Article

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Study of Nuclear Structure of nucleus $^{100}_{42}Mo$ Using Interacting Boson Model-1

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دراسة التركيب النووى لنواة $(^{100}_{42}Mo)$باستخدام النموذج البوزونات المتفاعلة (IBM-1)

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ABSTRACT

Background:

To study the nuclear structure, a novel model known as the interacting boson model-one (IBM-1) is utilized. It is hypothesized that low-lying collective quadrupole states can be produced as states of an N-boson system capable of occupying two levels, other with angular momentum J = 0 and one with angular momentum J = 2.

Materials and Methods:

We investigated the nuclear structure of Molybdenum in the theoretical context. We utilized the Interacting Boson Model (IBM-1) to discover the appropriate Hamiltonian for investigating $^{100}_{42}Mo$ -isotope in this work.

Results:

To generate the Hamiltonian, we calculated energy levels (ground, beta and gamma) bands and B(E2) transitions for the $^{100}_{42}Mo$-isotope using the best-suited parameter values.

Conclusions:

According to this study and the IBM test results, $^{100}_{42}Mo$ has a transition behavior of SU(5) limitations. When the results are compared with the experimental data, they show good agreement.

Keywords: Interacting Boson Model (IBM-1), $^{100}_{42}Mo$-isotope, energy levels, B(E2) transitions

الخلاصة

مقدمة:

لدراسة التركيب النووي ، يتم استخدام نموذج جديد يُعرف باسم نموذج البوزون المتفاعل −1. يُفترض أن الحالات الرباعية الجماعية المنخفضة يمكن إنتاجها كحالات لنظام N−boson قادر على احتلال مستويين ، أحدهما بزخم زاوي J = 0 والآخر بزخم زاوي J = 2.

طرق العمل:

لقد بحثنا في التركيب النووي للموليبدينوم في السياق النظري. استخدمنا نموذج البوزون المتفاعل (IBM−1) لاكتشاف هاملتونيان المناسب للتحقيق $^{100}_{42}Mo$ في هذا العمل.

الاستنتاجات:

لتوليد هاملتونيان ، قمنا بحساب مستويات الطاقة (الحالة الارضية ، بيتا وجاما) نطاقات وانتقالات(B(E2 لنظير $^{100}_{42}Mo$ باستخدام أفضل قيم المعلمات المناسبة. وفقًا لهذه الدراسة ونتائج اختبار IBM، لدى $^{100}_{42}Mo$ سلوك انتقالي لقيود SU(5) عند مقارنة النتائج بالبيانات التجريبية ، فإنها تظهر توافقًا جيدًا.

الكلمات المفتاحية: نموذج البوزونات المتفاعلة الاول IBM−1، $^{100}_{42}Mo$ ،مستويات الطاقة الطيفية، انتقالات(B(E2

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INTRODUCTION

A substantial amount of research has gone into studying the shape and properties of complex nuclei, which has led to the development of several nuclear models. As a result, approximations must be used to grasp the structure of complex atomic nuclei. The structure of the nuclei affects the selection of an appropriate estimate. To investigate the shape and activity of even-even nuclei, Arima and Iachello created the Interacting Boson Model (IBM) to study the structure and activity of even-even nuclei. The IBM is based on both the well-known shell model and mathematical group atomic nucleus models [1, 2].

Despite its comparatively basic construction, it has the potential to be a powerful instrument. It is also of considerable scientific importance since it illustrates the dynamical symmetries of multiple nuclei, which are shown via Lie algebras. The underlying idea is that proton and neutron bosons with spins of 0 and 2 can represent the low energy group degrees of freedom in nuclei. Because the bosons could be couples of holes or particles, the IBM-1's s (L=0) and d (L=2) bosons have six sub-states and thus form a six-dimensional universe. This leads to a definition in terms of the six-dimensional unitary group, U(6). As a consequence, using group-theoretical techniques, many of the IBM-1's distinguishing features can be found and expressed analytically. The primary distinguishing feature of IBM-1 is that both bosons (neutron and proton) are identical. The various reductions of U(6) result in three dynamical symmetries known as SU(5), SU(3), and O(6), which are pertinent to the spherical vibrator, deformed rotor, and asymmetric (γ-soft), in that sequence[3, 4].

The current study aims to evaluate the dynamical symmetry, energy levels, B(E2), and quadrupole moment values of the $^{100}_{42}Mo$ within the context of the (IBM-1) model.

Theoretical Basis of IBM-1 Model

The IBM-1 Hamiltonian takes the form [1-3].

$$\hat{H} = \varepsilon\,(n_d) + a_o\,(\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4) \quad \text{..... (1)}$$

Where ε, a_o, a_1, a_2, a_3 and a_4 are the model parameters, P, L, Q, T_3 and T_4 are the pairing, angular momentum, quadrupole, octopole and hexadecapole operators respectively. n_d is the d-boson number operator, and all operators in the Hamiltonian are the following[4, 5]

$$\hat{P} = \frac{1}{2}\left[\left(\tilde{d}\,.\tilde{d}\right) - (\tilde{s}.\tilde{s})\right] = \frac{1}{2}(\tilde{d}^2 - \tilde{s}^2) \quad \text{..... (2)}$$

$$T_l = [d^{\dagger}\otimes \tilde{d}]^l \quad l=0,1,2,3,4,\ldots \quad \text{..... (3)}$$

$$\hat{L} = \sqrt{10}\,[d^{\dagger}\otimes \tilde{d}]^1 = \sqrt{10}\hat{T}_1 \quad \text{..... (4)}$$

$$\hat{Q} = [d^{\dagger}\otimes \tilde{s} + s^{\dagger}\otimes \tilde{d}]^2 - \frac{\sqrt{7}}{2}[d^{\dagger}\otimes \tilde{d}]^2 = [d^{\dagger}\otimes \tilde{s} + s^{\dagger}\otimes \tilde{d}]^2 - \frac{\sqrt{7}}{2}T_2 \quad \text{..... (5)}$$

$$\hat{T}_3 = [d^{\dagger}\otimes \tilde{d}]^3, \quad \text{..... (6)}$$

$$\hat{T}_4 = [d^{\dagger}\otimes \tilde{d}]^4, \quad \text{..... (7)}$$

$$\hat{n}_d = \sqrt{5}\hat{T}_0 \quad , \quad \text{..... (8)}$$

The B(E2) values have been calculated using the E2 operator. The E2 transition operator (electric quadrupole transition operator) has to possess a Hermitian tensor of rank two in order to preserve the number of bosons. Because the universal E2 operator is capable of being expressed in these restrictions

$$T(E2) = e_B\ [(d^\dagger\ \tilde{s} + s^\dagger\ \tilde{d}) + \chi(d^\dagger\ \tilde{d})^{(2)}] = e_B Q \quad \text{----------} \quad (9)$$

Here e_B represents the effective boson charge. The metric χ specifies the relevance of the two terms. The E2 operator, which shares the same structure as the Q operator in the Hamiltonian, consists of a single component which alters n_d by unity and another that leaves n_d unaltered with the ratio of the two components determined by the χ variables [6, 7].

Results and Discussion

Using the parameters of Table (1) in the Hamiltonian equations of IBM-1, the result for a low-lying positive parity energy spectra have been obtained for $^{100}_{42}Mo$. These low-lying energy spectra which obtained by IBM-1 are within the SU(5) limit. The IBM-1 Hamiltonian is constructed and solved in the SU(5) basis using computer program code PHINT [8]. The variables are independent variables that have been established to recreate the excitation-energy of all positive parity levels literally perfectly.

Table-1: The IBM-1 Hamiltonian parameters used in the present study for IBM-1 calculations of $^{100}_{42}Mo$ isotope

$^{100}_{42}Mo$							
ϵ MeV	a_0 MeV	a_1 MeV	a_2 MeV	a_3 MeV	a_4 MeV	CHI	SO6
0.5400	0.0000	0.0170	-0.0160	0.0087	0.0067	-1.3500	1.0000

The most accurate fit to the Hamiltonian variables Eq.(1) utilized in this inquiry is shown in Table 1, producing an excellent match with the estimated energy levels in the project and the associated experimental data for $^{100}_{42}Mo$ [9, 10] as seen in figure (1).

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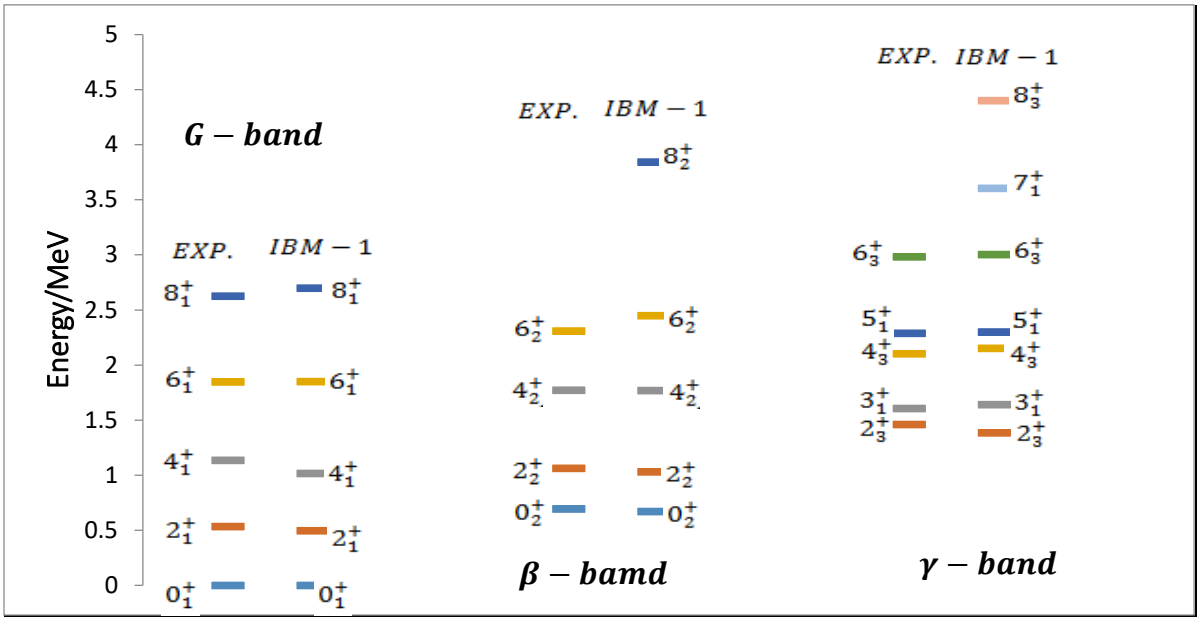

Figure (1): A comparison between the experimental and IBM-1 low–lying positive parity states of the ground, gamma, and beta bands for $^{100}_{42}Mo$

The boson E2 operator in IBM-1, for the beneath the excited states of the considered $^{100}_{42}Mo$, equation (9) is utilized to calculate the E2 transition rates and quadrupole moments. According to the concept, the value of the IBM-1's effective charge e_B is obtained by normalizing to $B(E2: 2_1^+ \rightarrow 0_1^+)$ experimental data. The B(E2) values employed during the present investigations obtained by the IBMT-code are shown in Table 2:

Table-2: The experimental and calculated values of B(E2) transition and quadrupole moments for $^{100}_{42}Mo$ [11-13].

$e_B = 0.1100\ e.b$	$\chi = -1.3500$	$SO(6) = 1.000$
$B(E2)\ e^2b^2$ / $Q(J)e.b$	Exp.	IBM-1
$B(E2: 2_1^+ \rightarrow 0_1^+)$	0.09374(0.55%)	0.09681
$B(E2: 2_1^+ \rightarrow 0_2^+)$		0.03390
$B(E2: 2_2^+ \rightarrow 0_1^+)$	0.00171(0.14%)	0.00012
$B(E2: 2_2^+ \rightarrow 0_2^+)$	0.01516(0.22%)	0.01760
$B(E2: 2_3^+ \rightarrow 0_2^+)$	0.03860(1.1%)	0.01016
$B(E2: 2_2^+ \rightarrow 2_1^+)$	0.1406(1.378%)	0.15940
$B(E2: 2_3^+ \rightarrow 2_1^+)$	0.00309(0.022%)	0.00085
$B(E2: 2_3^+ \rightarrow 2_2^+)$	0.06617(2.206%)	0.05140
$B(E2: 4_1^+ \rightarrow 2_1^+)$	0.19024(1.1%)	0.17940
$B(E2: 4_1^+ \rightarrow 2_2^+)$		0.00720
$B(E2: 4_1^+ \rightarrow 2_3^+)$		0.04150
$B(E2: 4_2^+ \rightarrow 2_2^+)$	0.08271(1.65%)	0.11040
$B(E2: 4_2^+ \rightarrow 2_3^+)$		0.01980
$B(E2: 4_3^+ \rightarrow 2_3^+)$		0.01920
$B(E2: 4_2^+ \rightarrow 4_1^+)$	0.0772(1.65%)	0.09710

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$B(E2: 6_1^+ \rightarrow 4_1^+)$	0.25917(3.86%)	0.21970
$B(E2: 8_1^+ \rightarrow 6_1^+)$	0.33912(4.963%)	0.28200
$Q2_1^+$	$-0.33^{+0.10}_{-0.10}$	-0.33210
$Q2_2^+$	$1.2^{+0.10}_{-0.08}$	1.4230
$Q2_3^+$	$-0.24^{+0.12}_{-0.07}$	-0.52180
$Q4_1^+$	$-0.35^{+0.18}_{-0.18}$	-0.66750
$Q4_2^+$		-0.24270
$Q4_3^+$		-0.96420

Our results reveal that the findings from experiments and theory are in great accord for B(E2) transitions and quadrupole moment values.

The energy ratios $(\frac{E4_1^+}{E2_1^+})$, $(\frac{E2_2^+}{E2_1^+})$, and $(\frac{E6_1^+}{E2_1^+})$ of the selected $^{100}_{42}Mo$, has been calculated in the frame work of IBM-1, together with their corresponding experimental values are plotted for $^{100}_{42}Mo$ and displayed in figure (2). Dynamical symmetry is a useful tool for illustrating the structure and behavior of nuclei. It may be computed by knowing the $\frac{E4_1^+}{E2_1^+}$ ratio. And the figure (2) show that the $^{100}_{42}Mo$ evidence are considering as vibrational symmetry of SU(5) chain group. Generally, the IBM-1 calculations of the values of above ratios are well-disposed with the experimental energy ratios values.

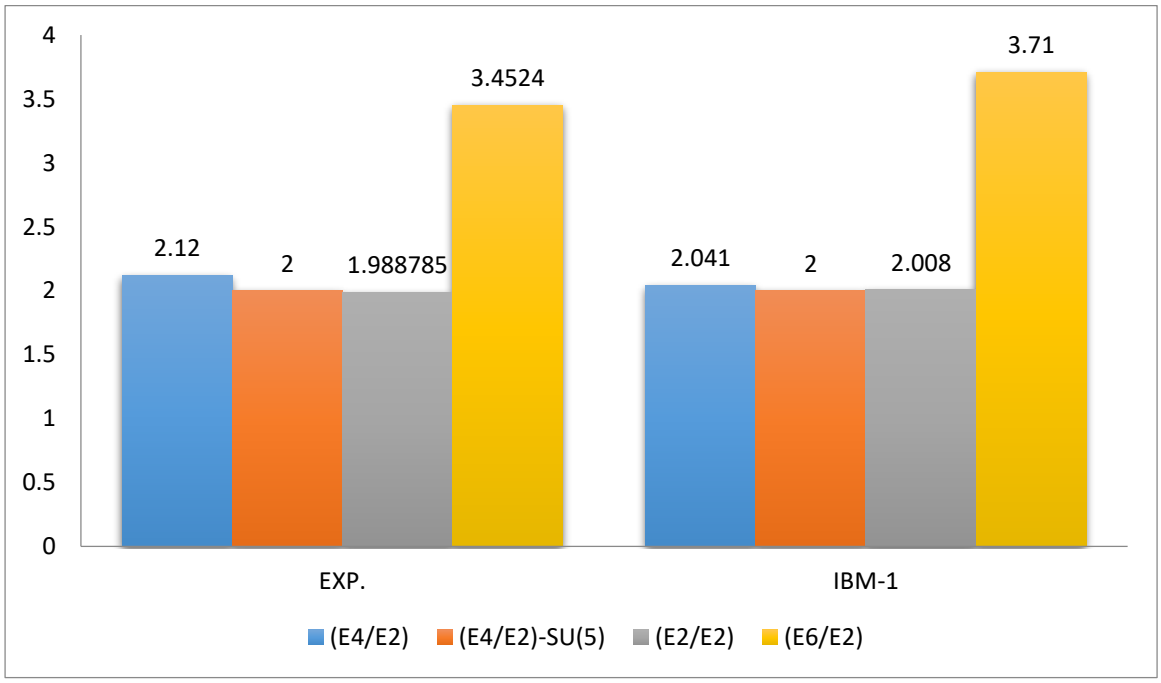

Figure (2): experimental and calculated values of the energy ratios $(\frac{E4_1^+}{E2_1^+})$, $(\frac{E2_2^+}{E2_1^+})$, and $(\frac{E6_1^+}{E2_1^+})$ for $^{100}_{42}Mo$.

The ratio $B(E2: 4_1^+ \rightarrow 2_1^+)/B(E2: 2_1^+ \rightarrow 0_1^+)$, on the other hand, is a component of nuclei structure. As a result, our present works ratio of 1.85312 is the best fit agreement to 1.8417 of experimental data [9]. The figure (3) shows the comparison between the experimental and theoretical ratio of B(E2) transition.[13]

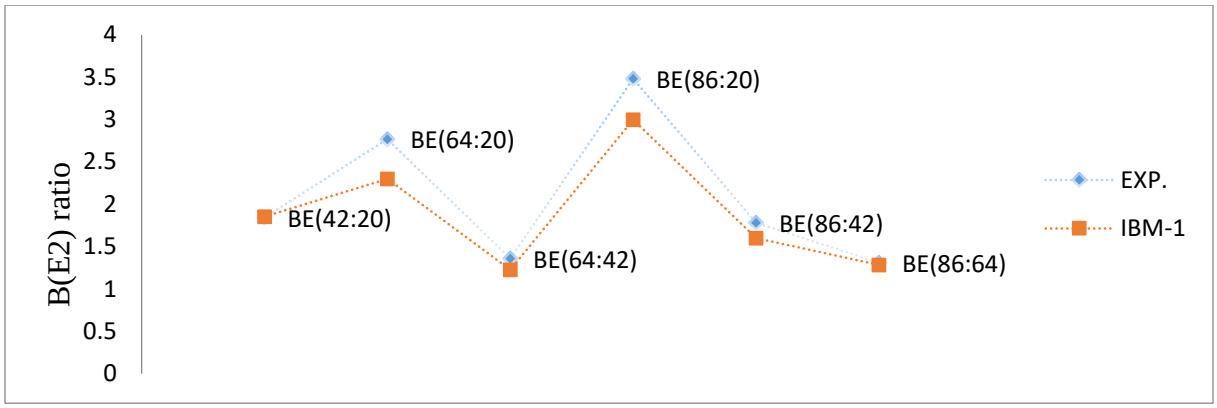

Figure (3): The comparison between experimental and IBM-1 ratios of $B(E2: j_i^+ \rightarrow j_f^+)/B(E2: j_i^+ \rightarrow j_f^+)$ for $^{100}_{42}Mo$

Conclusion

In this work, the interacting boson model (IBM-1) has been utilized for finding energy levels, B(E2) transition, quadrupole moment, and dynamical symmetry properties of $^{100}_{42}Mo$. According to the current study, the energy levels for the positive parity owing to IBM-1 demonstrates that $^{100}_{42}Mo$ is a molybdenum isotope in vibrational region with SU(5) dynamical symmetry which is distinguishing characteristics. IBM-1 accurately reproduces the experimental energy levels of the (ground, beta, and gamma) state bands. On the other hand, B(E2) transition and quadrupole moment values according to IBM-1 has a good acceptance with experimental data

Conflict of interests.

There are non-conflicts of interest.

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