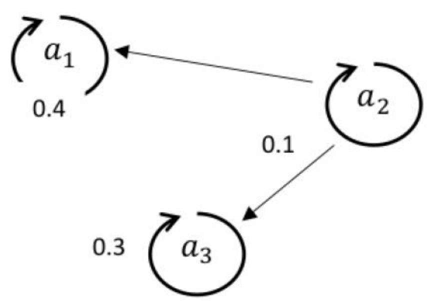
$$\begin{matrix} s_1 & s_2 & 1 & s_3 & 1 & 1 \end{matrix}$$

$$S = \begin{array}{c} s_1 \\ s_2 \\ s_3 \end{array}\begin{bmatrix} 0.1 & 0.3 & 0.6 \\ 0 & 1 & 0 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$$



Fig(2)

S is an absorbing matrix.



1.5.Example(5):

$$A = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array}\begin{array}{c} \begin{matrix} a_1 & a_2 & a_3 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0.4 & 0.1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

A is an absorbing matrix.

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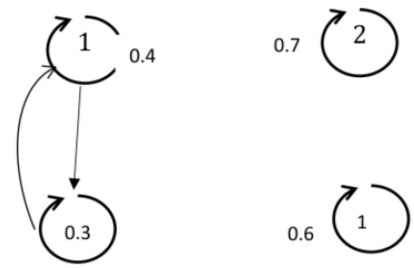
1.6.Example (6):

$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 0.3 & 0 & 0.7 & 0 \\ 0 & 1 & 0 & 0 \\ 0.4 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$



B is not absorbing matrix.

Conclusion

Because of changing technical and scientific demands, mathematical and statistical fields demand a high level of reliability. Most recent probability analyses have focused on the technique of independent trials. The present study found that when a series of chance tests are combined to establish an independent evaluation technique, the typical conclusion happens.

Conflict of interests.

There are non-conflicts of interest.

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