



Studying Potential Energy Surface for ^{230}Th , ^{232}Th and ^{234}Th Isotopes Using Interacting Boson Model-1

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دراسة سطح طاقة الجهد لنظائر ^{230}Th , ^{232}Th و ^{234}Th باستخدام نموذج البوزونات المتفاعلة (IBM-1)

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ABSTRACT

Background:

Studying of the potential energy surface by using the interacting bosons model for potential (IBMP) to determine the deformation in the nucleus of isotopes ^{230}Th , ^{232}Th , and ^{234}Th . This program helps to determine the deformation that takes place in the nucleus by analyzing the deflection of the contour lines and their aggregation in a specific region.

Materials and Methods:

The IBMP program was used to study the surface potential.

Results:

We obtained the results that show the nuclear structure of the radioactive isotopes, through which the limitations and distortions can be known

Conclusions:

The potential distribution on the surface of the core for rotational determination SU(3) and translational SU(3)-O(6) results in high distortion and asymmetric distribution of contour lines.

Key words: Interacting boson model-1, potential energy, thorium isotopes.

1. INTRODUCTION

The nuclear theory up to 1970 was based on the shell model and the collective model. Feshbach I. and Iachello F. introduced the interacting boson model (IBM) in 1973. Later, in 1974, Arima A. and Iachello F. developed a new model based on a three approach, which is a theoretical group or algebraic [1].

The IBM explains the behaviour of low-lying, positive parity quadrupole-collective states in even-even deformed nuclei for medium and heavy ones. The IBM-1 model does not differentiate between protons and neutrons. The IBM-1 framework has been extended to include IBM-2, which distinguishes between protons and neutrons, and IBM-3 and IBM-4, which can explain light nuclei [2,3].

The shell model suggests that the low-lying collective states of these nuclei come from interacting nucleon pairs coupled with angular momentum, according to their pretense.

L can be either zero or 2, representing the s boson and d boson, respectively. The energies of these bosons are (s, d) , with s often being equal to zero.

The Casten triangle, seen in figure (2.4), exhibits three dynamical symmetries and transitional regions (4)

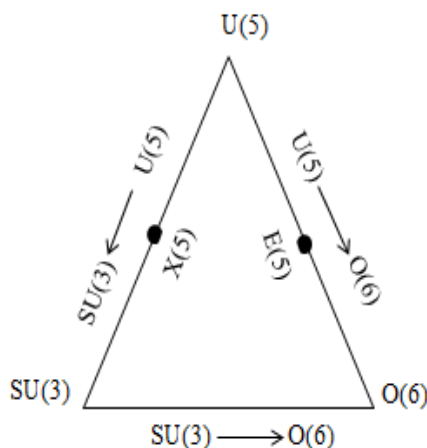


Figure (1) The Casten triangle



2. MATERIALS AND METHODS

IBM-1 Hamiltonian:

Equation gives IBM-1 Hamiltonian forms [5,6]:

$$\hat{H} = \varepsilon \hat{n}_d + a_0 \hat{P} \cdot \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4 \quad (1)$$

$$\hat{n}_d = [d^\dagger \cdot \tilde{d}] \quad \text{d bosons.}$$

$$\hat{P} = \frac{1}{2} (\tilde{d} \cdot d) - \frac{1}{2} (\tilde{s} \cdot s) \quad \text{d-s boson pairing operator}$$

$$\hat{L} = \sqrt{10} [d^\dagger \times \tilde{d}]^{(1)} \quad \text{angular momentum}$$

$$\hat{Q} = [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]^{(2)} + \chi [d^\dagger \times \tilde{d}]^{(2)} \quad \text{quadrupole operator}$$

where χ take the value of 0 to $\pm\sqrt{7}$.

$$\hat{T}_3 = [d^\dagger \times \tilde{d}]^{(3)} \quad \text{the operator of octupole}$$

$$\hat{T}_4 = [d^\dagger \times \tilde{d}]^{(4)} \quad \text{the operator of hexapole}$$

$$\epsilon = \epsilon_d - \epsilon_s \quad \text{the energy of boson}$$

The Hamiltonian of the transition area lies between those of the rotating limit and the unstable limit [7]

$$\hat{H} = a_0 \hat{P} \cdot \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} \quad (2)$$

Potential Equation (Deformation Equation):

The PES ($E(N, \beta, \gamma)$) generates the nucleus., which matches the Hamiltonian function [8,9]:

$$E(N, \beta, \gamma) = \frac{\langle N, \beta, \gamma | H | N, \beta, \gamma \rangle}{\langle N, \beta, \gamma | N, \beta, \gamma \rangle} \quad (3)$$

Calculated potential energy surface as a faction of (N), (β) and (γ) [10]:

$$E(N, \beta, \gamma) = \frac{N\epsilon_d}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)^2} (a_1\beta^4 + a_2\beta^3 \cos 3\gamma + a_3\beta^2 + a_4) \quad (4)$$

where (β) represents the nucleus' deformation, which is spherical when ($\beta = 0$) and distorted when ($\beta < 0$), and (γ) is the nucleus' measure of focus symmetry deviation, which is prolate when ($\gamma = 0$) and oblate when ($\gamma = 60$).

3. RESULTS AND DISCUSSION

Through equation (2), we obtained table (1), which includes the parameters of the energy levels:

Table (1) Values for Hamiltonian parameters

The Isotopes	N	E P S (MeV)	P . P (MeV)	L . L (MeV)	Q . Q (MeV)	T ₃ . T ₃ (MeV)	T ₄ . T ₄ (MeV)	CHI	SO 6
²³⁰ Th	11	0.0000	0.0000	0.0005	-0.0182	0.0000	0.1000	0.1250	1
²³² Th	12	0.0000	0.0000	0.0005	-0.0200	0.0000	0.1500	0.5000	1
²³⁴ Th	13	0.0000	0.0025	0.0069	-0.0026	0.0000	0.0000	0.0900	1

Figures (1 to 3) were compared using an available data. For all of the nuclei under examination, agreement between the ground-band and the actual results is very good, and a decent agreement for the other bands if there are data.

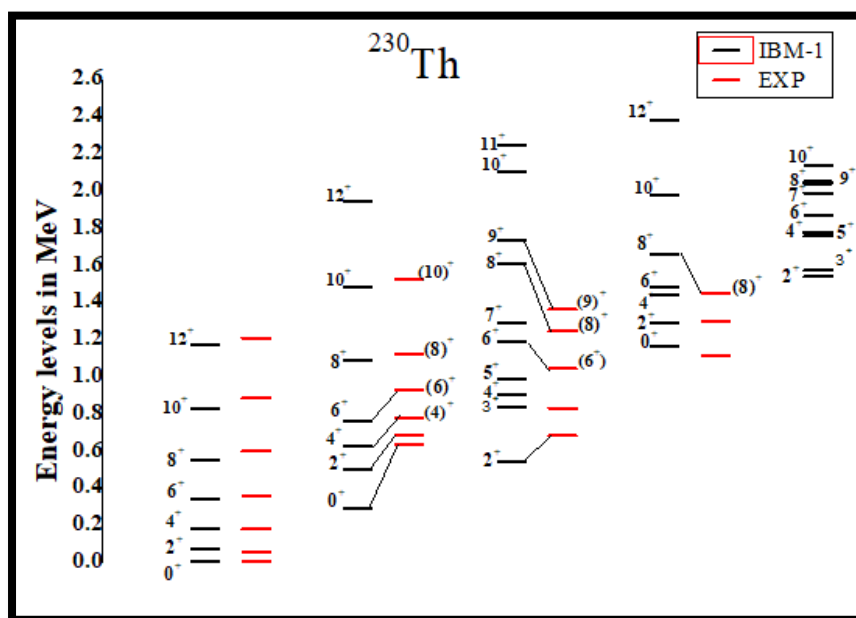


Figure (2) The arrangement of energy levels for ²³⁰Th

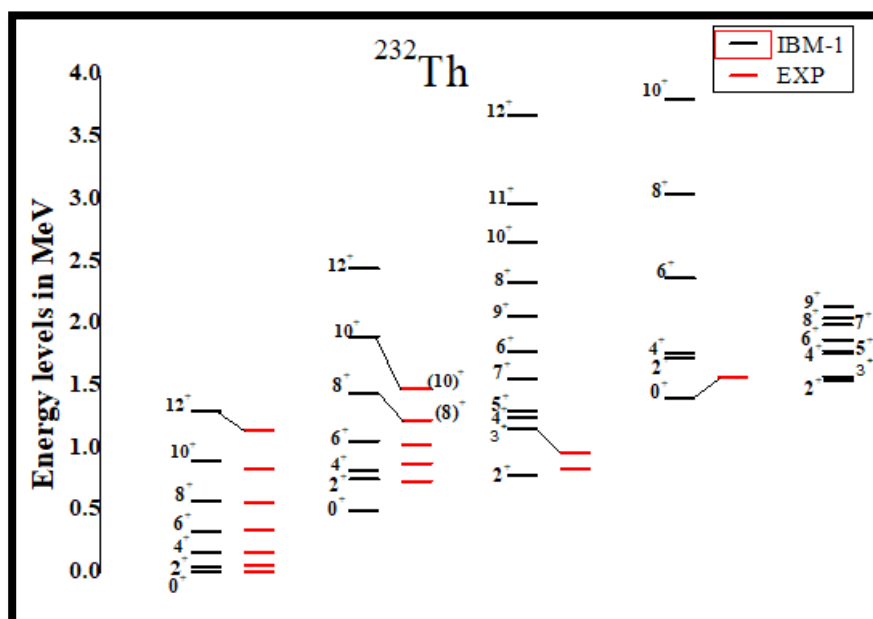


Figure (3) The arrangement of energy levels for ^{232}Th

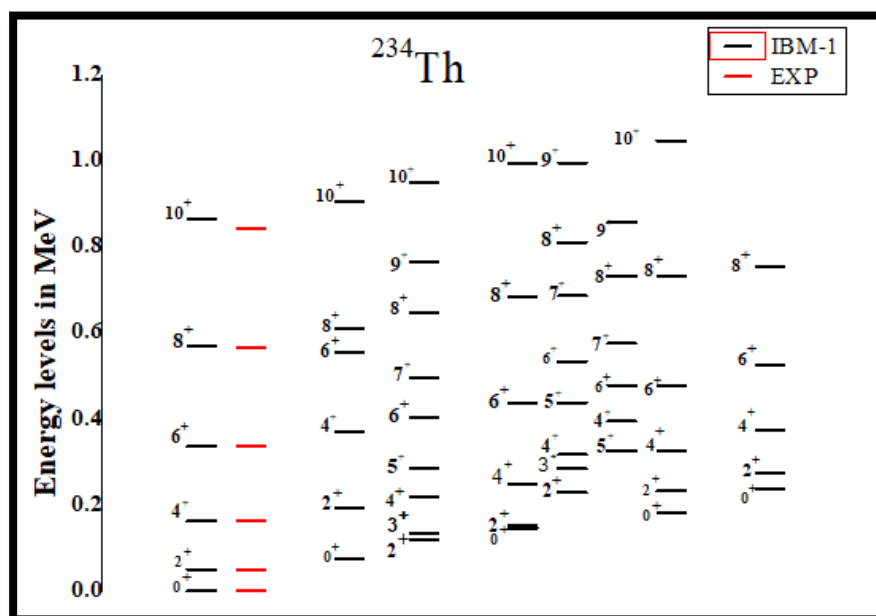


Figure (4) The arrangement of energy levels for ^{234}Th

The P.E.S.FOR program is applied to calculate P.E.S $E(N, \beta, \gamma)$. It was calculated from equation (4), where table (2) shows the parameters that are entered in the effort program.

Table (2) The coefficients of potential energy

The Isotopes	Boson Number	ES (MeV)	ED (MeV)	A1 (MeV)	A2 (MeV)	A3 (MeV)	A4 (MeV)
^{230}Th	11	-0.0910	0.1650	0.0510	0.0050	-0.0730	0.0000
^{232}Th	12	-0.1000	0.2480	0.0760	0.0210	-0.0800	0.0000
^{234}Th	13	-0.0130	0.0390	0.0010	0.0010	-0.0120	0.0000

The table (2) contains the number of bosons that increases with the increase in the mass number, and we note that the variable ES is a negative value, and the parameter A3 is also negative, while A4 for all isotopes has a value of zero.

Figures (4 to 6) represent the potential energy surface for (^{230}Th , ^{232}Th and ^{234}Th) respectively, as contour lines and symmetric shape of prolate if ($\gamma = 0$) right band, and oblate if ($\gamma = 60$) left band.

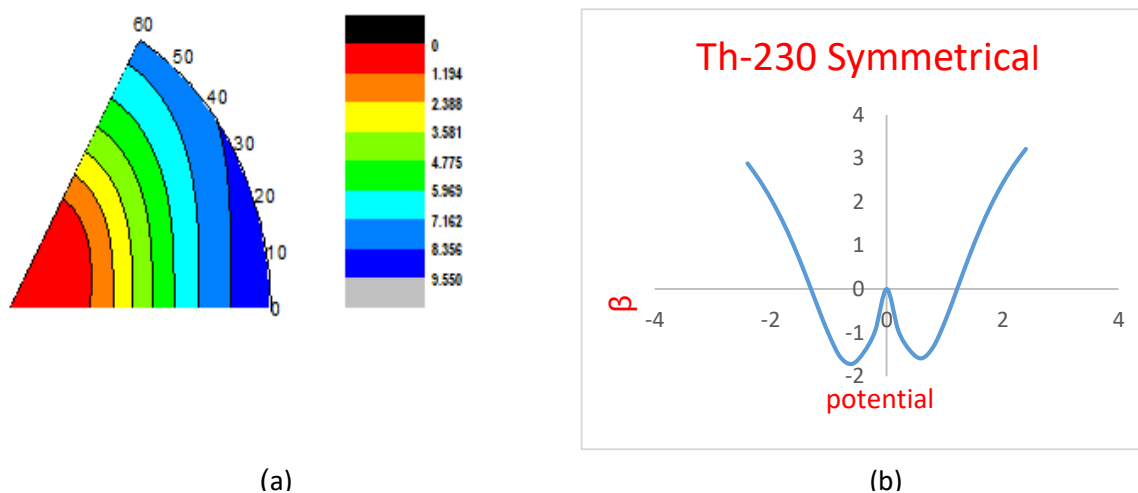


Figure (5) Symmetric shape and potential distribution for ^{230}Th

The symmetric shape for two sides: prolate and oblate for ^{230}Th and contour lines for the potential energy denoted in figure (4). There is asymmetry in both side because the deformation in the distribution of the potential in surface specially in $30 < \gamma < 60$ and the maximum value of potential reaches (4.939 MeV when $\beta = 2.4$ and $\gamma = 0^\circ$) and the minimum value of potential reaches (0 MeV when $\beta = 0.6$ and $\gamma = 60^\circ$).

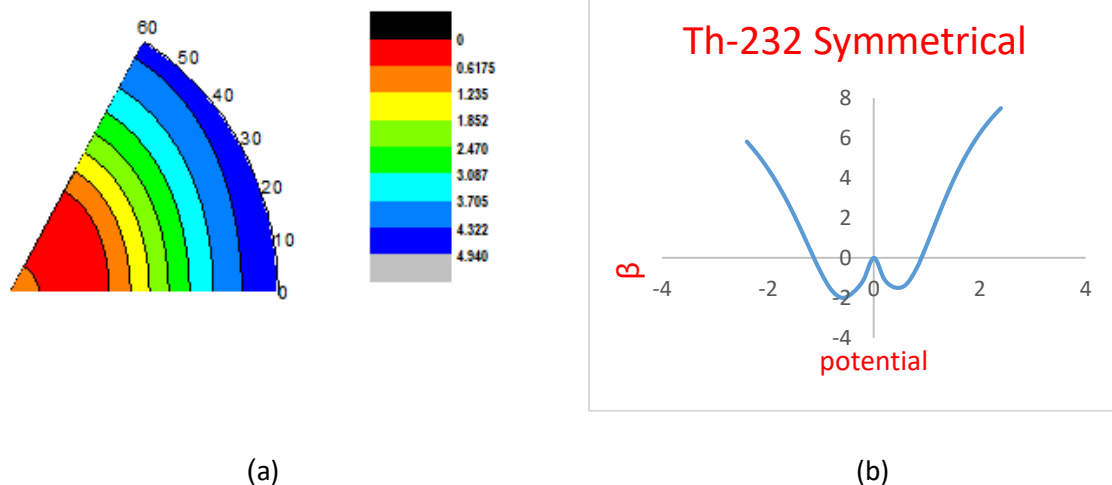


Figure (6) Symmetric shape and potential distribution for ^{232}Th

The symmetric shape for two sides: prolate and oblate for ^{232}Th and contour lines for the potential energy denoted in figure (5). There is asymmetry in both side because the deformation in the distribution of the potential in surface specially in $30 < \gamma < 60$ and the maximum value of potential reaches (9.510 MeV when $\beta = 2.4$ and $\gamma = 0^\circ$) and the minimum value of potential reaches (0 MeV when $\beta = 0.6$ and $\gamma = 60^\circ$).

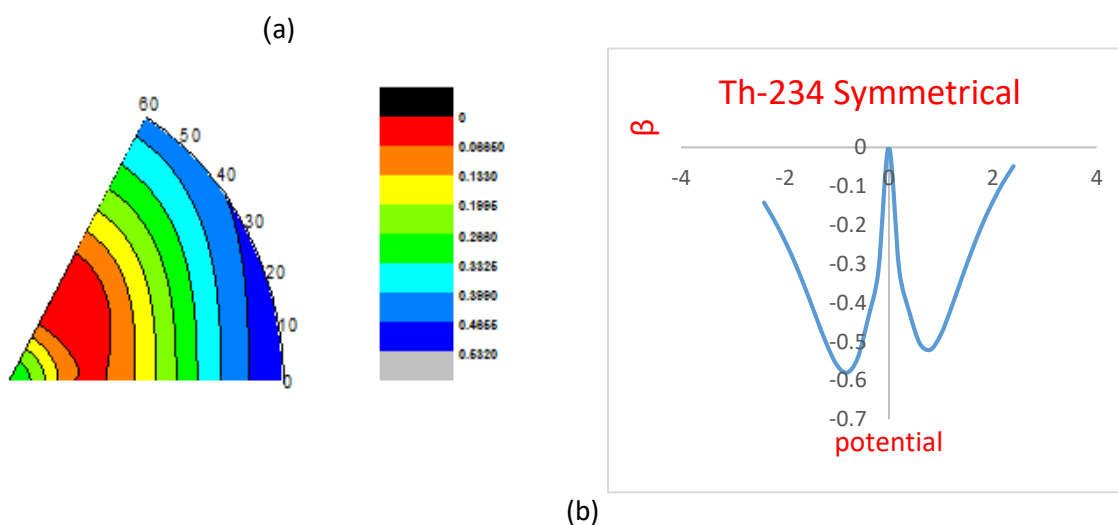


Figure (7) Symmetric shape and potential distribution for ^{234}Th



The symmetric shape for two sides: prolate and oblate for ^{232}Th and contour lines for the potential energy denoted in figure (6). There is asymmetry in both side because the deformation in the distribution of the potential in surface specially in " $30 < \gamma < 60$ " and the maximum value of potential reaches (0.532 MeV when $\beta = 2.4$ and $\gamma = 0^\circ$) and the minimum value of potential reaches (0 MeV when $\beta = 0.8$ and $\gamma = 60^\circ$)

4. CONCLUSIONS

The axial symmetry of the radioactive isotopes ^{230}Th , ^{232}Th and ^{234}Th is found to be compatible with the typical axial symmetry of the O(6) and SU(3) term, where its shape is irregular as the contour lines gather in one place and decrease in another, meaning that the distribution of the contour lines is uneven on the surface of the nucleus. The isotopes is highly distorted where the distribution of the contour lines on the surface of the nucleus is random and irregular, and has not symmetry both sides, this means that the potential distribution is not equal across the surface of the core and belong to rotational limit O(6)-SU(3).

Conflict of interests.

There are non-conflicts of interest.

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الخلاصة

مقدمة:

برنامج جهد لنموذج (IBMP) كان يستخدم لدراسة طاقة الجهد النظائر الثوريوم وهذا البرنامج يساعد لتحديد التشوه الحاصل في النواة بواسطة تحليل انحراف الخطوط الكنتورية وتكونه في منطقه معينة.

طرق العمل: تم استخدام برنامج IBMP لدراسة سطح الجهد.

النتائج: حصلنا على النتائج التي توضح التركيب النووي للنظائر المشعة، والتي من خلالها يمكن معرفة التحديدات والتشوهات

الحسابات: توزيع الجهد على سطح النواة لتحديد الدوراني $SU(3)$ والانتقالي $O(6) - SU(3)$ ينتج تشوه عالي وتوزيع غير متناظر للخطوط الكنتورية.

الكلمات المفتاحية: نموذج البوزونات المتفاعلة-1، الطاقة الكامنة، نظائر الثوريوم.