ARTICLE

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On the (τ^2, τ) −Arcs in $PG(2, qq)$ for $\tau \leq qq$

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حول الاقواس− (τ^2, τ) في $PG(2, qq)$ لقيم $\tau \leq qq$

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ABSTRACT

In this article the existence of the (τ^2, τ) −arcs in the projective plane of order qq has been proved. For $\tau = qq - 1$, $qq \geq 3$ we have shown that the $((qq-1)^2, qq-1)$ −arcs are complete. The existence of the completes (144,12)-arc in $PG(2,13)$, (256,16)-arc in $PG(2,17)$ and (324,18) -arc in $PG(2,19)$ have been found.

Keywords: Projective plane, Complete $(\check{k}, \tau)$ − arcs.

الخلاصة

مقدمة:

في هذا البحث تم البرهنة على وجود الاقواس− (τ^2, τ) في المستوي الاسقاطي من الرتبة qq. عندما $\tau = qq - 1$, $qq \geq 3$ برهنا ان الاقواس − $((qq-1)^2, qq-1)$ هي اقواس تامة. كذلك تم بيان وجود القوس-(144,12) التام في المستوي الاسقاطي $PG(2,13)$ و القوس-(256,16) التام في المستوي الاسقاطي $PG(2,17)$ و القوس-(324,18) التام في المستوي الاسقاطي $PG(2,19)$.

الكلمات المفتاحية:

المستوي الاسقاطي, الاقواس− $(\check{k}, \tau)$ التامة.

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INTRODUCTION

In (1947), Bose [8] introduced the concept of the $(\breve{k},\tau)$ −arcs in the projective plane of order qq as a set of $\breve{K}$ points in a projective plane $PG(2,qq)$ with the following properties: there is at least one line contains exactly τ points from the set $\breve{K}$ and every other line contains at most τ points from the set $\breve{K}$. The $\breve{k}$ −arc is a set of points $\breve{K}$ with τ=2. Bose showed that for qq even the maximal value for which the $\breve{k}$ −arc exists in $PG(2,qq)$ is equal to qq+2 and is equal to qq+1 for qq odd. A line that contains i points from the set $\breve{K}$ is called i −secant. If there is no $(\breve{k}+1,\tau)$ −arc containing the $(\breve{k},\tau)$ −arc in $PG(2,qq)$ then, a $(\breve{k},\tau)$ −arc is called complete. A point $\breve{Q} \in PG(2,qq)\backslash\breve{K}$ is called point of index zero if there is no τ −secant passing through it. In (1956), Barlotti [2] showed that the maximal value for which a $(\breve{k},\tau)$ −arc exists in $PG(2,qq)$ is less than or equal to $(\tau-1)qq+\tau$. Many researchers have been interested in finding the maximum value of the $(\breve{k},\tau)$ −arcs. Therefore, several methods were used to find the maximum values of the $(\breve{k},\tau)$ −arcs in $PG(2,qq)$. One of the methods to obtain the maximum value is the classification method of the $(\breve{k},\tau)$ −arcs. Because of the large number of the $(\breve{k},\tau)$ −arcs that appear from the study of the classification of the $(\breve{k},\tau)$ −arcs it is difficult to reach the maximum value. Therefore, it was important to determine the minimum and maximum values of completes $(\breve{k},\tau)$ −arcs in $PG(2,qq)$. There are many studies of the $(\breve{k},\tau)$ −arcs in different ways. In (1979), Hirschfeld [6] introduced a method of classification of the $(\breve{k},\tau)$ −arcs to reach to the maximum value. The method based on the projectively equivalent of the $(\breve{k},\tau)$ −arcs (two $(\breve{k},\tau)$ −arcs $\widehat{K}_1$and $\widehat{K}_2$ are projectively equivalent in $PG(2,qq)$ if there exists a projectivity matrix T such that $T(\widehat{K}_1)=\widehat{K}_2$). This method is the best method of classification for small qq. Yasin [4] has been studied the classification of the $(\breve{k},3)$ −arcs in $PG(2,8)$ up to projectively equivalent for $\breve{k}$=1,...,15. In [7] Abdul Hussain (1997) studied the classification of $(\breve{k},4)$ −arcs in the projective plane of order five based of the type of the i −secant distribution of the $(\breve{k},\tau)$ −arcs method. In his study, he found the number of inequivalent $(\breve{k},4)$ −arcs in $PG(2,5)$ for all $\breve{k}$ =6,...,16. In [13], Hamed and Hirschfeld showed that there exists a complete (48,4)-arc in the projective plane of order 17 while other studies of the $(\breve{k},\tau)$ −arcs in $PG(2,qq)$ can be found in ([7],[8],[9],[10],[11],[12],[13]). In this work, the researcher introduced a new kind of

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$(\breve{k}, \tau)$ −arcs when $\breve{k} = \tau^2$. Large completes (τ^2, τ) −arcs in $PG(2, qq)$ have been found for qq =13,17,19.

For a $(\breve{k}, \tau)$ −arcs $\breve{K}$ in $PG(2, qq)$, let r_i denotes the total number of i −secants of $\breve{K}$ and ρ_i denotes the number of i −secants through a point $\breve{Q} \in PG(2, qq) \setminus \breve{K}$.

Definition 1. [3] The i −secant distribution of the $(\breve{k}, \tau)$ −arc $\breve{K}$ is defined to be the $(\tau + 1)$ −tuple $(r_\tau, r_{\tau-1}, \dots, r_0)$.

Definition 2. [3] The i −secant distribution of the point $\breve{Q} \in PG(2, qq) \setminus \breve{K}$ is defined to be the $(\tau + 1)$ −tuple $(\rho_\tau, \rho_{\tau-1}, \dots, \rho_0)$.

Theorem 1. [3] If $\{\hat{P}_0, \hat{P}_1, \dots, \hat{P}_{\tau+1}\}$ and $\{\hat{Q}_0, \hat{Q}_1, \dots, \hat{Q}_{\tau+1}\}$ are two sets in $PG(\tau, qq)$ such that no $\tau + 1$ points chosen from the same set lie in a prime, then there exists a unique projectivity T such that $T\hat{P}_i = \hat{Q}_i$, $i = 0, \dots, \tau + 1$.

Theorem 2. [3] For a $(\breve{k}, \tau)$ −arcs $\breve{K}$ the following equations fulfilled,

$$\sum_{i=0}^{\tau} r_i = qq^2 + qq + 1, \quad (1.1)$$

$$\sum_{i=1}^{\tau} i\, r_i = \breve{k}(qq + 1), \quad (1.2)$$

$$\sum_{i=2}^{\tau} \frac{1}{2} i\,(i-1)\, r_i = \frac{1}{2}\breve{k}(\breve{k} - 1) \quad (1.3)$$

Theorem 3. [3] For a $(\breve{k}, \tau)$ −arcs $\breve{K}$ the following equations fulfilled,

$$\sum_{i=0}^{\tau} \rho_i = qq + 1, \quad (1.4)$$

$$\sum_{i=1}^{\tau} i\, \rho_i = \breve{k}, \quad (1.5)$$

$$\sum_{\breve{Q} \in PG(2,qq) \setminus \breve{K}} \rho_i = (qq + 1 - i)\, r_i \quad (1.6)$$

Lemma 1. [3] If a $(\breve{k}, \tau)$ −arc $\breve{K}$ is complete then $(qq + 1 - \tau) r_\tau \geq qq^2 + qq + 1 - \breve{k}$, with equality if and only if $\rho_\tau = 1$ for all $\breve{Q} \in PG(2, qq) \setminus \breve{K}$.

Theorem 4. [3] A $(\breve{k}, \tau)$ −arc $\breve{K}$ has maximal value if and only if every line in $PG(2, qq)$ is a 0 −secant or an τ −secant.

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2. The (τ^2, τ) −arcs in $PG(2, qq)$.

The researcher studied special kind of the $(\breve{k}, \tau)$ −arcs when $\breve{k} = \tau^2$ to find large completes (τ^2, τ) −arcs for some qq.

Theorem 5. There exists the (τ^2, τ) −arcs in $PG(2, qq)$.

Proof. Let $\breve{Q} \in PG(2, qq) \backslash \breve{K}$ any point has the i −secant distribution $\rho_\tau = \tau$, $\rho_{\tau-1} = \rho_{\tau-2} = \cdots = \rho_1 = 0$ and $\rho_0 = qq + 1 - \tau$. Then every line passing through the set $\breve{K}$ contains at most n points, counting the points on the i −secants passing through the point $\breve{Q}$ led to τ^2 points in the set $\breve{K}$. ■

Theorem 6. There exists the completes $((qq-1)^2, qq-1)$ −arcs in $PG(2, qq)$ for $qq \geq 3$.

Proof. The existence can be gotten from theorem (5). To prove the completeness note that from theorem (5) the number of $(qq-1)$ −secants passing through the point $\breve{Q}$ is $qq-1$ and the number of points not on the $((qq-1)^2, qq-1)$ −arc that incident with the $(qq-1)$ −secants is qq. The number of 0 −secants is equal to 2, so $2qq$ points external to the $((qq-1)^2, qq-1)$ −arc and belong to the two 0 −secants have been found. Hence, the total number of points not on the $((qq-1)^2, qq-1)$ −arc is $3qq$, through each point must passing at least one $(qq-1)$ −secant; therefore there exists no point of index zero. This proves that the $((qq-1)^2, qq-1)$ −arc is complete. ■

Theorem 7. For a $((qq-1)^2, qq-1)$ −arc in $PG(2, qq)$, $qq \geq 3$ the value of r_{qq-1} is greater than or equal to $\frac{3qq}{2}$.

Proof. From Lemma (1) we have $r_{qq-1} \geq \frac{qq^2+qq+1-(qq-1)^2}{2} = \frac{3qq}{2}$. ■

Theorem 8. If a (τ^2, τ) −arc $\breve{K}$ is maximal in $PG(2, qq)$ then, $\tau = qq$.

Proof. If $\breve{K}$ is maximal then, from theorem (4) every line is a 0 −secant or an τ −secant so $r_{\tau-1} = r_{\tau-2} = \cdots = r_1 = 0$. From equations (1.2) and (1.3) we have

$$\tau r_\tau = \tau^2(q+1) \text{ and } \tau(\tau-1)r_\tau = (\tau^2-1)\tau, \text{ it follows that } \tau = qq. \blacksquare$$

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Theorem 9. For a $(\breve{k},3)$ −arc in $PG(2,qq)$ the following hold:

$$r_0 \geq qq^2 + (1-\breve{k})(qq+1) + \frac{\breve{k}(\breve{k}-1)}{3}, \qquad r_1 \leq \breve{k}\left(q + \frac{3}{2} - \frac{\breve{k}}{2}\right),$$

$$r_2 \geq 0, \quad r_3 \leq \frac{\breve{k}(\breve{k}-1)}{6}.$$

Proof. From theorem (2) for $n = 3$

$$r_0 + r_1 + r_2 + r_3 = qq^2 + qq + 1, \tag{2.1}$$

$$r_1 + 2r_2 + 3r_3 = \breve{k}(qq+1), \tag{2.2}$$

$$r_2 + 3r_3 = \frac{\breve{k}(\breve{k}-1)}{2}. \tag{2.3}$$

The maximum value of r_3 is $\frac{\breve{k}(\breve{k}-1)}{6}$ it follows that $r_2 \geq 0$. Now, from equations (2.2) and (2.3),

$$r_1 + r_2 = \breve{k}(qq+1) - \frac{\breve{k}(\breve{k}-1)}{2}$$

This implies that $$r_1 \leq \breve{k}\left(qq + \frac{3}{2} - \frac{\breve{k}}{2}\right).$$

Equation (2.1) lead to

$$r_1 + r_2 + r_3 = qq^2 + qq + 1 - r_0$$

Substituting into the equation (2.2) we get that

$$r_1 + 2r_3 = \breve{k}(qq+1) - (qq^2 + qq + 1) + r_0$$

By subtracting equation (2.4) from equation (2.3) the result is

$$r_3 = \frac{\breve{k}(\breve{k}-1)}{2} + (qq^2 + qq + 1) - \breve{k}(qq+1) - r_0$$

Since

$$r_3 \leq \frac{\breve{k}(\breve{k}-1)}{6}$$

It follows that

$$r_0 \geq qq^2 + (1 - \breve{k})(qq + 1) + \frac{\breve{k}(\breve{k}-1)}{3}. \quad \blacksquare$$

The following result holds for $\breve{k} = \tau^2$and $\tau = 3$.

Corollary 1. For a $(9,3)$ $-$arc in $PG(2, qq)$, $qq \geq 4$ the following satisfied:

$$r_0 \geq qq^2 - 8qq + 16, \quad r_1 \leq 9qq - 18, \quad r_2 \geq 0, \quad r_3 \leq 12.$$

Known maximal values of the completes $((qq-1)^2, qq-1)$ $-$arc in $PG(2, qq)$ are (9,3)-arc in $PG(2,4)$, (16,4)-arc in in $PG(2,5)$, (36,6)-arc in in $PG(2,7)$ and (49,7)-arc in in $PG(2,8)$. The size of the largest $(\breve{k}, \tau)$ $-$arc in $PG(2, qq)$ for small qq is given in table (1).

Table (1)

τ \ qq	3	4	5	7	8	9	11	13	16	17	19
2	4	6	6	8	10	10	12	14	18	18	20
3		9	11	15	15	17	21	23	28	28-35	31-39
4			16	22	28	28	32	38-40	52	48-52	52-58
5				29	33	37	43-45	49-53	65	61-69	68-77
6				36	42	48	56	64-66	78-82	79-86	87-96
7					49	55	67	79	93-97	95-103	105-115
8						65	77-78	92	120	114-120	126-134
9							89-90	105	128-131	137	147-153
10							100-102	118-119	142-148	154	172
11								132-133	159-164	167-171	191
12								145-147	180-181	184-189	204-210
13									195-199	205-207	225-230
14									210-214	221-225	244-250
15									231	239-243	267-270
16										256-261	286-290
17											305-310
18											324-330

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To construct the complete (144,12)-arc in $PG(2,13)$. Let the irreducible polynomial over Galois field of order 13 is given by $f(x) = x^3 - x^2 - 2$. Then P_1=(1,0,0), P_2=(0,1,0), $P3$=(0,0,1), P_{46}=(1,1,1). Let the 12−secant ℓ_1 contains the two points P_1, P_2 and the 12−secant ℓ_2 contains the two points P_3, P_{46}. The common point of the two lines ℓ_1 and ℓ_2 is P_{42}=(1,1,0)∈ $PG(2,qq)\backslash\breve{K}$. Then the union of all points on the remaining ten 12−secants passing through the point P_{42} with the points on the two 12−secants ℓ_1 and ℓ_2 form (144,12)-arc in PG (2,13). There is one point on each 12−secant then, the number of points on the 12−secants is thirteen. The remaining points not on the (144,12)-arc are the points on the 0−secants passing through the point (1,1,0). The number of points on the two 0−secants is twenty six. Through every point on the 0−secant must passing at least one 12−secant. This show that there is no points of index zero so the (144,12)−arc is complete. We will refer to the point P_i by its index i.

The points that form a complete (144,12)-arc in $PG(2,13)$ with i −secant distribution (79,59,33,9,1,0,0,0,0,0,0,0,2) are given by the following set

$\breve{K}$ ={1,2,3,46,9,25,38,60,108,120,129,135,140,154,16,18,19,26,55,59,77,125,137,146,152,157,

14,44,45,52,68,81,85,103,151,163,172,178,54,63,69,74,88,116,118,119,126,142,155,159,30,51,

57,62,76,104,106,107,114,130,143,147,11,39,41,49,65,78,82,100,148,160,169,175,4,32,34,35,58 ,71,75,93,141,153,162,168,10,22,31,37,56,84,86,87,94,110,123,127,7,20,24,90,102,111,117,122, 136,164,166,167,15,27,36,47,61,89,91,92,99,115,128,132,21,33,48,53,67,95,97,98,105,121,134, 138,5,6,13,29,64,112,124,133,139,144 }.

Theorem 10. There exists a complete (144,12)−arc in $PG(2,13)$ with i −secant distribution (79,59,33,9,1,0,0,0,0,0,0,0,2).

Construction of the (256,16)−arc in $PG(2,17)$: The irreducible polynomial over Galois field of order 17 is given by $g(x) = x^3 - x^2 - 5x - 1$. Then C_1=(1,0,0), C_2=(0,1,0), C_3=(0,0,1), C_{95}=(1,1,1). The point of the intersection of the two 16−secants passing through the points C_1, C_2, C_3 and C_{95} is C_{62}=(1,1,0). Then the set of points

$\breve{K}_1$ ={1,2,3,95,28,31,74,82,84,175,186,199,203,224,238,243,261,293,16,19,50,70,72,163,174,

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187, 191,212,226,231,249,281,290,296,10,23,27,48,67,85,117,126,132,133,159,162,193,205,
213,215,7,12,30,71,77,78,104,107,138, 150,158,160,251,262,275,279,17, 26,32,33,59,93,105,
113,115,206,217,230,234,255,269,274,6,9,40,52,60,153,164,177,181,202,216,221, 239,271,280,
286,21,53,68,69,98,129,141,149,151,242,253,266,270,291,38,49,66,87,101,106,124,156,165,171
,172,198,201,232,244,252, ,18,22,43,57,80,112,121,127,128,154,157,188,200,208,210,34,45,58,
83,97,102,120,152,161,167,168,194,197,228,240,248,8,11,42,54,64,155,166,179,183,204,218,
223,241,273,282,288,14,46,55,61,88,91,122,134,142,144,235,246,259,263,284,298,51,75,79,100
,114,119,137,169,178,184,185,211,214,245,257,265,73,86,90,111,125,130,148,180,189,195,196,
222,225,256,268,276,13,24,37,41,76,81,99,131,140,146,147,173,176,207,219,227,20,29,35,36,
65,96,108,116,118,209,220,233,237,258,272,277 }.

form a complete (256,16)-arc in $PG(2,17)$ with i −secant distribution
(124,115,45,17,4,0,0,0,0,0,0,0,0,0,0,0,2).

Theorem 11. There exists a complete (256,16)−arc in PG(2,17) with i −secant distribution (124,115,45,17,4,0,0,0,0,0,0,0,0,0,0,0,2).

Construction of the (324,18)−arc in $PG(2,19)$: The polynomial $h(x) = x^3 - x^2 - x - 4$ is an irreducible over Galois field of order 19. Then T_1=(1,0,0), T_2=(0,1,0) , T_3=(0,0,1), T_{11}=(1,1,1). The point of the intersection of the two 18−secants passing through the points T_1, T_2, T_3 and T_{11} is T_{315}=(1,1,0). Then the set of points is

$\breve{K}_2$ ={1,2,3,11,100,124,130,143,155,171,175,189,225,227,262,267,289,306,372,15,24,81,84,91,
92,190,214,220,233,245,261,265,279,294,317,352,357,43,67,73,86,98,114,118,132,147,168,170,
205,210,232,249,258,318,325,8,29,31,66,71,93,110,119,176,179 ,186,187,285,309,328,340,356,
360,44,47,54,55,153,177,183,196,208,224,228,242,257,278,280,320,342,359,32,56,62,75,87,103
,107,121,136,157,159,194,199,221,238,247,304,307,14,35,37,72,77,99,116,125,182,185,192,193
,291,321,334,346,362,366,19,36,45,102,105,112,113,211,235,241,254,266,282,286,300,336,338,

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373,39,42,49,50,148,172,178,191,203,219,223,237,252,273,275,310,337,354,10,109,133,204,

139,152,164,180,184,198,213,234,236,271,276,298,324,5,9,23,38,59,61,96,101,123,140,149,206 ,209,216,217,339,345,358,25,30,52,69,78,135,138,145,146,244,268,274,287,299,319,333,348,

369,33,57,63,76,88,104,108,122,137,158,160,195,200,222,239,248,305,308,21,26,48,65,74,131, 134,141,142,240,264,270,283,295,311,329,344,365,40,64,70,83,95,111,115,129,144,165,167,

202,207,229,246,255,312,322,7,12,34,51,60,117,120,127,128,226,250,256,269,281,297,301,330, 351,13,22,79,82,89,90,188,212,218,231,243,259,263,277,292,313,350,355,4,6,41,46,68,85,94,

151,154,161,162,260,284,290,303,331,335,349 }.

represent a complete (324,18)-arc in $PG(2,19)$ with i −secant distribution

(157,129,72,15,6,0,0,0,0,0,0,0,0,0,0,0,0,0,2).

Theorem 12. There exists a complete (324,18)−arc in $PG(2,19)$ with i −secant distribution (157,129,72,15,6,0,0,0,0,0,0,0,0,0,0,0,0,0,2).

There is incomplete (τ^2,τ) −arc in $PG(2,qq)$ show in the example (1).

Example 1.

In this example incomplete (τ^2,τ) −arc in $PG(2,11)$ when τ =9 has been introduced. Let $h_1(x) = x^3 - x^2 - x - 3$ be an irreducible polynomial over Galois field of order 11, then D_1=(1,0,0), D_2=(0,1,0), D_3=(0,0,1), D_{55}=(1,1,1) and D_{19}=(1,1,0). The following set of points

A_1={1,2,3,4,5,6,7,9,10,11,12,13,14,16,17,18,20,21,23,24,25,26,27,28,29,30,31, 32,33,34,35,

36,37,39,40,44,45,46,47,49,52,53,54,55,57,58,60,61,63,65,67,69, 70,71,72,75,76,77,80,81,83,

84,85,86,87,90,92,93,94,95,96,97,98,99,100,103,105,107,108,110,117}

form incomplete (81,9)-arc in $PG(2,11)$ with i-secant distribution (30,38,36,16,10,0,0,0,0,3). The result of the classification of the (81,9)-arc led to one complete (82,9)-arc given by the set of points $M_2 = A_1 \cup \{104\}$ with i −secant distribution (35,38,32,15,10,0,0,0,1,2) and two completes (83,9)-arcs that are not projectively equivalent given by the sets of points $M_3 = A_1 \cup \{42\} \cup \{50\}$ with i −secant distribution (42,34,29,16,9,0,0,0,2,1) and $M_4 = A_1 \cup \{50\} \cup \{56\}$ with i −secant distribution (43,31,31,17,8,0,0,1,0,2).

Example 2. There exists a complete (100,10)-arc in $PG(2,11)$ given by the set of points

L_1={1,2,3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,20,21,22,23,24,25,26,27,28, 29,30,31,32,33, 34,35,36,37,38,39,40,42,44,45,46,47,49,52,53,54,55,57,58,60,61,63, 64,65,66,67,69,70,71, 72,73,74,75,76,77,79,80,81,83,84,85,86,87,88,90,91,92,93,94, 95,96,97,98,99,100,101,102, 103,105,107,108,110,111,116,117,122,124,125,126}

with i −secant distribution (56,49,18,7,1,0,0,0,0,0,2).

Conclusion

In this study a new type of the $(\check{k},\tau)$ −arcs in $PG(2,qq)$ for $\check{k}=\tau^2$ has been introduced. By studying this type of $(\check{k},\tau)$ −arcs we were able to find some maximum values of $\check{k}$ and some large complete (τ^2,τ) −arcs for q =11,13,17,19. To construct the $(\check{k},\tau)$ −arcs in $PG(2,qq)$ one can started from (τ^2,τ) −arcs to get large complete arcs and if possible get the maximum value for which $(\check{k},\tau)$ −arc exists in $PG(2,qq)$.

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Conflict of interests.

Not exists.

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