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On the Decomposition Matrix for the Spin Characters S_{29} Modulo $p = 11$

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حول مصفوفة التجزئة لشواخص التمثيل للزمرة المعيارية $\bar{S}_{29}$ مقياس $p = 11$

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ABSTRACT

Background:

In this work, we compute the decomposition matrix to the spin (projective) characters of S_{29}, which is the correlations between the irreducible spin characters and the irreducible modular spin characters of S_{29}, for a given field characteristic of $p = 11$. We may obtain it by figuring out all irreducible spin characters for S_{29}, $p = 11$ by fixing all bar partitions, as well as all irreducible modular spin characters for S_{29}, $p = 11$, where we generate projective character for S_{29} by projective character of S_{28} and used Maple program to see all the possible of columns to choose the possible the right columns of them. The aim of this study is to pave the way for finding general relationships and theorems to study irreducible modular spin characters.

Materials and Methods:

We have used the $(r, \bar{r})$-inducing to generate projective character S_{29} by projective character of S_{28} and Maple program to choose the possible the right columns

Results:

We find decomposition matrix to the spin (projective) characteristics of S_{29} for a given field characteristic of $p = 11$ which equals $\boldsymbol{B_1} \oplus \boldsymbol{B_2} \oplus \ldots \oplus \boldsymbol{B_{21}}$

Conclusions:

We have conducted multiple studies to get sufficient data to identify new characteristics and theorems if the field characteristic is prime because there is no standard approach for researching the issue, especially when we prove the field and the change of groups. Prior scholars achieved this when they looked at the division matrix in the field where the characteristic is 0, We used the $(r, \bar{r})$-inducing also, we used Maple programming to view every possible columns.

Keywords:

Irreducible modular spin characters, Representation group, Decomposition matrix to the spin characters, projective characters

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الخلاصة

مقدمة:

في هذا العمل، نحسب مصفوفة التجزئة للمشخصات الاسقاطية لـ S_{29}،والتي تربط بين المشخصات الاسقاطية الغير قابلة للاختزال والمشخصات الاسقاطية المعيارية غير القابلة للاختزال لـ S_{29} ، عند الحقل الذي مميزة $p = 11$. قد نحصل عليه من خلال اكتشاف جميع للمشخصات الاسقاطية غير القابلة للاختزال لـ S_{29}، $p = 11$ عن طريق تجزئة n الى أجزاء غير متساوية بالإضافة إلى جميع المشخصات الاسقاطية المعيارية غير القابلة للاختزال لـ S_{29} ، $p = 11$ ، حيث قمنا إنشاء مشخص إسقاطي لـ S_{29}باستخدام المشخص الإسقاطي لـ S_{28} واستخدام برنامج مابل لرؤية كل الأعمدة الممكنة ثم اختيار الأعمدة المناسبة لها. الهدف من هذا البحث هو تمهيد الطريق لإيجاد العلاقات العامة والنظريات لدراسة المشخصات المعيارية غير القابلة للاختزال.

طرق العمل:

استخدمنا $(r ، \bar{r})$–المستحثة و برنامج مابل

الاستنتاجات:

وجدنا مصفوفة التجزئة الاسقاطية لزمرة S_{29} عند الحقل الذي مميزة $p = 11$ والتي تساوي $\oplus$ $\boldsymbol{B_{21}B_1}$ $\oplus$...$\oplus$ $\boldsymbol{B_2}$

الكلمات المفتاحية:

المشخصات الاسقاطية الغير قابلة للتحليل, التمثيل الزمري, مصفوفة المسخصات الاسقاطية, المشخصات الاسقاطية

1. INTRODUCTION

As seen in[1], the spin (projective) representations of $S_n$are representations whose kernel does not include central$Z = \{-1,1\}$. The spin characters of the spin representations of S_n are labels for the various components of the n-partitions, and they are denoted by the symbol $\langle\alpha\rangle$.One irreducible spin character marked by $\langle\alpha\rangle^*$ is self-associate if $\alpha = (\alpha_1, \alpha_2, \dots \alpha_m)$ and $n - m$ is even. If $n - m$ is odd, the symbols $\langle\alpha\rangle$ and $\langle\alpha\rangle'$stand in for two associate spin characters; for more information, see[1][2].The relationships between the irreducible spin characters and the irreducible modular spin characters of $S_n$serve as the foundation for the spin character decomposition matrix.The number of rows corresponds to the number of projective characters, and the number of columns to the number of (p, α)-regular classes[3].The irreducible spin character's non-negative coefficients may be used to write every spin character of S_n as a liner combination[4]. The distribution of the spin characters into p-blocks is accomplished using the $(r, \bar{r})$-inducing (restricting) technique. see[5][6]. Several people have contributed to this field of study and do research on this topic[7][8][9]. In this research before we declare any results, let's define certain notations and terminology. The terms "p.s." (p.i.s.) and "m.s." (i.m.s.) stand for "principal spin character" (indecomposable) and "modular spin character" (irreducible), respectively. "d_i" stands for "p.i.s." of S_n, "D_i" stands for "p.i.s." of S_{n-1}, and $\langle\ \rangle^{no}$ is the number of i.m.s.

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2. Preliminaries

Theorem 2.1[1] Degree of the spin character $\langle \alpha_1, \dots, \alpha_m \rangle = 2^{[(n-m)/2]} \frac{n!}{\prod_{i=1}^{m} \alpha_i!} \prod_{1 \le i < j \le m} \frac{(\alpha_i - \alpha_j)}{(\alpha_i + \alpha_j)}$

Theorem 2.2[10]Let G be a group of order $m_0 p^a$, B be a p-block G of defect one, and let b be the number of p-conjugate characters to the irreducible ordinary character χ of G then:

1. There exists a positive integer number N such that the irreducible ordinary characters lying in the block B can be partitioned into two disjoint classes:
   $B_1 = \{\chi \in B | b, \quad deg\chi \equiv N mod p^a\}, B_2 = \{\chi \in B | b, \quad deg\chi \equiv -N mod p^a\}$.
2. Each coefficient of the decomposition matrix of the block B is 1 or 0.

Theorem 2.3[11] Let G be a group of order $m_0 p^a$, where $(p, m_0) = 0$. If c is a principal character of sub group H of G, then deg c $\equiv 0$ mod p^a

Theorem 2.4[2] If n is odd and $p \nmid n$ or $p \nmid (n-1)$, then $\langle n-1,1 \rangle$ and $\langle n-1,1 \rangle'$ are distinct irreducible modular spin characters of degree $2^{[(n-3)/2]} \times (n-2)$ which are denoted by $\varphi\langle n-1,1 \rangle$ and $\varphi\langle n-1,1 \rangle'$respectively.

3. Decomposition Matrix for the Spin Characters of S_{29}

Decomposition matrix of degree (384,300), has decomposed in to 51 blocks which are $\boldsymbol{B_1}$, $\boldsymbol{B_2}$,…, $\boldsymbol{B_5}$ of defect two, $\boldsymbol{B_6}$, $\boldsymbol{B_7}$, ⋯ , $\boldsymbol{B_{21}}$ are of defect one and the others of defect zero.

Case 1. Table 1 displays the decomposition matrices for the double type spin block $\boldsymbol{B_1}$.

Table 1. Block $\boldsymbol{B_1}$

Spin character	Decomposition matrix																			
$\langle 29 \rangle^*$	1																			
$\langle 22,7 \rangle$	1	1																		
$\langle 21,7,1 \rangle^*$		1	1																	
$\langle 20,7,2 \rangle^*$			1	1																
$\langle 19,7,3 \rangle^*$				1	1															
$\langle 18,11 \rangle$	1	1				1														
$\langle 18,10,1 \rangle^*$	2	1	1			1	1													
$\langle 18,9,2 \rangle^*$			1	1			1	1												
$\langle 18,8,3 \rangle^*$				1	1			1	1											
$\langle 18,7,4 \rangle^*$					1				1	1										
$\langle 18,6,5 \rangle^*$										1										
$\langle 17,7,5 \rangle^*$									1	1	1									
$\langle 16,7,6 \rangle^*$									1		1	1								

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$\langle \mathbf{14,8,7} \rangle^*$								1	1			1	1							
$\langle \mathbf{13,9,7} \rangle^*$							1	1					1	1						
$\langle \mathbf{12,10,7} \rangle^*$	2					2	1							1	2					
$\langle \mathbf{11,10,7,1} \rangle$						1								1	1	1				
$\langle \mathbf{11,9,7,2} \rangle$													1	1	1	1	1			
$\langle \mathbf{11,8,7,3} \rangle$												1	1				1	1		
$\langle \mathbf{11,7,6,5} \rangle$												1						1		
$\langle \mathbf{10,9,7,2,1} \rangle^*$															1		1		1	
$\langle \mathbf{10,8,7,3,1} \rangle^*$																1	1	1	1	1
$\langle \mathbf{10,7,6,5,1} \rangle^*$																		1		1
$\langle \mathbf{9,8,7,3,2} \rangle^*$																1				1
$\langle \mathbf{9,7,6,5,2} \rangle^*$																			1	1
$\langle \mathbf{8,7,6,5,3} \rangle^*$																			1	
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}

Proof. Using (5,7)-inducing of p.i.s. method on D_1 for S_{28} to S_{29} we have

$D_1 \uparrow^{(5,7)} S_{29}$=$(\langle 28 \rangle + \langle 22,6 \rangle^* + \langle 17,11 \rangle^* + \langle 17,10,1 \rangle + \langle 17,10,1 \rangle' + 2\langle 12,10,6 \rangle) \uparrow^{(5,7)} S_{29}$

$=\langle 29 \rangle^* + \langle 22,7 \rangle + \langle 22,7 \rangle' + \langle 18,11 \rangle + \langle 18,11 \rangle' + 2\langle 18,10,1 \rangle^* + 2\langle 12,10,7 \rangle^*$

$=d_1$.

similarly, using $(\mathrm{r},\bar{\mathrm{r}})$ −inducing of p.i.s. for S_{28} to S_{29} gives

$D_3 \uparrow^{(5,7)} S_{29} = d_2$, $D_5 \uparrow^{(5,7)} S_{29} = d_3$, $D_7 \uparrow^{(5,7)} S_{29} = d_4$, $D_{228} \uparrow^{(4,8)} S_{29} = d_5$, $D_{13} \uparrow^{(5,7)} S_{29} = d_6$, $D_{15} \uparrow^{(5,7)} S_{29} = d_7$, $D_{17} \uparrow^{(5,7)} S_{29} = d_8$, $D_{19} \uparrow^{(5,7)} S_{29} = d_9$, $D_{11} \uparrow^{(5,7)} S_{29} = d_{10}$, $D_{233} \uparrow^{(6,6)} S_{29} = d_{11}$, $D_{23} \uparrow^{(5,7)} S_{29} = d_{12}$, $D_{25} \uparrow^{(5,7)} S_{29} = d_{13}$, $D_{27} \uparrow^{(5,7)} S_{29} = d_{14}$, $D_{29} \uparrow^{(5,7)} S_{29} = d_{15}$, $D_{31} \uparrow^{(5,7)} S_{29} = d_{16}$, $D_{33} \uparrow^{(5,7)} S_{29} = d_{17}$, $D_{35} \uparrow^{(5,7)} S_{29} = d_{18}$, $D_{37} \uparrow^{(5,7)} S_{29} = d_{19}$, $D_{39} \uparrow^{(5,7)} S_{29} = d_{20}$, and on $(11,\alpha)$-regular classes:

1. $\langle 22,7 \rangle = \langle 22,7 \rangle'$
2. $\langle 18,11 \rangle = \langle 18,11 \rangle'$
3. $\langle 11,10,7,1 \rangle = \langle 11,10,7,1 \rangle'$
4. $\langle 11,9,7,2 \rangle = \langle 11,9,7,2 \rangle'$
5. $\langle 11,8,7,3 \rangle = \langle 11,8,7,3 \rangle'$
6. $\langle 11,7,6,5 \rangle = \langle 11,7,6,5 \rangle'$
7. $\langle 18,7,4 \rangle^* = \langle 18,6,5 \rangle^* + \langle 18,8,3 \rangle^* - \langle 18,9,2 \rangle^* + \langle 18,10,1 \rangle^* - \langle 18,11 \rangle - \langle 29 \rangle^*$
8. $\langle 11,8,7,3 \rangle = \langle 11,7,6,5 \rangle + \langle 11,9,7,2 \rangle - \langle 11,10,7,1 \rangle + \langle 18,11 \rangle - \langle 22,7 \rangle$
9. $\langle 10,8,7,3,1 \rangle^* = \langle 10,7,6,5,1 \rangle^* + \langle 10,9,7,2,1 \rangle^* + \langle 11,10,7,1 \rangle - \langle 12,10,7 \rangle^* + \langle 18,10,1 \rangle^* - \langle 21,7,1 \rangle^*$
10. $\langle 10,7,6,5,1 \rangle^*$=$\langle 9,7,6,5,2 \rangle^* - \langle 8,7,6,5,3 \rangle^* + \langle 11,7,6,5 \rangle - \langle 16,7,6 \rangle^* + \langle 17,7,5 \rangle^* - \langle 18,6,5 \rangle^*$
11. $\langle 8,7,6,5,3 \rangle^* = \langle 9,7,6,5,2 \rangle^* - \langle 10,8,7,3,1 \rangle^* + \langle 10,9,7,2,1 \rangle^* + \langle 11,7,6,5 \rangle + \langle 11,10,7,1 \rangle - \langle 21,10,7 \rangle^* - \langle 16,7,6 \rangle^* + \langle 17,7,5 \rangle^* - \langle 18,6,5 \rangle^* + \langle 18,10,1 \rangle^* - \langle 21,7,1 \rangle^*$
12. $\langle 9,7,6,5,2 \rangle^* = \langle 9,8,7,3,2 \rangle^* + \langle 10,9,7,2,1 \rangle^* - \langle 11,9,7,2 \rangle + \langle 13,9,7 \rangle^* - \langle 18,9,2 \rangle^* + \langle 20,7,2 \rangle^*$

hence, since the number of i.m.s is equal to or fewer than the number of spin characters, the matrix can only have 32 columns. **Table 1** can only have a maximum of 20 columns due to the fact that it has 12 equations that match the spin characters S_{29} in $\boldsymbol{B_2}$.

case 2. Decomposition matrix for the blocks $\boldsymbol{B_2}$ of type associative as shown in the **Tables 2**.

Table 2. Block $\boldsymbol{B_2}$

Spin character	Decomposition matrix																																							
$\langle 28,1\rangle$	1																																							
$\langle 28,1\rangle'$		1																																						
$\langle 23,6\rangle$	1		1																																					
$\langle 23,6\rangle'$		1		1																																				
$\langle 22,6,1\rangle^*$			1	1	1	1																																		
$\langle 20,6,2,1\rangle$					1		1																																	
$\langle 20,6,2,1\rangle'$						1		1																																
$\langle 19,6,3,1\rangle$							1		1																															
$\langle 19,6,3,1\rangle'$								1		1																														
$\langle 18,6,4,1\rangle$									1		1																													
$\langle 18,6,4,1\rangle'$										1		1																												
$\langle 17,12\rangle$			1										1																											
$\langle 17,12\rangle'$				1										1																										
$\langle 17,11,1\rangle^*$	1	1	1	1	1	1							1	1	1	1																								
$\langle 17,9,2,1\rangle$					1		1								1		1																							
$\langle 17,9,2,1\rangle'$						1		1								1		1																						
$\langle 17,8,3,1\rangle$							1		1								1		1																					
$\langle 17,8,3,1\rangle'$								1		1								1		1																				
$\langle 17,7,4,1\rangle$									1		1								1		1																			
$\langle 17,7,4,1\rangle'$										1		1								1		1																		
$\langle 17,6,5,1\rangle$											1										1																			
$\langle 17,6,5,1\rangle'$												1										1																		
$\langle 15,7,6,1\rangle$																			1		1		1																	
$\langle 15,7,6,1\rangle'$																				1		1		1																
$\langle 14,8,6,1\rangle$																	1		1				1		1															
$\langle 14,8,6,1\rangle'$																		1		1				1		1														
$\langle 13,9,6,1\rangle$															1		1								1		1													
$\langle 13,9,6,1\rangle'$																1		1								1		1												
$\langle 12,11,6\rangle^*$	1	1											1	1	1	1													1	1										
$\langle 12,10,6,1\rangle$													1	1	1												1		1	1	1	1								
$\langle 12,10,6,1\rangle'$													1	1		1												1	1	1	1	1								
$\langle 12,9,6,2\rangle$																									1		1		1		1	1	1							
$\langle 12,9,6,2\rangle'$																										1		1		1	1	1		1						

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$\langle 12,8,6,3\rangle$																							1		1								1		1					
$\langle 12,8,6,3\rangle'$																								1		1								1		1				
$\langle 12,7,6,4\rangle$																							1												1					
$\langle 12,7,6,4\rangle'$																								1												1				
$\langle 11,9,6,2,1,\rangle^*$																													1	1	1	1	1	1			1	1		
$\langle 11,8,6,3,1\rangle^*$																															1	1	1	1	1	1	1	1	1	1
$\langle 11,7,6,4,1\rangle^*$																																			1	1			1	1
$\langle 9,8,6,3,2,1\rangle$																															1								1	
$\langle 9,8,6,3,2,1\rangle'$																																1								1
$\langle 9,7,6,4,2,1\rangle$																																					1		1	
$\langle 9,7,6,4,2,1\rangle'$																																						1		1
$\langle 8,7,6,4,3,1\rangle$																																					1			
$\langle 8,7,6,4,3,1\rangle'$																																						1		
	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}	d_{37}	d_{38}	d_{39}	d_{40}	d_{41}	d_{42}	d_{43}	d_{44}	d_{45}	d_{46}	d_{47}	d_{48}	d_{49}	d_{50}	d_{51}	d_{52}	d_{53}	d_{54}	d_{55}	d_{56}	d_{57}	d_{58}	d_{59}	d_{60}

Proof. The following values are obtained by using $(r,\bar{r})$-inducing of p.i.s. D_{41}, D_3, D_5, D_6, ..., D_{13},D_{15}, D_{16},...,D_{22},D_{52},D_{53}, D_{54}, D_{29}, D_{31}, D_{32}, D_{33}, D_{35},D_{37}, D_{38}, D_{39}, D_{40} for S_{28} to S_{29} to give k_1,k_2,d_{25},d_{26},..., d_{32},k_3, d_{35}, d_{36}, …, d_{42},k_4,k_5,…,k_7, d_{51}, d_{52},k_8,k_9,d_{57},d_{58},…, d_{60} respectively. Since$\langle 28,1\rangle$ and $\langle 28,1\rangle'$ are distinct irreducible modular spin characters (**Theorem 2.4**), k_1 must divide into d_{21},d_{22}. Since $\langle 17,12\rangle \neq \langle 17,12\rangle'$ so k_2 split to d_{23}, d_{24} or k_3 is split to d_{33}, d_{34}. Suppose k_3 is split, but $\langle 23,6\rangle \neq \langle 23,6\rangle'$ then k_2 split. If k_2 split, and frome $(11,\alpha)$-regular classes

$$\langle 17,12\rangle - \langle 23,6\rangle + \langle 28,1\rangle \neq \langle 17,12\rangle' - \langle 23,6\rangle' + \langle 28,1\rangle' \qquad (1)$$

then k_3 also split so in both cases we get k_2 and k_3 are splits. A split is made in k_4 or k_5 because of $\langle 14,8,6,1\rangle \neq \langle 14,8,6,1\rangle'$. If k_5 is split into d_{45},d_{46},however, $\langle 12,7,6,4\rangle \neq \langle 12,7,6,4\rangle'$ split k_4 to d_{43}, d_{44} after that. If k_4 split, and frome$(11,\alpha)$-regular classes

$$\langle 14,8,6,1\rangle - \langle 15,7,6,1\rangle + \langle 17,4,1\rangle - \langle 17,8,3,1\rangle + \langle 19,6,3\rangle - \langle 18,6,4,1\rangle \neq$$

$$\langle 14,8,6,1\rangle' - \langle 15,7,6,1\rangle' + \langle 17,4,1\rangle' - \langle 17,8,3,1\rangle' + \langle 19,6,3\rangle' - \langle 18,6,4,1\rangle' \qquad (2)$$

then k_5 split so in both cases we get k_4 and k_5 are splits. Since $\langle 12,9,6,2\rangle \neq \langle 12,9,6,2\rangle'$ so k_6 or k_7 is split. Suppose k_7 is split to d_{49}, d_{50}, but $\langle 12,10,6,1\rangle \neq \langle 12,10,6,1\rangle'$ then k_6 split to d_{47}, d_{48}. If k_6 split, and frome $(11,\alpha)$-regular classes

$$\langle 12,9,6,2\rangle - \langle 13,9,6,1\rangle + \langle 17,9,2,1\rangle - \langle 20,6,2,1\rangle \neq$$

$$\langle 12,9,6,2\rangle' - \langle 13,9,6,1\rangle' + \langle 17,9,2,1\rangle' - \langle 20,6,2,1\rangle' \qquad (3)$$

then k_7 split so in both cases we get k_6 and k_7 are splits. Since $\langle 12,8,6,3\rangle \neq \langle 12,8,6,3\rangle'$ it follows that either k_8 or k_9 is split. Suppose k_8 is split to d_{53}, d_{54}, but $\langle 12,7,6,4\rangle \neq \langle 12,7,6,4\rangle'$ then k_9 split to d_{55}, d_{56}. If k_9 split, and frome $(11,\alpha)$-regular classes

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$$\langle 12,8,6,3\rangle - \langle 14,8,6,1\rangle + \langle 17,8,3,1\rangle - \langle 19,6,3,1\rangle - \langle 12,7,6,4\rangle + \langle 15,7,6,1\rangle - \langle 17,7,4,1\rangle + \langle 18,6,4,1\rangle \neq \langle 12,8,6,3\rangle' - \langle 14,8,6,1\rangle' + \langle 17,8,3,1\rangle' - \langle 19,6,3,1\rangle' - \langle 12,7,6,4\rangle' + \langle 15,7,6,1\rangle' - \langle 17,7,4,1\rangle' + \langle 18,6,4,1,\rangle' \qquad (4)$$

then k_8 split so in both cases we get k_8 and k_9 are splits. **Tables 2** is the outcome of the information presented previously.

Case 3. The block $\boldsymbol{B_4}$ of type associate's decomposition matrix, as given in **Tables 3.**

Table 3. Block $\boldsymbol{B_4}$

Spin character	Decomposition matrix																																							
$\langle 26,3\rangle$	1																																							
$\langle 26,3\rangle'$		1																																						
$\langle 25,4\rangle$	1		1																																					
$\langle 25,4\rangle'$		1		1																																				
$\langle 22,4,3\rangle^*$			1	1	1	1																																		
$\langle 21,4,3,1\rangle$					1		1																																	
$\langle 21,4,3,1\rangle'$						1		1																																
$\langle 20,4,3,2\rangle$							1		1																															
$\langle 20,4,3,2\rangle'$								1		1																														
$\langle 17,5,4,3\rangle$									1		1																													
$\langle 17,5,4,3\rangle'$										1		1																												
$\langle 16,6,4,3\rangle$									1		1		1																											
$\langle 16,6,4,3\rangle'$										1		1		1																										
$\langle 15,14\rangle$			1												1																									
$\langle 15,14\rangle'$				1												1																								
$\langle 15,11,3\rangle^*$	1	1	1	1	1	1									1	1	1	1																						
$\langle 15,10,3,1\rangle$					1		1										1		1																					
$\langle 15,10,3,1\rangle'$						1		1										1		1																				
$\langle 15,9,3,2\rangle$							1		1										1		1																			
$\langle 15,9,3,2\rangle'$								1		1										1		1																		
$\langle 15,7,4,3\rangle$									1				1									1		1																	
$\langle 15,7,4,3\rangle'$										1				1									1		1																
$\langle 15,6,5,3\rangle$													1											1																	
$\langle 15,6,5,3\rangle'$														1											1																
$\langle 14,11,4\rangle^*$	1	1													1	1	1	1								1	1														
$\langle 14,10,4,1\rangle$															1	1	1		1							1		1													
$\langle 14,10,4,1\rangle'$															1	1		1		1							1		1												
$\langle 14,9,4,2\rangle$																			1		1						1		1											
$\langle 14,9,4,2\rangle'$																				1		1						1		1										
$\langle 14,8,4,3\rangle$																					1		1						1		1									
$\langle 14,8,4,3\rangle'$																						1		1						1		1								

$\langle \mathbf{14,6,5,4} \rangle$																							1								1									
$\langle \mathbf{14,6,5,4} \rangle'$																								1								1									
$\langle \mathbf{13,9,4,3} \rangle$																											1		1		1		1								
$\langle \mathbf{13,9,4,3} \rangle'$																												1		1		1		1							
$\langle \mathbf{12,10,4,3} \rangle$															1	1											1	1	1							1		1	1				
$\langle \mathbf{12,10,4,3} \rangle'$															1	1											1	1		1						1	1	1				
$\langle \mathbf{11,10,4,3,1} \rangle^{*}$																									1	1							1	1	1	1	1	1		
$\langle \mathbf{11,9,4,3,2} \rangle^{*}$																															1	1	1	1	1	1	1	1	1	1
$\langle \mathbf{11,6,5,4,3} \rangle^{*}$																															1	1							1	1
$\langle \mathbf{10,9,4,3,2,1} \rangle$																																			1				1	
$\langle \mathbf{10,9,4,3,2,1} \rangle'$																																				1				1
$\langle \mathbf{10,6,5,4,3,1} \rangle$																																					1		1	
$\langle \mathbf{10,6,5,4,3,1} \rangle'$																																						1		1
$\langle \mathbf{9,6,5,4,3,2} \rangle$																																					1			
$\langle \mathbf{9,6,5,4,3,2} \rangle'$																																						1		
	d_{101}	d_{102}	d_{103}	d_{104}	d_{105}	d_{106}	d_{107}	d_{108}	d_{109}	d_{110}	d_{111}	d_{112}	d_{113}	d_{114}	d_{115}	d_{116}	d_{117}	d_{118}	d_{119}	d_{120}	d_{121}	d_{122}	d_{123}	d_{124}	d_{125}	d_{126}	d_{127}	d_{128}	d_{129}	d_{130}	d_{131}	d_{132}	d_{133}	d_{134}	d_{135}	d_{136}	d_{137}	d_{138}	d_{139}	d_{140}

Proof. The following values are obtained by using $(r,\bar{r})$-inducing of p.i.s. :D_{61}, D_{62}, D_{24}, D_{25}, D_{64},D_{65}, ..., D_{68}, D_{153}, D_{154}, D_{123}, D_{71},D_{125}, D_{155}, D_{156}, D_{75}, D_{73}, D_{77}, D_{189}, D_{157}, D_{158}, D_{79}, D_{159}, D_{160} for S_{28} to S_{29} to give k_1, k_2, d_{105}, d_{106},k_3,k_4,…,k_7, d_{117}, d_{118},k_8, k_9,k_{10}, d_{125}, d_{126}, k_{11}, k_{12},k_{13}, k_{14}, d_{135}, d_{136},k_{15}, d_{139}, d_{140} respectively. Since $\langle 15,14\rangle \neq \langle 15,14\rangle'$ so k_2 or k_7 is split. Suppose k_7 is split to d_{115}, d_{116}, but $\langle 25,4\rangle \neq \langle 25,4\rangle'$ then k_2 split to d_{103}, d_{104}. If k_2 split, and frome $(11,\alpha)$-regular classes

$$\langle 15,14\rangle - \langle 25,4\rangle + \langle 26,3\rangle \neq \langle 15,14\rangle' - \langle 25,4\rangle' + \langle 26,3\rangle' \quad (5)$$

then k_7 also split, so in both cases we get k_2 and k_7 are splits.Since $\langle 26,3\rangle \neq \langle 26,3\rangle'$so k_1 divided or there are two columns φ_1,φ_2, but block $\boldsymbol{B_4}$ is associate so there must be another column associate with k_1 then $\langle 26,3\rangle$ has 3 i.m.s., but $\langle 26,3\rangle \downarrow S_{28} = \langle 25,3\rangle^{*1} + \langle 26,2\rangle^{*1}$ has only two of i.m.s. so this contradicts the hypothesis, then $k_1 = d_{101} + d_{102}$. Since $\langle 20,4,3,2\rangle \neq \langle 20,4,3,2\rangle'$ so k_3 or k_4 is split. Suppose k_3 is split to d_{107}, d_{108}, but $\langle 17,5,4,3\rangle \neq \langle 17,5,4,3\rangle'$ then k_4 split to d_{109}, d_{110}. If k_4 split, and frome $(11,\alpha)$-regular classes

$$\langle 20,4,3,2\rangle + \langle 16,6,4,3\rangle - \langle 17,5,4,3\rangle \neq \langle 20,4,3,2\rangle' + \langle 16,6,4,3\rangle' - \langle 17,5,4,3\rangle' \quad (6)$$

then k_3 also split so in both cases we get k_3 and k_4 are splits. Since $\langle 16,6,4,3\rangle \neq \langle 16,6,4,3\rangle'$ so k_5 or k_6 is split. Suppose k_6 is split to d_{113}, d_{114}, but $\langle 15,6,5,3\rangle \neq \langle 15,6,5,3\rangle'$ then k_5 split to d_{111}, d_{112}. If k_5 split, and frome $(11,\alpha)$-regular classes

$$\langle 16,6,4,3\rangle - \langle 17,5,4,3\rangle \neq \langle 16,6,4,3\rangle' - \langle 17,5,4,3\rangle' \quad (7)$$

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then k_6 also split so in both cases we get k_5 and k_6 are splits. Since $\langle 14,6,5,4\rangle \neq \langle 14,6,5,4\rangle'$ so k_{10} or k_{13} is split. Suppose k_{10} is split to d_{123}, d_{124}, but $\langle 13,9,4,3\rangle \neq \langle 13,9,4,3\rangle'$ then k_{13} split to d_{131}, d_{132}. If k_{13} split, and frome $(11,\alpha)$-regular classes

$$\langle 14,6,5,4\rangle + \langle 12,10,4,3\rangle - \langle 13,9,4,3\rangle + \langle 14,9,4,2\rangle - \langle 15,9,3,2\rangle + \langle 20,4,3,2\rangle \neq \langle 14,6,5,4\rangle' + \langle 12,10,4,3\rangle' - \langle 13,9,4,3\rangle' + \langle 14,9,4,2\rangle' - \langle 15,9,3,2\rangle' + \langle 20,4,3,2\rangle' \quad (8)$$

then k_{10} also split so in both cases we get k_{10} and k_{13} are splits. Since $\langle 15,9,3,2\rangle \neq \langle 15,9,3,2\rangle'$ so k_8 or k_9 is split. Suppose k_8 is split to d_{119}, d_{120}, but $\langle 14,8,4,3\rangle \neq \langle 14,8,4,3\rangle'$ then k_9 split to d_{121}, d_{122}. If k_9 split, and frome $(11,\alpha)$-regular classes

$$\langle 15,9,3,2\rangle - \langle 20,4,3,2\rangle - \langle 14,8,4,3\rangle + \langle 14,6,5,4\rangle + \langle 13,9,4,3\rangle \neq \langle 15,9,3,2\rangle' - \langle 20,4,3,2\rangle' - \langle 14,8,4,3\rangle' + \langle 14,6,5,4\rangle' + \langle 13,9,4,3\rangle' \quad (9)$$

then k_8 also split so in both cases we get k_8 and k_9 are splits. Since $\langle 12,10,4,3\rangle \neq \langle 12,10,4,3\rangle'$ so k_{11} or k_{14} is split. Suppose k_{14} is split to d_{133}, d_{134}, but $\langle 13,9,4,3\rangle \neq \langle 13,9,4,3\rangle'$ then k_{11} split to d_{127}, d_{128}. If k_{11} split,frome $(11,\alpha)$-regular classes

$$\langle 13,9,4,3\rangle - \langle 14,6,5,4\rangle + \langle 15,6,5,3\rangle - \langle 16,6,4,3\rangle + \langle 17,5,4,3\rangle - \langle 14,9,4,2\rangle + \langle 15,9,3,2\rangle - \langle 20,4,3,2\rangle \neq \langle 13,9,4,3\rangle' - \langle 14,6,5,4\rangle' + \langle 15,6,5,3\rangle' - \langle 16,6,4,3\rangle' + \langle 17,5,4,3\rangle' - \langle 14,9,4,2\rangle' + \langle 15,9,3,2\rangle' - \langle 20,4,3,2\rangle' \quad (10)$$

then k_{14} also split so in both cases we get k_{11} and k_{14} are splits.Since $\langle 14,9,4,2\rangle \neq \langle 14,9,4,2\rangle'$ so k_{12} divided or there are two columns φ_1, φ_2 such that $\varphi_1 = a_1\langle 14,9,4,2\rangle + a_2\langle 14,8,4,3\rangle + a_3\langle 14,6,5,4\rangle + a_4\langle 13,9,4,3\rangle + a_5\langle 12,10,4,3\rangle + a_6\langle 11,10,4,3,1\rangle^* + a_7\langle 11,9,4,3,2\rangle^* + a_8\langle 11,6,5,4,3\rangle^* + a_9\langle 10,9,4,3,2,1\rangle + a_{10}\langle 10,6,5,4,3,1\rangle + a_{11}\langle 9,6,5,4,3,2\rangle, \varphi_2 = a_1\langle 14,9,4,2\rangle' + a_2\langle 14,8,4,3\rangle' + a_3\langle 14,6,5,4\rangle' + a_4\langle 13,9,4,3\rangle' + a_5\langle 12,10,4,3\rangle' + a_6\langle 11,10,4,3,1\rangle^* + a_7\langle 11,9,4,3,2\rangle^* + a_8\langle 11,6,5,4,3\rangle^* + a_9\langle 10,9,4,3,2,1\rangle' + a_{10}\langle 10,6,5,4,3,1\rangle' + a_{11}\langle 9,6,5,4,3,2\rangle'$, to find are since $\langle 14,9,4,2\rangle \downarrow S_{28} = \langle 13,9,4,2\rangle^{*4} + \langle 14,8,4,2\rangle^{*4} + \langle 14,9,3,2\rangle^{*2} + \langle 14,9,4,1\rangle^{*2} = 12$ of i.m.s. then we have $a_1 \in \{0,1,\ldots,8\}$, in the same way we got $a_3, a_8, a_{11} \in \{0,1\}$, $a_9, a_{10} \in \{0,1,2\}$, $a_6, a_7 \in \{0,1,\ldots,4\}$, $a_2, a_4, a_5 \in \{0,1,\ldots,6\}$. let $a_1 \in \{1,2,\ldots,8\}$(if $a_1 = 0$ then k_{12} is split), since $\langle 14,9,4,2\rangle \downarrow S_{28} \cap \langle 11,6,5,4,3\rangle^* \downarrow S_{28}$ has no i.m.s so $a_8 = 0$, the same way we get a_9, a_{10}, a_{11} equals to zero, we must discuss all probabilities such that the degree $\varphi_1, \varphi_2 \equiv 0 \; mod 11^2$, that's difficult, so we do the algorithm by maple program in appendix to help us and we find it are equal to **1156** probabilities, therefore, we aim to limit the number of possibilities, since inducing m.s. is m.s. we have:

$$(\langle 14,7,4,3\rangle^* - \langle 14,6,5,3\rangle^* - \langle 14,9,3,2\rangle^* + \langle 14,10,3,1\rangle) \uparrow^{(4,8)} S_{29} \; hence \; a_2 \geq a_1 + a_3 \quad (11)$$

$$(\langle 14,9,3,2\rangle^* - \langle 14,7,4,3\rangle^* + \langle 14,6,5,3\rangle^*) \uparrow^{(4,8)} S_{29} \; hence \; a_1 + a_3 \geq a_2 \; \therefore a_2 = a_1 + a_3 \quad (12)$$

$$(\langle 11,10,4,3\rangle^* - \langle 10,9,4,3,2\rangle - \langle 10,9,4,3,2\rangle' + \langle 10,6,5,4,3\rangle + \langle 10,6,5,4,3\rangle') \uparrow^{(0,1)} S_{29} \; hence \; a_5 + a_6 \geq 2a_7 \quad (13)$$

$$(\langle 10,9,4,3,2\rangle + \langle 10,9,4,3,2\rangle' - \langle 11,10,4,3\rangle^* + \langle 14,10,4\rangle + \langle 14,10,4\rangle') \uparrow^{(0,1)} S_{29} \; hence \; 2a_7 \geq a_5 + a_6 \; \therefore 2a_7 = a_5 + a_6 \quad (14)$$

$$(\langle 12,9,4,3\rangle^* - \langle 14,9,4,1\rangle^* + \langle 15,9,3,1\rangle - \langle 11,9,4,3,1\rangle + \langle 9,6,5,4,3,1\rangle^*) \uparrow^{2,10} S_{29}$$

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$$hence\ a_4 + a_5 \geq a_1 + a_6 + a_7 \tag{15}$$

$$(\langle 14,9,4,1\rangle^* - \langle 12,9,4,3\rangle^* + \langle 11,9,4,3,1\rangle) \uparrow^{(2,10)} S_{29}$$

$$hence\ a_1 + a_6 + a_7 \geq a_4 + a_5\ \therefore a_4 + a_5 = a_1 + a_6 + a_7 \tag{16}$$

we get degrees $\varphi_1, \varphi_2 \equiv 0 \ mod\ 11^2$ only when $\varphi_1 + \varphi_2 = mk_{12} + nk_{14}$ such that $m \in \{1,2,\ldots,6\}$, $n \in \{0,1,\ldots,4\}$ where is split to k_{12} and k_{14} then k_{12} splits to d_{129}, d_{130}. then turn to **Table 3**.

Case 4. Table 4 shows the decomposition matrices for the spin block $\boldsymbol{B_5}$ of the double type.

Table 4. Block $\boldsymbol{B_5}$

Spin character	Decomposition matrix																			
$\langle 26,2,1\rangle^*$	1																			
$\langle 24,4,1\rangle^*$	1	1																		
$\langle 23,4,2\rangle^*$		1	1																	
$\langle 22,4,2,1\rangle$			1	1																
$\langle 19,4,3,2,1\rangle^*$				1	1															
$\langle 17,5,4,2,1\rangle^*$					1	1														
$\langle 16,6,4,2,1\rangle^*$					1	1	1													
$\langle 15,13,1\rangle^*$		1						1												
$\langle 15,12,2\rangle^*$	1	1	1					1	1											
$\langle 15,11,2,1\rangle$			1	1					1	1										
$\langle 15,8,3,2,1\rangle^*$				1	1					1	1									
$\langle 15,7,4,2,1\rangle^*$				1	1		1				1	1								
$\langle 15,6,5,2,1\rangle^*$							1					1								
$\langle 14,8,4,2,1\rangle^*$										1	1	1	1							
$\langle 13,12,4\rangle^*$	1								1					1						
$\langle 13,11,4,1\rangle$								1	1	1				1	1					
$\langle 13,9,4,2,1\rangle^*$										1			1		1	1				
$\langle 13,8,4,3,1\rangle^*$												1	1			1	1			
$\langle 13,6,5,4,1\rangle^*$												1					1			
$\langle 12,11,4,2\rangle$								1						1	1			1		
$\langle 12,10,4,2,1\rangle^*$														2	1	1		2	2	
$\langle 12,8,4,3,2\rangle^*$																1	1	1	2	1
$\langle 12,6,5,4,2\rangle^*$																	1			1
$\langle 11,8,4,3,2,1\rangle$																		1	1	1
$\langle 11,6,5,4,2,1\rangle$																			1	1
$\langle 8,6,5,4,3,2,1\rangle^*$																			1	
	d_{141}	d_{142}	d_{143}	d_{144}	d_{145}	d_{146}	d_{147}	d_{148}	d_{149}	d_{150}	d_{151}	d_{152}	d_{153}	d_{154}	d_{155}	d_{156}	d_{157}	d_{158}	d_{159}	d_{160}

Proof. Using $(\mathrm{r}, \overline{\mathrm{r}})$- inducing of p.i.s. D_{81}, D_{83}, D_{63}, D_{64}, …, D_{67}, D_{97}, D_{69}, D_{70},…,D_{75}, D_{111}, D_{113}, D_{78}, D_{79}, D_{80}, for S_{27} to S_{28} ,and on $(11, \alpha)$-regular classes:

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1. $\langle 22,4,2,1\rangle = \langle 22,4,2,1\rangle'$
2. $\langle 15,11,2,1\rangle = \langle 15,11,2,1\rangle'$
3. $\langle 13,11,4,1\rangle = \langle 13,11,4,1\rangle'$
4. $\langle 12,11,4,2\rangle = \langle 12,11,4,2\rangle'$
5. $\langle 11,8,4,3,2,1\rangle = \langle 11,8,4,3,2,1\rangle'$
6. $\langle 11,6,5,4,2,1\rangle = \langle 11,6,5,4,2,1\rangle'$
7. $\langle 15,7,4,2,1\rangle^* = \langle 15,6,5,2,1\rangle^* + \langle 15,8,3,2,1\rangle^* - \langle 15,11,2,1\rangle + \langle 15,12,2\rangle^* - \langle 15,13,1\rangle^* + \langle 22,4,2,1\rangle - \langle 23,4,2\rangle^* + \langle 24,4,1\rangle^* - 2\langle 26,2,1\rangle^*$
8. $\langle 13,8,4,3,1\rangle^* = \langle 13,6,5,4,1\rangle^* + \langle 13,9,4,2,1\rangle^* - \langle 13,11,4,1\rangle + \langle 13,12,4\rangle^* + \langle 15,11,2,1\rangle - \langle 15,12,2\rangle^* + 2\langle 15,13,1\rangle^* - \langle 22,4,2,1\rangle + \langle 23,4,2\rangle^* - 2\langle 24,4,1\rangle^* + 2\langle 26,2,1\rangle^*$
9. $\langle 12,10,4,2,1\rangle^* = 2\langle 8,6,5,4,3,2,1\rangle^* + 2\langle 12,11,4,2\rangle - \langle 13,9,4,2,1\rangle^* + \langle 14,8,4,2,1\rangle^* - \langle 15,7,4,2,1\rangle^* - 2\langle 15,13,1\rangle^* + \langle 16,6,4,2,1\rangle^* - \langle 17,5,4,2,1\rangle^* + \langle 19,4,3,2,1\rangle^* + 2\langle 24,4,1\rangle^* - 2\langle 26,2,1\rangle^*$
10. $\langle 12,8,4,3,2\rangle^* = \langle 8,6,5,4,3,2,1\rangle^* + \langle 11,6,5,4,2,1\rangle + \langle 12,11,4,2\rangle + \langle 13,8,4,3,1\rangle^* - \langle 13,11,4,1\rangle - \langle 14,8,4,2,1\rangle^* + \langle 15,8,3,2,1\rangle^* + \langle 15,11,2,1\rangle - \langle 19,4,3,2,1\rangle^* - \langle 22,4,2,1\rangle$
11. $\langle 11,8,4,3,2,1\rangle = \langle 11,6,5,4,2,1\rangle + \langle 12,11,4,2\rangle - \langle 13,11,4,1\rangle + \langle 15,11,2,1\rangle - \langle 22,4,2,1\rangle$
12. $\langle 126,5,4,2\rangle^* = \langle 11,6,5,4,2,1\rangle - \langle 8,6,5,4,3,2,1\rangle^* + \langle 13,6,5,4,1\rangle^* - \langle 15,6,5,2,1\rangle^* + \langle 16,6,4,2,1\rangle^* - \langle 17,5,4,2,1\rangle^*$

Then the matrix contains at most 20 columns since there are 12 equations corresponding the spin characters of S_{29} in $\boldsymbol{B_5}$. Then we get **Table 4**.

Case 5. Decomposition matrices for the blocks $\boldsymbol{B_6}, \boldsymbol{B_7}, \boldsymbol{B_8}$ of type duble as shown in **Tables 5**.

Table 5. Blocks $\boldsymbol{B_6}, \boldsymbol{B_7}, \boldsymbol{B_8}$

Block	spin character	Decomposition matrix														
$\boldsymbol{B_6}$	$\langle \mathbf{25,3,1}\rangle^*$	1														
	$\langle \mathbf{14,12,3}\rangle^*$	1	1													
	$\langle \mathbf{14,11,3,1}\rangle$		1	1												
	$\langle \mathbf{14,9,3,2,1}\rangle^*$			1	1											
	$\langle \mathbf{14,7,4,3,1}\rangle^*$				1	1										
	$\langle \mathbf{14,6,5,3,1}\rangle^*$					1										
$\boldsymbol{B_7}$	$\langle \mathbf{24,3,2}\rangle^*$						1									
	$\langle \mathbf{14,13,2}\rangle^*$						1	1								
	$\langle \mathbf{13,11,3,2}\rangle$							1	1							
	$\langle \mathbf{13,10,3,2,1}\rangle^*$								1	1						
	$\langle \mathbf{13,7,4,3,2}\rangle^*$									1	1					
	$\langle \mathbf{13,6,5,3,2}\rangle^*$										1					
$\boldsymbol{B_8}$	$\langle \mathbf{23,5,1}\rangle^*$											1				
	$\langle \mathbf{16,12,1}\rangle^*$											1	1			
	$\langle \mathbf{12,11,5,1}\rangle$												1	1		
	$\langle \mathbf{12,9,5,2,1}\rangle^*$													1	1	
	$\langle \mathbf{12,8,5,3,1}\rangle^*$														1	1
	$\langle \mathbf{12,7,5,4,1}\rangle^*$															1
		d_{161}	d_{162}	d_{163}	d_{164}	d_{165}	d_{166}	d_{167}	d_{168}	d_{169}	d_{170}	d_{171}	d_{172}	d_{173}	d_{174}	d_{175}

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Proof. By using $(r, \bar{r})$-inducing of p.i.s. of $D_{83}, D_{85}, D_{87}, D_{95}, D_{93}$, D_{81} , D_{99}, D_{101}, D_{103}, D_{105}, D_{126}, D_{128}, D_{130}, D_{132}, D_{134} for S_{28} to S_{29} gives d_{161}, d_{162},…,d_{175} respictively.Since

- degree $\{\langle 25,3,1\rangle^*, \langle 14,11,3,1\rangle + \langle 14,11,3,1\rangle', \langle 14,7,4,3,1\rangle^*\} \equiv 99\ mod\ 11^2$,
- degree $\{\langle 14,12,3\rangle^*, \langle 14,9,3,2,1\rangle^*, \langle 14,6,5,3,1\rangle^*\} \equiv -99\ mod\ 11^2$,
- degree $\{\langle 24,3,2\rangle^*, \langle 13,11,3,2\rangle + \langle 13,11,3,2\rangle', \langle 13,7,4,3,2\rangle^*\} \equiv 66\ mod\ 11^2$,
- degree $\{\langle 14,13,2\rangle^*, \langle 13,10,3,2,1\rangle^*, \langle 13,6,5,3,2\rangle^*\} \equiv -66\ mod\ 11^2$,
- degree $\{\langle 16,12,1\rangle^*, \langle 12,9,5,2,1\rangle^*, \langle 12,7,5,4,1\rangle^*\} \equiv 110\ mod\ 11^2$,
- degree $\{\langle 23,5,1\rangle^*, \langle 12,11,5,1\rangle + \langle 12,11,5,1\rangle', \langle 12,8,5,3,1\rangle^*\} \equiv -110\ mod\ 11^2$,

and on$(11, \alpha)$-regular classes:

1. $\langle 14,11,3,1\rangle = \langle 14,11,3,1\rangle'$
2. $\langle 14,9,3,2,1\rangle^* = \langle 114,7,4,3,1\rangle^* - \langle 14,6,5,3,1\rangle^* + \langle 14,11,3,1\rangle - \langle 14,12,3\rangle^* + \langle 25,3,1\rangle^*$
3. $\langle 13,11,3,2\rangle = \langle 13,11,3,2\rangle'$
4. $\langle 13,10,3,2,1\rangle^* = \langle 13,7,4,3,2\rangle^* - \langle 13,6,5,3,2\rangle^* + \langle 13,11,3,2\rangle - \langle 14,13,2\rangle^* + \langle 24,3,2\rangle^*$
5. $\langle 12,11,5,1\rangle = \langle 12,11,5,1\rangle'$
6. $\langle 12,9,5,2,1\rangle^* = \langle 12,8,5,3,1\rangle^* - \langle 12,7,5,4,1\rangle^* + \langle 12,11,5,1\rangle - \langle 16,12,1\rangle^* + \langle 23,5,1\rangle^*$

based on the above, each block contains 5 columns.frome above we get **Table 5**

Case 6. The decomposition matrix for the blocks $\boldsymbol{B_9}$, $\boldsymbol{B_{10}}$ of type associte as shown in **Tables 6**.

Table 6. Blocks $\boldsymbol{B_9}, \boldsymbol{B_{10}}$

Block	spin character	Decomposition matrix																			
B_9	$\langle 23,3,2,1\rangle$	1																			
	$\langle 23,3,2,1\rangle'$		1																		
	$\langle 14,12,2,1\rangle$	1		1																	
	$\langle 14,12,2,1\rangle'$		1		1																
	$\langle 13,12,3,1\rangle$			1		1															
	$\langle 13,12,3,1\rangle'$				1		1														
	$\langle 12,11,3,2,1\rangle^*$					1	1	1	1												
	$\langle 12,7,4,3,2,1\rangle$							1		1											
	$\langle 12,7,4,3,2,1\rangle'$								1		1										
	$\langle 12,6,5,3,2,1\rangle$									1											
	$\langle 12,6,5,3,2,1\rangle'$										1										
B_{10}	$\langle 21,8\rangle$											1									
	$\langle 21,8\rangle'$												1								
	$\langle 19,10\rangle$											1		1							
	$\langle 19,10\rangle'$												1		1						
	$\langle 11,10,8\rangle^*$													1	1	1	1				
	$\langle 10,9,8,2\rangle$															1		1			
	$\langle 10,9,8,2\rangle'$																1		1		
	$\langle 10,8,7,4\rangle$																	1		1	
	$\langle 10,8,7,4\rangle'$																		1		1
	$\langle 10,8,6,5\rangle$																			1	
	$\langle 10,8,6,5\rangle'$																				1
		d_{176}	d_{177}	d_{178}	d_{179}	d_{180}	d_{181}	d_{182}	d_{183}	d_{184}	d_{185}	d_{186}	d_{187}	d_{188}	d_{189}	d_{190}	d_{191}	d_{192}	d_{193}	d_{194}	d_{195}

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Proof. Using $(r,\bar{r})$-inducing of p.i.s. $D_{83}, D_{84}, D_{81}, D_{82}, D_{111}, D_{112}, D_{113}, D_{114}$, D_{136}, D_{137}, D_{138}, D_{139}, D_{140} of S_{28} to S_9 gives d_{176},d_{177}, d_{178}, d_{179}, k_1, , d_{182}, d_{183}, d_{184}, d_{185}, k_2, k_3,k_4,k_5, k_6 respectively. Since $\boldsymbol{B_9}$ of defect one then from (**Theorem 2.2**) k_1 must split to d_{180}, d_{181}. Since $\langle 21,8\rangle \neq \langle 21,8\rangle'$so k_2 divided or there are two columns is split, but block $\boldsymbol{B}_{10}$ is associate so there must be another column associate with k_2 then $\langle 26,3\rangle$ has 3 i.m.s. but $\langle 21,8\rangle \downarrow S_{28} = \langle 20,8\rangle^{*1} + \langle 21,7\rangle^{*1}$ has only two of i.m.s. so this contradicts the hypothesis, then $k_2 = d_{186} + d_{187}$. As since $\boldsymbol{B_{10}}$ of defect one then k_3, k_4 must splits to $d_{188} + d_{189}$ and $d_{190} + d_{191}$, respectively. Since $\langle 10,8,7,4\rangle \neq \langle 10,8,7,4\rangle'$ so k_5 or k_6 is split. Suppose k_4 is split to d_{192}, d_{193}, but $\langle 10,8,6,5\rangle \neq \langle 10,8,6,5\rangle'$ then k_6 split to d_{194}, d_{195}. If k_6 split, and frome $(11,\alpha)$-regular classes

$$\langle 10,8,7,4\rangle - \langle 10,8,6,5\rangle \neq \langle 10,8,7,4\rangle' - \langle 10,8,6,5\rangle' \quad (17)$$

then k_5 also split so in both cases we get k_5 and k_6 are splits. Then we get **Table 6**.

Case 7. Decomposition matrices for the duble-type blocks $\boldsymbol{B_{11}}$,$\boldsymbol{B_{12}}$,. . ,$\boldsymbol{B_{15}}$ are displayed in **Tables** 7.

Table 7. Blocks $\boldsymbol{B_{11}}$,$\boldsymbol{B_{12}}$, $\boldsymbol{B_{13}}$, $\boldsymbol{B_{14}}$, $\boldsymbol{B_{15}}$

Block	spin character	Decomposition matrix																								
$\boldsymbol{B_{11}}$	$\langle 21,6,2\rangle^*$	1																								
	$\langle 17,10,2\rangle^*$	1	1																							
	$\langle 13,10,6\rangle^*$		1	1																						
	$\langle 11,10,6,2\rangle$			1	1																					
	$\langle 10,8,6,3,2\rangle^*$				1	1																				
	$\langle 10,7,6,4,2\rangle^*$					1																				
$\boldsymbol{B_{12}}$	$\langle 21,5,3\rangle^*$						1																			
	$\langle 16,10,3\rangle^*$						1	1																		
	$\langle 14,10,5\rangle^*$							1	1																	
	$\langle 11,10,5,3\rangle$								1	1																
	$\langle 10,9,5,3,2\rangle^*$									1	1															
	$\langle 10,7,5,4,3\rangle^*$										1															
$\boldsymbol{B_{13}}$	$\langle 20,8,1\rangle^*$											1														
	$\langle 19,9,1\rangle^*$											1	1													
	$\langle 12,9,8\rangle^*$												1	1												
	$\langle 11,9,8,1\rangle$													1	1											
	$\langle 9,8,7,4,1\rangle^*$														1	1										
	$\langle 9,8,6,5,1\rangle^*$															1										
$\boldsymbol{B_{14}}$	$\langle 20,6,3\rangle^*$																1									
	$\langle 17,9,3\rangle^*$																1	1								
	$\langle 14,9,6\rangle^*$																	1	1							
	$\langle 11,9,6,3\rangle$																		1	1						
	$\langle 10,9,6,3,1\rangle^*$																			1	1					
	$\langle 9,7,6,4,3\rangle^*$																				1					
$\boldsymbol{B_{15}}$	$\langle 20,5,4\rangle^*$																					1				

	$\langle 16,9,4\rangle^*$																					1	1			
	$\langle 15,9,5\rangle^*$																						1	1		
	$\langle 11,9,5,4\rangle$																							1	1	
	$\langle 10,9,5,4,1\rangle^*$																								1	1
	$\langle 9,8,5,4,3\rangle^*$																									1
		d_{196}	d_{197}	d_{198}	d_{199}	d_{200}	d_{201}	d_{202}	d_{203}	d_{204}	d_{205}	d_{206}	d_{207}	d_{208}	d_{209}	d_{210}	d_{211}	d_{212}	d_{213}	d_{214}	d_{215}	d_{216}	d_{217}	d_{218}	d_{219}	d_{220}

Proof. By using $(r,\bar{r})$-inducing of p.i.s. D_3 , $D_{17}, D_{25}, D_{33}, D_{35}$, $D_{141}, D_{143}, D_{145}, D_{147}, D_{149}$, D_{166}, D_{168}, $D_{170}, D_{172}, D_{174}$, D_5, , $D_{15}, D_{27}, D_{29}, D_{39}$, D_{177} , $D_{179}, D_{181}, D_{183}, D_{185}$ for S_{28} to S_{29} gives d_{196}, d_{197},…,d_{220} respictively, since

- degree $\{\langle 17,10,2\rangle^*, \langle 11,10,6,2\rangle + \langle 11,10,6,2\rangle', \langle 10,7,6,4,2\rangle^*\} \equiv 110 \; mod \; 11^2$,
- degree $\{\langle 21,6,2\rangle^*, \langle 13,10,6\rangle^*, \langle 10,8,6,3,2\rangle^*\} \equiv -110 \; mod \; 11^2$,
- degree $\{\langle 21,5,3\rangle^*, \langle 14,10,5\rangle^*, \langle 10,9,5,3,2\rangle^*\} \equiv 77 \; mod \; 11^2$,
- degree $\{\langle 16,10,3\rangle^*, \langle 11,10,5,3\rangle + \langle 11,10,5,3\rangle', \langle 10,7,5,4,3\rangle^*\} \equiv -77 \; mod \; 11^2$,
- degree $\{\langle 20,8,1\rangle^*, \langle 12,9,8\rangle^*, \langle 9,8,7,4,1\rangle^*\} \equiv 99 \; mod \; 11^2$,
- degree $\{\langle 19,9,1\rangle^*, \langle 11,9,8,1\rangle + \langle 11,9,8,1\rangle', \langle 9,8,6,5,1\rangle^*\} \equiv -99 \; mod \; 11^2$,
- degree $\{\langle 17,9,3\rangle^*, \langle 11,9,6,3\rangle + \langle 11,9,6,3\rangle', \langle 9,7,6,4,3\rangle^*\} \equiv 66 \; mod \; 11^2$,
- degree $\{\langle 20,6,3\rangle^*, \langle 14,9,6\rangle^*, \langle 10,9,6,3,1\rangle^*\} \equiv -66 \; mod \; 11^2$,
- degree $\{\langle 20,5,4\rangle^*, \langle 15,9,5\rangle^*, \langle 10,9,5,4,1\rangle^*\} \equiv 110 \; mod \; 11^2$,
- degree $\{\langle 16,9,4\rangle^*, \langle 11,9,5,4\rangle + \langle 11,9,5,4\rangle', \langle 9,8,5,4,3\rangle^*\} \equiv -110 \; mod \; 11^2$,

and on$(11,\alpha)$-regular classes:

1. $\langle 11,10,6,2\rangle = \langle 11,10,6,2\rangle'$
2. $\langle 13,10,6\rangle^* = \langle 17,10,2\rangle^* - \langle 21,6,2\rangle^* + \langle 11,10,6,2\rangle - \langle 10,8,6,3,2\rangle^* + \langle 10,7,6,4,2\rangle^*$
3. $\langle 11,10,5,3\rangle = \langle 11,10,5,3\rangle'$
4. $\langle 14,10,5\rangle^* = \langle 16,10,3\rangle^* - \langle 21,5,3\rangle^* + \langle 11,10,5,3\rangle - \langle 10,9,5,3,2\rangle^* + \langle 10,7,5,4,3\rangle^*$
5. $\langle 11,9,8,1\rangle = \langle 11,9,8,1\rangle'$
6. $\langle 12,9,8\rangle^* = \langle 19,9,1\rangle^* - \langle 20,8,1\rangle^* + \langle 11,9,8,1\rangle - \langle 9,8,7,4,1\rangle^* + \langle 9,8,6,5,1\rangle^*$
7. $\langle 11,9,8,1\rangle = \langle 11,9,8,1\rangle'$
8. $\langle 14,9,6\rangle^* = \langle 17,9,3\rangle^* - \langle 20,6,3\rangle^* + \langle 11,9,6,3\rangle - \langle 10,9,6,3,1\rangle^* + \langle 9,7,6,4,3\rangle^*$
9. $\langle 11,9,5,4\rangle = \langle 11,9,5,4\rangle'$
10. $\langle 15,9,5\rangle^* = \langle 14,9,4\rangle^* - \langle 20,5,4\rangle^* + \langle 11,9,5,4\rangle - \langle 10,9,5,4,1\rangle^* + \langle 9,8,5,4,3\rangle^*$

based on the above, each blocks contains 5 columns , we get the **Table 7.**

Case 8. Decomposition matrix for the associte blocks $\boldsymbol{B_{16}}$ and $\boldsymbol{B_{17}}$ as displayed in **Tables 8.**

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Table 8. Blocks $\boldsymbol{B_{16}}$, $\boldsymbol{B_{17}}$

Block	spin character	Decomposition matrix																			
	$\langle 20,5,3,1\rangle$	1																			
	$\langle 20,5,3,1\rangle'$		1																		
	$\langle 16,9,3,1\rangle$	1		1																	
	$\langle 16,9,3,1\rangle'$		1		1																
	$\langle 14,9,5,1\rangle$			1		1															
$\boldsymbol{B_{16}}$	$\langle 14,9,5,1\rangle'$				1		1														
	$\langle 12,9,5,3\rangle$					1		1													
	$\langle 12,9,5,3\rangle'$						1		1												
	$\langle 11,9,5,3,1\rangle^*$							1	1	1	1										
	$\langle 9,7,5,4,3,1\rangle$									1											
	$\langle 9,7,5,4,3,1\rangle'$										1										
	$\langle 19,7,2,1\rangle$											1									
	$\langle 19,7,2,1\rangle'$												1								
	$\langle 18,8,2,1\rangle$											1		1							
	$\langle 18,8,2,1\rangle'$												1		1						
	$\langle 13,8,7,1\rangle$													1		1					
$\boldsymbol{B_{17}}$	$\langle 13,8,7,1\rangle'$														1		1				
	$\langle 12,8,7,2\rangle$															1		1			
	$\langle 12,8,7,2\rangle'$																1		1		
	$\langle 11,8,7,2,1\rangle^*$																	1	1	1	1
	$\langle 8,7,6,5,2,1\rangle$																			1	
	$\langle 8,7,6,5,2,1\rangle'$																				1
		d_{221}	d_{222}	d_{223}	d_{224}	d_{225}	d_{226}	d_{227}	d_{228}	d_{229}	d_{230}	d_{231}	d_{232}	d_{233}	d_{234}	d_{235}	d_{236}	d_{237}	d_{238}	d_{239}	d_{240}

Proof. By$(\mathrm{r},\bar{\mathrm{r}})$-inducing of p.i.s. D_{176},D_{177},D_{178},D_{179}, D_{52}, D_{55}, D_{184}, D_{185},D_{191},D_{192},D_{193}, D_{194}, D_{203}, D_{197}, D_{199}, D_{200} to S_{28} gives d_{221}, d_{222}, d_{223}, d_{224}, k_1, k_2 d_{229}, d_{230}, d_{231}, d_{232}, d_{233}, d_{234}, k_3, k_4, d_{239}, d_{240} respectively. Since $\langle 12,9,5,3\rangle \neq \langle 12,9,5,3\rangle'$ and $\langle 12,9,5,3\rangle \downarrow S_{28} \cap \langle 9,7,5,4,3,1\rangle \downarrow S_{28}$ has no i.m.s then k_1 divided or there are two columns:

$\phi_1 = a_1\langle 14,9,5,1\rangle + a_2\langle 12,9,5,3\rangle + a_3\langle 11,9,5,3,1\rangle^*$,$\phi_2 = a_1\langle 14,9,5,1\rangle' + a_2\langle 12,9,5,3\rangle' + a_3\langle 11,9,5,3,1\rangle^*$, to describe columns since $\boldsymbol{B_{16}}$ of defect one then we have $a_1, a_2, a_3 \in \{0,1\}$, suppose $a_2 = 1$ ($a_2 = 0$ hence k_1 is split) then by algorithm maple program in appendix degrees $\phi_1, \phi_2 \equiv 0 \; mod \; 11^2$ only when

- $a_1 = a_2 = 1$ and $a_3 = 0 \Rightarrow k_1 = d_{225} + d_{226}$ or ,
- $a_1 = 0$ and $a_2 = a_3 = 1 \Rightarrow k_2 = d_{227} + d_{228}$,

so k_1 is split. As since $\boldsymbol{B_{16}}$ of defect one then k_2 must splits to d_{227}, d_{228}. Since $\langle 12,8,7,2\rangle \neq \langle 12,8,7,2\rangle'$ and $\langle 12,9,5,3\rangle \downarrow S_{28} \cap \langle 8,7,6,5,2,1\rangle \downarrow S_{28}$ has no i.m.s then k_3 divided or there are two columns:

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$\phi_1 = a_4\langle 13,8,7,1\rangle + a_5\langle 12,8,7,2\rangle + a_6\langle 11,8,7,2,1\rangle^*, \phi_2 = a_4\langle 13,8,7,1\rangle' + a_5\langle 12,8,7,2\rangle' + a_6\langle 11,8,7,2,1\rangle^*$, to describe columns since $\boldsymbol{B_{17}}$ of defect one then we have $a_4, a_5, a_6 \in \{0,1\}$. Suppose $a_5 = 1$ ($a_5 = 0$ hence k_3 is split), but degrees $\phi_1, \phi_2 \equiv 0 \; mod \; 11^2$ only when

- $a_4 = a_5 = 1$ and $a_6 = 0 \Rightarrow k_3 = d_{235} + d_{236}$ or
- $a_4 = 0$ and $a_5 = a_6 = 1 \Rightarrow k_4 = d_{237} + d_{238}$

So k_3 is split. As since $\boldsymbol{B_{17}}$ of defect one then k_4 must splits to $d_{337} + d_{338}$. Hence the decomposition matrix for this block is **Table 8**.

Case 9. The decomposition matrix for the blocks $\boldsymbol{B_{18}}$ of type duble and $\boldsymbol{B_{19}}$ of type associte as shown in the **Tables 9**.

Table 9. Block $\boldsymbol{B_{18}}, \boldsymbol{B_{19}}$

Block	spin character	Decomposition matrix														
$\boldsymbol{B_{18}}$	$\langle 19,6,4\rangle^*$	1														
	$\langle 17,8,4\rangle^*$	1	1													
	$\langle 15,8,6\rangle^*$		1	1												
	$\langle 11,8,6,4\rangle$			1	1											
	$\langle 10,8,6,4,1\rangle^*$				1	1										
	$\langle 9,8,6,4,2\rangle^*$					1										
	$\langle 19,5,4,1\rangle$						1									
$\boldsymbol{B_{19}}$	$\langle 19,5,4,1\rangle'$							1								
	$\langle 16,8,4,1\rangle$						1		1							
	$\langle 16,8,4,1\rangle'$							1		1						
	$\langle 15,8,5,1\rangle$								1		1					
	$\langle 15,8,5,1\rangle'$									1		1				
	$\langle 12,8,5,4\rangle$										1		1			
	$\langle 12,8,5,4\rangle'$											1		1		
	$\langle 11,8,5,4,1\rangle^*$												1	1	1	1
	$\langle 9,8,5,4,2,1\rangle$														1	
	$\langle 9,8,5,4,2,1\rangle'$															1
		d_{241}	d_{242}	d_{243}	d_{244}	d_{245}	d_{246}	d_{247}	d_{248}	d_{249}	d_{250}	d_{251}	d_{252}	d_{253}	d_{254}	d_{255}

Proof. Since

- degree $\{\langle 17,8,4\rangle^*, \langle 11,8,6,4\rangle + \langle 11,8,6,4\rangle', \langle 9,8,6,4,2\rangle^*\} \equiv 77 \; mod \; 11^2$
- degree $\{\langle 19,6,4\rangle^*, \langle 15,8,6\rangle^*, \langle 10,8,6,4,1\rangle^*\} \equiv -77 \; mod \; 11^2$

using(4,8)-inducing of p.i.s., $D_7, D_{17}, D_{25}, D_{33}, D_{31}$ for S_{28} to S_{29}, and on$(11, \alpha)$-regular classes:

1. $\langle 11,8,6,4\rangle = \langle 11,8,6,4\rangle'$
2. $\langle 15,8,6\rangle^* = \langle 17,8,4\rangle^* - \langle 19,6,4\rangle^* + \langle 11,8,6,4\rangle - \langle 10,8,6,4,1\rangle^* + \langle 9,8,6,4,2\rangle^*$

then the matrix contains at most 5 columns of S_{29} in $\boldsymbol{B_{18}}$. To find block $\boldsymbol{B_{19}}$ using $(\mathrm{r}, \bar{\mathrm{r}})$-inducing of p.i.s. D_{206}, D_{207},D_{208}, D_{209},D_{214}, D_{215} to S_{28} gives $d_{246}, d_{247}, d_{248}, d_{249}$,$d_{254}$, d_{255}, k_1, k_2,

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respectively. Since $\langle 12,8,5,4\rangle \neq \langle 12,8,5,4\rangle'$ and $\langle 12,8,5,4\rangle \downarrow S_{28} \cap \langle 9,8,5,4,2,1\rangle \downarrow S_{28}$ has no i.m.s then k_1 divided or there are two columns. Suppose there are two columns:

$\phi_1 = a_1\langle 15,8,5,1\rangle + a_2\langle 12,8,5,4\rangle + a_3\langle 11,8,5,4,1\rangle^*$,$\phi_2 = a_1\langle 15,8,5,1\rangle' + a_2\langle 12,8,5,4\rangle' + a_3\langle 11,8,5,4,1\rangle^*$, to describe columns since $\boldsymbol{B_{19}}$ of defect one then we have $a_1, a_2, a_3 \in \{0,1\}$. Suppose $a_2 = 1$ ($a_2 = 0$ hence c_1 is split). But degrees $\phi_1, \phi_2 \equiv 0 \; mod \; 11^2$ only when

- $a_1 = a_2 = 1$ and $a_3 = 0 \Rightarrow k_1 = d_{250} + d_{251}$ or ,
- $a_1 = 0$ and $a_2 = a_3 = 1 \Rightarrow k_2 = d_{252} + d_{253}$,

so k_1 is split. As since $\boldsymbol{B_{19}}$ of defect one then k_2 must splits to $d_{252} + d_{253}$. Hence the decomposition matrix for this block is **Table** 9.

Case 10. According to **Tables 10**, there is a decomposition matrix for the blocks $\boldsymbol{B_{20}}$ of type associte and $\boldsymbol{B_{21}}$ of type duble**.**

Table 10. Blocks $\boldsymbol{B_{20}}, \boldsymbol{B_{21}}$

Block	spin character	Decomposition matrix														
$\boldsymbol{B_{20}}$	$\langle 18,6,3,2\rangle$	1														
	$\langle 18,6,3,2\rangle'$		1													
	$\langle 17,7,3,2\rangle$	1		1												
	$\langle 17,7,3,2\rangle'$		1		1											
	$\langle 14,7,6,2\rangle$			1		1										
	$\langle 14,7,6,2\rangle'$				1		1									
	$\langle 13,7,6,3\rangle$					1		1								
	$\langle 13,7,6,3\rangle'$						1		1							
	$\langle 11,7,6,3,2\rangle^*$							1	1	1	1					
	$\langle 10,7,6,3,2,1\rangle$									1						
	$\langle 10,7,6,3,2,1\rangle'$										1					
$\boldsymbol{B_{21}}$	$\langle 18,5,3,2,1\rangle^*$											1				
	$\langle 16,7,3,2,1\rangle^*$											1	1			
	$\langle 14,7,5,2,1\rangle^*$												1	1		
	$\langle 13,7,5,3,1\rangle^*$													1	1	
	$\langle 12,7,5,3,2\rangle^*$														1	1
	$\langle 11,7,5,3,2,1\rangle$															1
		d_{256}	d_{257}	d_{258}	d_{259}	d_{260}	d_{261}	d_{262}	d_{263}	d_{264}	d_{265}	d_{266}	d_{267}	d_{268}	d_{269}	d_{270}

Proof. To find $\boldsymbol{B_{20}}$ we using $(r,\bar{r})$-inducing of p.i.s. $D_{216}, D_{217}, D_{218}, D_{219}, D_{273}, D_{274}$ to S_{28} gives $k_1, k_2, k_3, k_4, d_{264}, d_{265}$, respectively. Since $\langle 17,7,3,2\rangle \neq \langle 17,7,3,2\rangle'$ so k_1 or k_2 is split. Suppose k_2 is split to d_{258}, d_{259}, but $\langle 18,6,3,2\rangle \neq \langle 18,6,3,2\rangle'$ then k_1 split to d_{256}, d_{257}. If k_1 split to d_{256}, d_{257}, and frome $(11,\alpha)$-regular classes

$$\langle 17,7,3,2\rangle - \langle 18,6,3,2\rangle \neq \langle 17,7,3,2\rangle' - \langle 18,6,3,2\rangle' \tag{18}$$

then k_2 also split so in both cases we get k_1 and k_2 are splits. Since $\langle 13,7,6,3\rangle \neq \langle 13,7,6,3\rangle'$ and $\langle 13,7,6,3\rangle \downarrow S_{28} \cap \langle 10,7,6,3,2,1\rangle \downarrow S_{28}$ has no i.m.s then k_3 divided or there are two columns: $\phi_1 = a_1\langle 14,7,6,2\rangle + a_2\langle 13,7,6,3\rangle + a_3\langle 11,7,6,3,2\rangle^*$,$\phi_2 = a_1\langle 14,7,6,2\rangle' + a_2\langle 13,7,6,3\rangle' + a_3\langle 11,7,6,3,2\rangle^*$, to describe columns since $\boldsymbol{B_{20}}$ of defect one then we have $a_1, a_2, a_3 \in \{0,1\}$. Suppose $a_2 = 1$ ($a_2 = 0$ hence k_3 is split), but degrees $\phi_1, \phi_2 \equiv 0 \; mod \; 11^2$ only when

- $a_1 = a_2 = 1$ and $a_3 = 0 \Rightarrow k_3 = d_{260} + d_{261}$ or,
- $a_1 = 0$ and $a_2 = a_3 = 1 \Rightarrow k_4 = d_{262} + d_{263}$,

so k_3 is split. As since $\boldsymbol{B_{19}}$ of defect one then k_4 must splits to d_{262}, d_{263}. For block $\boldsymbol{B_{20}}$ since

- degree $\{\langle 16,7,3,2,1\rangle^*, \langle 13,7,5,3,1\rangle^*, \langle 11,7,5,3,2,1\rangle + \langle 11,7,5,3,2,1\rangle'\} \equiv 66 \; mod \; 11^2$
- degree $\{\langle 18,5,3,2,1\rangle^*, \langle 14,7,5,2,1\rangle^*, \langle 12,7,5,3,2\rangle^*\} \equiv -66 \; mod \; 11^2$

using (5,7)-inducing of p.i.s.$D_{87}, D_{95}, D_{103}, D_{111}, D_{115}$, and on $(11, \alpha)$-regular classes:

1. $\langle 11,7,5,3,2,1\rangle = \langle 11,7,5,3,2,1\rangle'$
2. $\langle 12,7,5,3,2\rangle^* = \langle 13,7,5,3,1\rangle^* + \langle 11,7,5,3,2,1\rangle - \langle 14,7,5,2,1\rangle^* + \langle 16,7,3,2,1\rangle^* - \langle 18,5,3,2,1\rangle^*$

then matrixcontains at most 5 columns of the spin characters of S_{29} in $\boldsymbol{B_{21}}$.Then get **Table 10**.

Case 11. The block $\boldsymbol{B_3}$ of type associate's decomposition matrix, as given in **Tables 11.**

Table 11. Block $\boldsymbol{B_3}$

spin character	Decomposition matrix																																							
$\langle 27,2\rangle$	1																																							
$\langle 27,2\rangle'$		1																																						
$\langle 24,5\rangle$	1		1																																					
$\langle 24,5\rangle'$		1		1																																				
$\langle 22,5,2\rangle^*$			1	1	1	1																																		
$\langle 21,5,2,1\rangle$					1		1																																	
$\langle 21,5,2,1\rangle'$						1		1																																
$\langle 19,5,3,2\rangle$							1		1																															
$\langle 19,5,3,2\rangle'$								1		1																														
$\langle 18,5,4,2\rangle$									1		1																													
$\langle 18,5,4,2\rangle'$										1		1																												
$\langle 16,13\rangle$			1										1																											
$\langle 16,13\rangle'$				1										1																										
$\langle 16,11,2\rangle^*$	1	1	1	1	1	1							1	1	1	1																								
$\langle 16,10,2,1\rangle$					1		1								1		1																							
$\langle 16,10,2,1\rangle'$						1		1								1		1																						
$\langle 16,8,3,2\rangle$							1		1								1		1																					
$\langle 16,8,3,2\rangle'$								1		1								1		1																				
$\langle 16,7,4,2\rangle$									1		1								1		1																			
$\langle 16,7,4,2\rangle'$										1		1								1		1																		
$\langle 16,6,5,2\rangle$											1										1																			

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$\langle 16,6,5,2\rangle'$												1										1																		
$\langle 15,7,5,2\rangle$																			1		1		1																	
$\langle 15,7,5,2\rangle'$																				1		1		1																
$\langle 14,8,5,2\rangle$																	1		1				1		1															
$\langle 14,8,5,2\rangle'$																		1		1				1		1														
$\langle 13,11,5\rangle^{*}$	1	1											1	1	1	1											1	1												
$\langle 13,10,5,1\rangle$													1	1	1		1										1		1											
$\langle 13,10,5,1\rangle'$													1	1		1		1										1		1										
$\langle 13,9,5,2\rangle$																	1								1				1		1									
$\langle 13,9,5,2\rangle'$																		1								1				1		1								
$\langle 13,8,5,3\rangle$																							1		1						1		1							
$\langle 13,8,5,3\rangle'$																								1		1						1		1						
$\langle 13,7,5,4\rangle$																							1										1							
$\langle 13,7,5,4\rangle'$																								1										1						
$\langle 12,10,5,2\rangle$													1	1													1	1	1		1				1	1				
$\langle 12,10,5,2\rangle'$													1	1													1	1		1		1			1	1				
$\langle 11,10,5,2,1\rangle^{*}$																											1	1			1	1			1	1	1	1		
$\langle 11,8,5,3,2\rangle^{*}$																															1	1	1	1	1	1	1	1	1	1
$\langle 11,7,5,4,2\rangle^{*}$																																	1	1					1	1
$\langle 10,8,5,3,2,1\rangle$																																			1				1	
$\langle 10,8,5,3,2,1\rangle'$																																				1				1
$\langle 10,7,5,4,2,1\rangle$																																					1		1	
$\langle 10,7,5,4,2,1\rangle'$																																						1		1
$\langle 8,7,5,4,3,2\rangle$																																					1			
$\langle 8,7,5,4,3,2\rangle'$																																						1		
	d_{61}	d_{62}	d_{63}	d_{64}	d_{65}	d_{66}	d_{67}	d_{68}	d_{69}	d_{70}	d_{71}	d_{72}	d_{73}	d_{74}	d_{75}	d_{76}	d_{77}	d_{78}	d_{79}	d_{80}	d_{81}	d_{82}	d_{83}	d_{84}	d_{85}	d_{86}	d_{87}	d_{88}	d_{89}	d_{90}	d_{91}	d_{92}	d_{93}	d_{94}	d_{95}	d_{96}	d_{97}	d_{98}	d_{99}	d_{100}

Proof. The following values are obtained by using $(r,\bar{r})$-inducing of p.i.s. for S_{28} to S_{29}: D_{41}, D_{42}, D_{141}, D_{142}, D_{44},D_{45}, D_{46}, D_{47}, D_{143}, D_{144}, D_{49},D_{50}, D_{51}, D_{52}, D_{73}, D_{145}, D_{146}, D_{75}, D_{57}, D_{58},D_{147}, D_{148}, D_{59}, D_{149}, D_{150} respectively. Since $\langle 16,13\rangle \neq \langle 16,13\rangle'$ so k_2 or k_6 is split. Suppose k_6 is split to d_{73}, d_{74}, but $\langle 24,5\rangle \neq \langle 24,5\rangle'$ then k_2 split to d_{63}, d_{64}. If k_2 split to and frome $(11,\alpha)$-regular classes

$$\langle 16,13\rangle + \langle 27,2\rangle - \langle 24,5\rangle \neq \langle 16,13\rangle' + \langle 27,2\rangle' - \langle 24,5\rangle' \qquad (19)$$

then k_6 also split, so in both cases we get k_2 and k_6 are splits.Since $\langle 27,2\rangle \neq \langle 27,2\rangle'$so k_1 divided or there are two columns is split, but block $\boldsymbol{B}_3$ is associate so there must be another column associate with k_1 then $\langle 27,2\rangle$ has 3 i.m.s, but $\langle 27,2\rangle \downarrow S_{28} = \langle 26,2\rangle^{*1} + \langle 27,1\rangle^{*1}$ has only two of i.m.s. So this contradicts the hypothesis, then $k_1 = d_{61} + d_{62}$.Since $\langle 19,5,3,2\rangle \neq \langle 19,5,3,2\rangle'$ so k_3 or k_4 is split. Suppose k_3 is split to d_{67}, d_{68}, but $\langle 18,5,4,2\rangle \neq \langle 18,5,4,2\rangle'$ then k_4 split to d_{69}, d_{70}. If k_4 split, and frome $(11,\alpha)$-regular classes

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$$\langle 19,5,3,2\rangle + \langle 16,6,5,2\rangle - \langle 18,5,4,2\rangle \neq \langle 19,5,3,2\rangle' + \langle 16,6,5,2\rangle' - \langle 18,5,4,2\rangle' \qquad (20)$$

then k_3 also split, so in both cases we get k_3 and k_4 are splits.Since $\langle 16,6,5,2\rangle \neq \langle 16,6,5,2\rangle'$ so k_5 or k_9 is split. Suppose k_5 is split to d_{71}, d_{72}, but $\langle 15,7,5,2\rangle \neq \langle 15,7,5,2\rangle'$ then k_9 split to d_{81}, d_{82}. If k_9 split, and frome $(11, \alpha)$-regular classes

$$\langle 16,5,2\rangle - \langle 15,7,5,2\rangle + \langle 14,8,5,2\rangle \neq \langle 16,5,2\rangle' - \langle 15,7,5,2\rangle' + \langle 14,8,5,2\rangle' \qquad (21)$$

then k_5 also split , so in both cases we get k_5 and k_9 are splits.Since $\langle 16,8,3,2\rangle \neq \langle 16,8,3,2\rangle'$ so k_7 or k_8 is split. Suppose k_7 is split to d_{77}, d_{78}, but $\langle 15,7,5,2\rangle \neq \langle 15,7,5,2\rangle'$ then k_8 split to d_{79}, d_{80}. If k_8 split, and frome $(11, \alpha)$-regular classes

$$\langle 14,8,5,2\rangle - \langle 16,8,3,2\rangle + \langle 19,5,3,2\rangle \neq \langle 14,8,5,2\rangle' - \langle 16,8,3,2\rangle' + \langle 19,5,3,2\rangle' \qquad (22)$$

then k_7 also split,so in both cases we get k_7 and k_8 are splits.Since $\langle 13,8,5,3\rangle \neq \langle 13,8,5,3\rangle'$ so k_{13} or k_{14} is split. Suppose k_{13} is split to d_{91}, d_{92}, but $\langle 13,7,5,4\rangle \neq \langle 13,7,5,4\rangle'$ then k_{14} split to d_{93}, d_{94}. If k_{14} split, and frome $(11, \alpha)$-regular classes

$$\langle 13,8,5,3\rangle - \langle 13,7,5,4\rangle \neq \langle 13,8,5,3\rangle' - \langle 13,7,5,4\rangle' \qquad (23)$$

then k_{13} also split , so in both cases we get k_{13} and k_{14} are splits.Since $\langle 14,8,5,2\rangle \neq \langle 14,8,5,2\rangle'$ so k_{10} or k_{11} is split. Suppose $k_{11}$is split to d_{85}, d_{86}, but $\langle 13,8,5,3\rangle \neq \langle 13,8,5,3\rangle'$ then k_{10} split to d_{83}, d_{84}. If k_{10} split, and frome $(11, \alpha)$-regular classes

$$\langle 14,8,5\rangle + \langle 16,7,4,2\rangle + \langle 19,5,3,2\rangle - \langle 15,7,5,2\rangle - \langle 16,8,3,2\rangle - \langle 18,5,4,2\rangle$$

$$\neq \langle 14,8,5\rangle' + \langle 16,7,4,2\rangle' + \langle 19,5,3,2\rangle' - \langle 15,7,5,2\rangle' - \langle 16,8,3,2\rangle' - \langle 18,5,4,2\rangle' \qquad (24)$$

Then k_{11} also split , so in both cases we get k_{10} and k_{11} are splits. When there are **299** columns and $\langle 13,8,3\rangle \neq \langle 13,8,3\rangle'$ on $(7, \alpha)$-regular classes, $k_{12} = d_{89} + d_{90}$. As a result, **Table 11** represents the decomposition matrix for block B_3.

4. Appendix (Maple Programming)

To discuss all probabilities such that the degree $\varphi_1, \varphi_2 \equiv 0\ mod 11^2$, where ($A_i$ = degree of Spin characters, $a_i \in \mathbb{N}, \forall i$).

>P:=11^2;
$D1 := A_1$;
$D2 := A_2$;
.
.
$Di := A_i$;
$S := 0$;
$j := 1$;
for a_1 from 0 to n_1 do
for a_2 from 0 to n_2 do
.

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.
for a_i from 0 to n_i do
$S := D1 * A_1 + D2 * A_2 + \cdots + Di * A_i$;
$G := modp(S, P)$;
if $G = 0$ then
print(j,'a_1'=a_1,'a_2'=a_2,...,'a_i'=a_i);
$j := j + 1$;
fi;
$S := S$;
od;
od;
.
.
od;

Conflict of interests.

There are non-conflicts of interest.

References

[1] J. Schur, "Uber die Darstellung der symmetrischen und der alternierenden gruppe durch gebrochene lineare subtituttionen," *J. Reine ang.Math.,* pp. 155-250, 1911.

[2] A. O. Morris and A. K. Yaseen, "Decomposition matrices for spin characters of symmetric group," *Royal Society of Edinburgh,* vol. 108, no. 1-2, pp. 145-164, 1988.

[3] J. F. Humphreys, "Projective modular representations of finite groups," *Journal of the London Mathematical Society,* vol. 16, no. 1, pp. 51-66, 1977.

[4] A. O. Morris, "The spin representation of the symmetric group," *London Mathematical Society,* Vols. s3-12, no. 1, pp. 55-76, 1962.

[5] A. K. Yaseen, Modular spin representations of the symmetric groups, United Kingdom: The University of Wales, Aberystwyth, 1987.

[6] J. F. Humphreys, "Blocks of the Projective representations of symmetric groups," *London Mathematical Society,* Vols. s2-33, no. 3, pp. 441-452, 1986.

[7] A. H. Jassim, "11- Modular Characters for $\bar{S}_{27}$," *Journal of University of Babylon for Pure and Applied Sciences,* vol. 27, no. 6, pp. 303-317, 2019.

[8] A. A. Yaseen and M. B. Tahir, "17-DECOMPOSITION MATRICES FOR THE SPIN CHARACTERS OF SYMMETRIC GROUP S_N, $17 \leq N \leq 22$," *Advances in Mathematics: Scientific Journal,* vol. 12, no. 9, pp. 10541-10561, 2020.

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[9] A. H. Jassim, "Decomposition Matrix for the Spin Characters S_{28} Modulo p=11," *to appear in Journal of Kufa for Mathematics and Computer , 2023.*

[10] B. M. Puttaswamaiah and J. D. Dixon, Modular representation of finite groups, New York: Academic Press, 1977.

[11] J. D. James, "The modular characters of the Mathieu groups," *Journal of Algebra,* vol. 27, pp. 57-111, 1973.