Article

JOURNAL OF UNIVERSITY OF BABYLON
For Pure and Applied Sciences (JUBPAS)

Vol.32; No.1.| 2024

ISSN: 2312-8135 | Print ISSN: 1992-0652

info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

Exact and Approximate Solution of Multi-Higher Order Fractional Differential Equations Via Sawi Transform and Sequential Approximation Method

Hozan Hilmi[1*], Shabaz Jalil MohammedFaeq[2], Shwan Swara Fatah[3]

[1] College of Science, University of Sulaimani hozan.mhamadhilmi@univsul.edu.iq Sulaymaniyah-Department of Mathematics, Kurdistan Region, sulaymaniyah-Iraq.

[2] College of Science, University of Sulaimanah shabaz.mohammedfaeq@univsul.edu.iq Sulaymaniyah-Department of Mathematics, Kurdistan Region, sulaymaniyah-Iraq

[3] College of Science, University of Charmo shwan.swara@charmouniversity.org -Department of Physics, and Haibat Sultan Technical Institute, shwanswara02@gmail.com Department of Business Information Technology Kurdistan Region, Sulaymaniyah-Iraq

***Corresponding author email:** hozan.mhamadhilmi@univsul.edu.iq; **mobile: 07736990981**

الحلول التحليلية و التقريبية للمعادلات التفاضلية ذات الرتبة الكسرية باستخدام تحويلات ساوي وطريقة تقريب المتتابع

هۆزان حلمی[1*]، شاباز جليل محمد فائق[2]، شوان سواره فتاح[3]

1 كليةالعلوم، جامعةالسليمانية hozan.mhamadhilmi@univsul.edu.iq ، السليمانية ،العراق

2 كليةالعلوم، جامعةالسليمانية shabaz.mohammedfaeq @univsul.edu.iq ، السليمانية ،العراق

3 كليةالعلوم، جامعة جرمو shwan.swara@charmouniversity.org، جمجمال ،العراق

كلية تكنولوجيا المعلومات التجارية ، معهد تكنيكى هيبت سلْطان shwanswara02@gmail.com ، كويسنجاق ،العراق

Accepted:	18/12/2023		Published:	31/3/2024

ABSTRACT

Background

In this paper, we propose two new techniques called the Sawi transformations and Sequential approximations method, which are applied to solve multi-higher order linear fractional differential equations with constant coefficients. In which Riemann-Liouville and Caputo define the fractional derivatives and fractional integral, fractional formula for all types have been derived, we first developed the Sawi transform of foundational mathematical functions for this purpose and then described the important characteristics of the Sawi transform, which may be applied to solve ordinary differential equations and fractional differential equations. Following that, the authors found an exact solution to a particular example of fractional differential equations.

Materials and Methods

With these methods good exact and approximate solutions can be obtained with only a few iterations for the Sequential approximations method, and the approximate solutions guarantee the desired accuracy. For more validation of the methods, and fractional formula of sawi transformation method work such as other transformation.

Results: The exact and approximation solutions for some fractional differential equations are obtained, and several examples are explained to demonstrate the efficiency and implementation of the proposed methods.

Conclusions

It is clear from reading the literature on fractional differential equations (FDEs) that the Sawi transformation method is a workable solution to these problems. To use this approach, which also completely resolved fractional differential equations, the FDEs must be simplified.

Keywords: Sawi transformation, Sequential approximation, Linear fractional differential equations, Riemann-Liouville’s fractional derivative, Caputo’s fractional derivative.

مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية

info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com ISSN: 2312-8135 | Print ISSN: 1992-0652

1. INTRODUCTION

Fractional calculus is a useful tool for describing the memory and inherited traits of various materials and processes.[1,2] It can be applied to a wide range of technical and scientific fields, including biology, chemistry, viscoelasticity, anomalous diffusion, fluid mechanics, acoustics, control theory, and others. In these applications, fractional differential equations were used to solve a family of integro-differential equations with singularities [3,4].

There have previously been presented a number of analytical or numerical methods for solving fractional differential equations, including [2,5]. Integral transforms are currently researchers' first preference among other mathematical techniques for solving issues in science, social science, and engineering because of their three key qualities: simplicity, accuracy, and providing results without the need for time-consuming calculation work, practical techniques of mathematical modeling and fractional mathematical modeling in science were applied [6]

Kamal[7], Aboodh[8], Sumudu[9], Sawi [11,12] Rishi [12], [13]are a few examples of recent developments in integral transforms and fractional differential equations. The renowned issues were examined by Aggarwal and other scholars [12] using a variety of integral transforms .

The creation of the "Sawi Transform", a novel integral transform with crucial properties for fractional and fractional differential equations, and the determination of the fractional differential equations' solutions are the aims of this article. Since it provides the accurate solutions to the problems without necessitating time-consuming computations, the "Sawi transform" that has been proposed is superior to the other transforms that have already been established. The Sawi transform is dualistic, as is the well-known and frequently applied Laplace transform.

We have derived the root of the transformation and utilize it to evaluate specific situations, just as most transformations have fractional formulas. We developed the Sawi transform and came up with a fractional formula that is some other transformation for fractional derivative. Also, we will use new methods Sawi Transform Method (STM) and Sequential Approximation Method (SAM), this method is used to solve mathematical problems by constructing a recursive sequence of approximations that converge to the solution. Each new approximation is computed on the basis of the previous approximation [13–17]

The aim of this work is to implement two novel methodologies Sawi Transform Method (STM) and Sequential Approximation Method (SAM), to compute the exact and approximate solution for the high order linear fractional differential equations of Riemann-Liouville and multi-high order linear fractional differential equations of Caputo sense. The form of the equations is as follows, respectively:

$$D_t^i[{}_a^RD_t^\alpha y(t)+py(t)]=g(t), \qquad t\in[a,b]$$
$$y^{(i)}(a)=d_i, {}_a^RD_t^{\alpha-j}y(t)|_{t=a}=c_j \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$
for all $i=0,1,2,\dots,n,$ and $j=1,2,\dots\mu-1,$ where $\mu=[\alpha]$

مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

where the fractional order: $\alpha > 0$, and $y, g \in C[a,b]$, $p \in R$.

And, another type of problem is:

$$ {}_{a}^{C}D_{t}^{\alpha_n} y(t) + \sum_{l=0}^{n-1} z_l \, {}_{a}^{C}D_{t}^{\alpha_l} y(t) = g(t), \qquad t \in [a,b] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2) $$
$$ y^{(i)}(a) = d_i , \quad \text{for all } i = 1,2,\ldots,\mu-1, \text{where } \mu = \lceil \alpha_n \rceil $$

where the fractional orders: $\alpha_n > \alpha_{n-1} > \cdots > \alpha_1 > \alpha_0 = 0$, and $y(t)$ is unknown function $g(t) \in C[a,b]$, $z_l \in R$ for all, $l = 0,1,\ldots,n-1$, $n \in Z^+$are constants.

2. Some basic definitions and lemmas of fractional calculus

In this section, we write some definitions, lemmas and propositions for the different types of fractions that we need for our purpose.

Definition 1[17,18]**:** A real-valued continuous function h defined on the closed bounded interval $[a,b]$, $C_\omega([a,b])$, $\omega \in \mathbb{R}$, if there exist a real number $k > \omega$, such that $h(t) = (t-a)^k h_0(t)$, where $h_0(t) \in C([a,b])$, and it is said to be in the space $C_\omega^n([a,b])$ iff $h^{(n)}(t) \in C_\omega([a,b])$, where $n \in \mathbb{Z}^+ \cup \{0\}$.

Definition 2 [21] **(Riemann-Liouville fractional integral):** For every $\alpha > 0$ and a local integrable function $h(t)$, the right fractional integral of order α is defined:

$$ {}_{a}I_{t}^{\alpha} h(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-s)^{\alpha-1} h(s)ds, \qquad -\infty \le a < t < \infty \ . $$

Definition 3 [5], [22] **(Riemann-Liouville fractional derivative):** For every α, and $m = \lceil \alpha \rceil$ the Riemann-Liouville derivative of order α can be defined as:

$$ {}_{a}^{R}D_{t}^{\alpha} h(t) = \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_a^t (t-s)^{m-\alpha-1} h(s)ds \ . $$

Definition 4 [5,20] **(Caputo fractional derivative)**: Let $\alpha > 0$, $m = \lceil \alpha \rceil$. The Caputo derivative operator of order α and $h(t)$ be m −times differentiabl function, $t > a$ is defined as:

$$ {}_{a}^{C}D_{t}^{\alpha} h(t) = \frac{1}{\Gamma(m-\alpha)} \int_a^t (t-s)^{m-\alpha-1} \left(\frac{d}{ds}\right)^m h(s)ds \ . $$

مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية

Lemma 1[17,18]**: (Mixed Riemann-Liouville fractional integration and differentiation)**

1. Let $\alpha \geq \beta \geq 0$, if the fractional derivative ${}_a^R D_t^{\beta}, (\lceil\beta\rceil - 1 < \beta \leq \lceil\beta\rceil)$, of a function $f(t)$ is integrable, (or, if $f(t) \in C[a,b]$ and ${}_a I_t^{\lceil\beta\rceil-\beta} f(t) \in C^{\lceil\beta\rceil}[a,b]$), then:
$$ {}_a I_t^{\alpha}\, {}_a^R D_t^{\beta} f(t) = {}_a I_t^{\alpha-\beta} f(t) - \sum_{k=1}^{\lceil\beta\rceil} \left[{}_a^R D_t^{\beta-k} f(t) \right]_{t=a} \frac{(t-a)^{\alpha-k}}{\Gamma(1+\alpha-k)} \quad . $$

2. Let $\alpha \geq 0$, if the fractional derivative ${}_a^R D_t^{\alpha}, (\lceil\alpha\rceil - 1 < \alpha \leq \lceil\alpha\rceil)$, of a function $f(t)$ is integrable, (or, if $f(t) \in C[a,b]$ and ${}_a I_t^{\lceil\alpha\rceil-\alpha} f(t) \in C^{\lceil\alpha\rceil}[a,b]$), then:
$$ {}_a I_t^{\alpha}\, {}_a^R D_t^{\alpha} f(t) = f(t) - \sum_{k=1}^{\lceil\alpha\rceil} [{}_a^R D_t^{\alpha-k} f(t)]_{t=a} \frac{(t-a)^{\alpha-k}}{\Gamma(1+\alpha-k)} \quad . $$

Lemma 2[17,18,21]**: (Mixed Caputo fractional integration and differentiation)**

1. Let $\alpha > \beta \geq 0, \lceil\alpha\rceil - 1 < \alpha \leq \lceil\alpha\rceil$ and $\lceil\beta\rceil - 1 < \beta \leq \lceil\beta\rceil$ where $\lceil\alpha\rceil, \lceil\beta\rceil \in \mathbb{N}$ be such that $f(t) \in C_{-1}^{\lceil\beta\rceil}[a,b]$, then
$$ {}_a I_t^{\alpha}\, {}_a^C D_t^{\beta} f(t) = {}_a I_t^{\alpha-\beta} f(t) - \sum_{k=0}^{\lceil\beta\rceil-1} \frac{f^{(k)}(a)}{\Gamma(k+\alpha-\beta+1)} (t-a)^{k+\alpha-\beta} \quad . $$

2. Let $\alpha \geq 0, \lceil\alpha\rceil - 1 < \alpha \leq \lceil\alpha\rceil$ where $\lceil\alpha\rceil \in \mathbb{N}$ be such that $f(t) \in C_{-1}^{\lceil\alpha\rceil}[a,b]$, then
$$ {}_a I_t^{\alpha}\, {}_a^C D_t^{\alpha} f(t) = f(t) - \sum_{k=0}^{\lceil\alpha\rceil-1} \frac{f^{(k)}(a)}{k!} (t-a)^k . $$

Remark:[22-24]

1. The fractional integration and fractional differentiation Operator is linear operator.
2. The relation between two Riemann-integration of orders α and β are given as shown:
$ {}_a I_t^{\alpha}\, {}_a I_t^{\beta} f(t) = {}_a I_t^{\beta}\, {}_a I_t^{\alpha} f(t) = {}_a I_t^{\alpha+\beta} f(t). $
3. Mixed integer order and fractional differentiation of Riemann-Liouville operator of order α, we can say: $D_t^n [{}_a^R D_t^{\alpha} f(t)] = {}_a^R D_t^{n+\alpha} f(t)$.
4. The Relation between fractional integration and fractional differentiation of Caputo operator of order α are given as shown: The Caputo derivative of fractional integral is
$$ {}_a^C D_t^{\alpha} ({}_a I_t^{\alpha} f(t)) = f(t). $$
5. From the above we got ${}_a^C D_t^{\alpha} ({}_a I_t^{\alpha} f(t)) \neq {}_a I_t^{\alpha} ({}_a^C D_t^{\alpha} f(t))$.
6. We have ${}_a I_t^{\alpha} t^n = \frac{\Gamma(1+n)}{\Gamma(1+n+\alpha)} t^{n+\alpha}$ and ${}_a^C D_t^{\alpha} t^n = \frac{\Gamma(1+n)}{\Gamma(1+n-\alpha)} t^{n-\alpha}$.

info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com ISSN: 2312-8135 | Print ISSN: 1992-0652

3. Fundamental properties of Sawi transform [12,25]

In this section, we have defined the Sawi transform and its properties. Using several other researches, we will learn an introduction to the fractional formula of the new transform.

Definition 5 [11]: The Sawi transform of exponential order piecewise continuous function, $f(t)$ defined in the interval $[0,\infty)$ is given by

$$S\{f(t)\} = \left(\frac{1}{v^2}\right)\int_0^\infty f(t)e^{-\left(\frac{1}{v}\right)t}dt = F(v), \qquad v > 0 .$$

Proposition 1 [11]**:** Some fundamental functions and their Sawi transform

$\boldsymbol{f(t), t > 0}$	$\boldsymbol{S\{f(t)\} = F(v)}$	$\boldsymbol{f(t), t > 0}$	$\boldsymbol{S\{f(t)\} = F(v)}$
1	$\frac{1}{v}$	e^{kt}	$\frac{1}{v(1-kv)}$
$\boldsymbol{t}$	1	sinkt	$\frac{k}{1+k^2v^2}$
$\boldsymbol{t^2}$	$2!\,v$	coskt	$\frac{1}{v(1+k^2v^2)}$
$\boldsymbol{t^n, n \in \mathbb{N}}$	$n!\,v^{n-1}$	sinhkt	$\frac{k}{(1-k^2v^2)}$
$\boldsymbol{t^\beta, \beta > -1, \beta \in R}$	$v^{\beta-1}\Gamma(\beta+1)$	coshkt	$\frac{1}{v(1-k^2v^2)}$

Proposition 2:[11] Inverse Sawi transformations for some fundamental functions.

$\boldsymbol{S^{-1}\{F(v)\} = f(t)}$	$\boldsymbol{f(t), t > 0}$	$\boldsymbol{S^{-1}\{F(v)\} = f(t)}$	$\boldsymbol{f(t), t > 0}$
$\boldsymbol{\frac{1}{v}}$	1	$\frac{1}{v(1-kv)}$	e^{kt}
1	t	$\frac{k}{1+k^2v^2}$	sinkt
$\boldsymbol{2!\,v}$	t^2	$\frac{1}{v(1+k^2v^2)}$	coskt
$\boldsymbol{n!\,v^{n-1}}$	$t^n, n \in \mathbb{N}$	$\frac{k}{(1-k^2v^2)}$	sinhkt
$\boldsymbol{v^{\beta-1}\Gamma(\beta+1)}$	$t^\beta, \beta > -1, \beta \in R$	$\frac{1}{v(1-k^2v^2)}$	coshkt

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

Property 1 [11] **(Convolution property):** If $S\{f(t)\} = F(v)$ and $S\{g(t)\} = G(v)$ then

$S\{f(t) * g(t)\} = v^2 F(v) G(v),$

Where * denotes convolution of f and g, then $f(t) * g(t) = \int_0^t f(t-u)g(u)du$

4. Sawi transform of Fractional Integrals and Derivatives

In this section we conclude fractional formula of Sawi transform for fractional integral and fractional derivative by using those properties and the property of convolution.

Preposition 3: If $\alpha \in [n-1, n)$, Using the Sawi transform of the fractional integral is

$$S\{{}_0I_t^\alpha f(t)\} = v^\alpha F(v)\ .$$

Proof: apply Sawi transform of the fractional integral

$S\{{}_0I_t^\alpha f(t)\} = S\{{}_0D_t^{-\alpha} f(t)\} = \frac{1}{\Gamma(\alpha)} S\left\{\int_0^t (t-u)^{\alpha-1} f(u)du\right\} = \frac{1}{\Gamma(\alpha)}\ v^2 S\{t^{\alpha-1}\}F(v) =$
$\frac{1}{\Gamma(\alpha)} v^2 v^{\alpha-2}\Gamma(\alpha)F(v) = v^\alpha F(v).$

Property 2: The Sawi transform for integer order derivative is

$$S\{f^{(m)}(t)\} = \frac{1}{v^m}F(v) - \sum_{k=0}^{m-1}\frac{1}{v^{m-k+1}}f^{(k)}(0).$$

We can use the above formula to derive Sawi transform for fractional derivative for both types (Riemann- Liouville, Caputo).

Preposition 4: If $f(t)$ is a continuous function, and $F(v)$ is a Sawi transform for Riemann-Liouville fractional derivative of order α is

$S\{{}_0^RD_t^\alpha f(t)\} = \ v^{-\alpha}F(v) - \sum_{k=0}^{[\alpha]-1} v^{k-[\alpha]-1}[{}_0^RD_t^{\alpha-k-1} f(t)]_{t=0}\ .$

Proof: From relation fractional integral and Riemann-Liouville fractional derivative we have

$S\{{}_0^RD_t^\alpha f(t)\} = S\{D_t^{[\alpha]} I_t^{[\alpha]-\alpha} f(t)\},$

And from property (2), Sawi Transform for integer derivative we have:

مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية
ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

$$S\{f^{(m)}(t)\} = \frac{1}{v^m} F(v) - \sum_{k=0}^{m-1} \frac{1}{v^{m-k+1}} f^{(k)}(0),$$

So, and let $m = \lceil \alpha \rceil$, we get:

$$S\{D_t^{\lceil \alpha \rceil} I_t^{\lceil \alpha \rceil - \alpha} f(t)\} = \frac{1}{v^{\lceil \alpha \rceil}} S\{I_t^{\lceil \alpha \rceil - \alpha} f(t)\} - \sum_{k=0}^{\lceil \alpha \rceil - 1} v^{k - \lceil \alpha \rceil - 1} \frac{d^{\lceil \alpha \rceil - k - 1}}{dt^{\lceil \alpha \rceil - k - 1}} I_t^{\lceil \alpha \rceil - \alpha} f(0) =$$

$$\frac{1}{v^{\lceil \alpha \rceil}} v^{\lceil \alpha \rceil - \alpha} F(v) - \sum_{k=0}^{\lceil \alpha \rceil - 1} v^{k - \lceil \alpha \rceil - 1} [{}_0^R D_t^{\alpha - k - 1} f(t)]_{t=0} = v^{-\alpha} F(v) -$$

$$\sum_{k=0}^{\lceil \alpha \rceil - 1} v^{k - \lceil \alpha \rceil - 1} [{}_0^R D_t^{\alpha - k - 1} f(t)]_{t=0} \;.$$

Preposition 5: If $f(t)$ is a function and $F(v)$ is a Sawi transform then the Sawi transform for Caputo fractional derivative of order α is

$$S\{{}_0^C D_t^{\alpha} f(t)\} = v^{-\alpha} F(v) - \sum_{k=0}^{\lceil \alpha \rceil - 1} v^{k - \alpha - 1} f^{(k)}(0).$$

Proof: from relation fractional integral and Caputo fractional derivative, we have:

$$S\{{}_0^C D_t^{\alpha} f(t)\} = S\{I_t^{\lceil \alpha \rceil - \alpha} D_t^{\lceil \alpha \rceil} f(t)\},$$

we can prove, by using property (3) and proposition (3), we obtain:

$$S\{I_t^{\lceil \alpha \rceil - \alpha} D_t^{\lceil \alpha \rceil} f(t)\} = v^{\lceil \alpha \rceil - \alpha} \left\{ \frac{1}{v^{\lceil \alpha \rceil}} F(v) - \sum_{k=0}^{\lceil \alpha \rceil - 1} \frac{f^{(k)}(0)}{v^{\lceil \alpha \rceil - k + 1}} \right\} = v^{-\alpha} F(v) - \sum_{k=0}^{\lceil \alpha \rceil - 1} v^{k - \alpha - 1} f^{(k)}(0).$$

5. METHOD OF SOLUTION

In this section, we have used two new techniques (Sawi transform and Sequential approximation) method for find exact and approximation solution of equations (1) and (2).

5.1 Sawi Transform Method (STM):

The first type of problem: This method is used to find an exact solution for solving problem of type (1). The equation is as follows:

$$D_t^i [{}_a^R D_t^{\alpha} y(t) + p y(t)] = g(t),$$

With the initial conditions: $y^{(i)}(a) = d_i, {}_a^R D_t^{\alpha - j} y(t)|_{t=a} = c_j$

for all $i = 0,1,2, \dots, n,$ and $j = 1,2, \dots \mu - 1, where \;\; \mu = \lceil \alpha \rceil$

ISSN: 2312-8135 | Print ISSN: 1992-0652

info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية

We can solve the above fractional differential equations by Sawi transformation method. First of all, by using relation of composition of fractional derivative and integer derivative

$D_t^n[{}_a^RD_t^\alpha y(t)] = {}_a^RD_t^{n+\alpha}y(t)$, and integer derivative is linear operator, we get:

$${}_a^RD_t^{i+\alpha}y(t) + pD_t^iy(t) = g(t)$$

Take Sawi transform for both sides, and by using proposition (4) and property (2), we get:

$S\{{}_0^RD_t^{i+\alpha}y(t)\} + S\{pD_t^iy(t)\} = S\{g(t)\}$,

$v^{-i-\alpha}Y(v) - \sum_{k=0}^{\lceil\alpha\rceil-1} v^{k-\lceil\alpha\rceil-1}\left[{}_0^RD_t^{i+\alpha-k-1}y(t)\right]_{t=0} + p\left[\frac{1}{v^i}Y(v) - \sum_{k=0}^{i-1}\frac{1}{v^{i-k+1}}y^{(k)}(0)\right] = G(v)$,

Where $S\{y(t)\} = Y(v)$ *and* $S\{g(t)\} = G(v)$

$$v^{-i-\alpha}Y(v) + p\frac{1}{v^i}Y(v) = G(v) + \sum_{k=0}^{\lceil\alpha\rceil-1} v^{k-\lceil\alpha\rceil-1}\left[{}_0^RD_t^{i+\alpha-k-1}y(t)\right]_{t=0} + p\sum_{k=0}^{i-1}\frac{1}{v^{i-k+1}}y^{(k)}(0)$$

$$Y(v)\left(v^{-i-\alpha} + p\frac{1}{v^i}\right) = G(v) + \sum_{k=0}^{\lceil\alpha\rceil-1} v^{k-\lceil\alpha\rceil-1}\left[{}_0^RD_t^{i+\alpha-k-1}y(t)\right]_{t=0} + p\sum_{k=0}^{i-1}\frac{1}{v^{i-k+1}}y^{(k)}(0)$$

$$Y(v) = \frac{1}{\left(v^{-i-\alpha} + p\frac{1}{v^i}\right)}\left(G(v) + \sum_{k=0}^{\lceil\alpha\rceil-1} v^{k-\lceil\alpha\rceil-1}\left[{}_0^RD_t^{i+\alpha-k-1}y(t)\right]_{t=0} + p\sum_{k=0}^{i-1}\frac{1}{v^{i-k+1}}y^{(k)}(0)\right) \ldots\ldots(3)$$

We take Sawi inverse for both sides to get exact solutions

$$y(t) = S^{-1}\left\{\frac{1}{\left(v^{-i-\alpha}+p\frac{1}{v^i}\right)}\left(G(v) + \sum_{k=0}^{\lceil\alpha\rceil-1} v^{k-\lceil\alpha\rceil-1}\left[{}_0^RD_t^{i+\alpha-k-1}y(t)\right]_{t=0} + p\sum_{k=0}^{i-1}\frac{1}{v^{i-k+1}}y^{(k)}(0)\right)\right\} \ldots.(4)$$

The second type of problem: This method is used to find an exact solution for solving problem of type (2). The equation is as follows:

$${}^CD_t^{\alpha_n}y(t) + \sum_{l=0}^{n-1} z_l\ {}^CD_t^{\alpha_l}y(t) = g(t),$$

with the initial conditions: $y^{(i)}(0) = d_i$, for all $i = 1, 2, \ldots, \mu - 1$, where $\mu = \lceil\alpha_n\rceil$

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

We can solve the above multi-high order fractional differential equations by Sawi transformation method. Firstly, take Sawi Transform for both sides, by using proposition (5), we get, as follows:

$$S\{{}^{C}D_t^{\alpha_n}y(t)\} + S\{\sum_{l=0}^{n-1} z_l\,{}^{C}D_t^{\alpha_l}y(t)\} = S\{g(t)\},$$

$$v^{-\alpha_n}Y(v) - \sum_{k=0}^{\mu-1} v^{k-\alpha_n-1}y^{(k)}(0) + \sum_{l=0}^{n-1} z_l\left\{v^{-\alpha_l}Y(v) - \sum_{k=0}^{\mu-1} v^{k-\alpha_l-1}y^{(k)}(0)\right\} = G(v),$$

where

$$S\{y(t)\} = Y(v) \ \ and \ \ S\{g(t)\} = G(v)$$

$$v^{-\alpha_n}Y(v) + \sum_{l=0}^{n-1} z_l\, v^{-\alpha_l}Y(v) = G(v) + \sum_{k=0}^{\mu-1} v^{k-\alpha_n-1}y^{(k)}(0) + \sum_{l=0}^{n-1} z_l\left(\sum_{k=0}^{\mu-1} v^{k-\alpha_l-1}y^{(k)}(0)\right)$$

$$Y(v)\left(v^{-\alpha_n} + \sum_{l=0}^{n-1} z_l\, v^{-\alpha_l}\right) = G(v) + \sum_{k=0}^{\mu-1} v^{k-\alpha_n-1}y^{(k)}(0) + \sum_{l=0}^{n-1} z_l\left(\sum_{k=0}^{\mu-1} v^{k-\alpha_l-1}y^{(k)}(0)\right)$$

Hence, Implies that

$$Y(v) = \frac{1}{(v^{-\alpha_n}+\sum_{l=0}^{n-1} z_l v^{-\alpha_l})}\left(G(v) + \sum_{k=0}^{\mu-1} v^{k-\alpha_n-1}y^{(k)}(0) + \sum_{l=0}^{n-1} z_l\left(\sum_{k=0}^{\mu-1} v^{k-\alpha_l-1}y^{(k)}(0)\right)\right) \ldots\ldots\ldots (5)$$

So, we can take Sawi inverse for both sides to get exact solutions

$$y(t) = S^{-1}\left\{\frac{1}{(v^{-\alpha_n}+\sum_{l=0}^{n-1} z_l v^{-\alpha_l})}\left(G(v) + \sum_{k=0}^{\mu-1} v^{k-\alpha_n-1}y^{(k)}(0) + \sum_{l=0}^{n-1} z_l\left(\sum_{k=0}^{\mu-1} v^{k-\alpha_l-1}y^{(k)}(0)\right)\right)\right\} (6)$$

5.2 Sequential Approximation Method (SAM):

The first type of problem: This method is used to find an approximate solution for solving problem of type (1) The equation is as follows:

$$D_t^i[{}_a^RD_t^\alpha y(t) + py(t)] = g(t), \qquad t \in [a,b]$$

with the initial conditions: $y^{(i)}(a) = d_i, {}_a^RD_t^{\alpha-j}y(t)|_{t=a} = c_j$

for all $i = 0,1,2,\ldots,n,$ and $j = 1,2,\ldots\mu-1,$ where $\mu = [\alpha]$

Now, taking i^{th} integration to above equation, we obtain:

$${}_a^RD_t^\alpha y(t) + py(t) = {}_aI_t^i g(t) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

After that, taking Riemann-Liouville fractional integral for fractional order α to equation (7), and using lemma 1, we obtain:

$$y(t) - \sum_{j=1}^{[\alpha]}\left[{}_a^RD_t^{\alpha-j}y(t)\right]_{t=a}\frac{(t-a)^{\alpha-k}}{\Gamma(1+\alpha-k)} + p\,{}_aI_t^{\alpha}y(t) = {}_aI_t^{\alpha}I_t^{i}g(t),$$

$$y(t) = \sum_{j=1}^{[\alpha]}\left[{}_a^RD_t^{\alpha-j}y(t)\right]_{t=a}\frac{(t-a)^{\alpha-k}}{\Gamma(1+\alpha-k)} - p\,{}_aI_t^{\alpha}y(t) + {}_aI_t^{\alpha+i}g(t).$$

So, we can construct a sequence of function $\{y_r(t)\}_{r=0}^{\infty}$ with the aid of the following recursion formula:

$$\left.\begin{aligned} y_0(t) &= \sum_{j=1}^{[\alpha]}\left[{}_a^RD_t^{\alpha-j}y(t)\right]_{t=a}\frac{(t-a)^{\alpha-j}}{\Gamma(1+\alpha-j)} + {}_aI_t^{\alpha+i}g(t), \text{for all } i = 0,1,2,\dots,n \\ y_{r+1}(t) &= y_0(t) - p\,{}_aI_t^{\alpha}y_r(t), \text{for all } \ r \in \{0\}\cup\mathbb{Z}^+ \end{aligned}\right\} \dots\dots\dots\dots\dots\dots\dots\dots (8)$$

The second type of problem: This method is used to find an approximate solution for solving problem of type (2) the following equation is the form:

$${}_a^CD_t^{\alpha_n}y(t) + \sum_{l=0}^{n-1} z_l\,{}_a^CD_t^{\alpha_l}y(t) = g(t), \qquad t \in [a,b] \ ,$$

With the initial conditions: $y^{(i)}(a) = d_i$, for all $i = 1, 2, \dots, \mu-1$, where $\mu = \lceil \alpha_n \rceil$

Now, taking Riemann-Liouville fractional integral for fractional order α_n to above equation, and using lemma (2), we obtain:

$$y(t) = \sum_{k=0}^{\lceil\alpha_n\rceil-1}\frac{y^{(k)}(a)}{k!}(t-a)^k + {}_aI_t^{\alpha_n}g(t) - {}_aI_t^{\alpha_n}\left[\sum_{l=1}^{n-1} z_l\,{}_a^CD_t^{\alpha_l}y(t) + z_0y(t)\right],$$

also, we can write

$$y(t) = \sum_{k=0}^{\lceil\alpha_n\rceil-1}\frac{y^{(k)}(a)}{k!}(t-a)^k + {}_aI_t^{\alpha_n}g(t) - \sum_{l=1}^{n-1} z_l\,{}_aI_t^{\alpha_n}\,{}_a^CD_t^{\alpha_l}y(t) - z_0\,{}_aI_t^{\alpha_n}y(t) \dots\dots\dots\dots (9)$$

So, by using lemma (2) we can write the equation (9) as follows:

$$y(t) = \sum_{k=0}^{\lceil\alpha_n\rceil-1}\frac{y^{(k)}(a)}{k!}(t-a)^k + {}_aI_t^{\alpha_n}g(t)$$

$$-\sum_{l=1}^{n-1} z_l\left[{}_aI_t^{\alpha_n-\alpha_l}y(t) - \sum_{k=0}^{\lceil\alpha_l\rceil-1}\frac{y^{(k)}(a)}{\Gamma(k+\alpha_n-\alpha_l+1)}(t-a)^{k+\alpha_n-\alpha_l}\right] - z_0\,{}_aI_t^{\alpha_n}y(t) \dots\dots\dots\dots\dots (10)$$

So, we can construct a sequence of function $\{y_r(t)\}_{r=0}^{\infty}$ with the aid of the following recursion formula:

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

$$\left.\begin{array}{c} y_0(t) = \sum_{k=0}^{[\alpha_n]-1} \frac{y^{(k)}(a)}{k!}(t-a)^k + {}_aJ_t^{\alpha_n} g(t)\,, \\ y_{r+1}(t) = y_0(t) - \sum_{l=1}^{n-1} z_l \left[{}_aI_t^{\alpha_n-\alpha_l} y_r(t) - \sum_{k=0}^{[\alpha_l]-1} \frac{y^{(k)}(a)}{\Gamma(k+\alpha_n-\alpha_l+1)} (t-a)^{k+\alpha_n-\alpha_l} \right] - z_0\, {}_aI_t^{\alpha_n} y_r(t) \\ \text{for all } \; r \in \{0\} \cup \mathbb{Z}^+ \end{array}\right\} \ldots\ldots (11)$$

Or, simplify

$$\left.\begin{array}{c} y_0(t) = \sum_{k=0}^{[\alpha_n]-1} \frac{y^{(k)}(a)}{k!}(t-a)^k + {}_aI_t^{\alpha_n} g(t) + \sum_{l=1}^{n-1} \sum_{k=0}^{[\alpha_l]-1} \frac{z_l y^{(k)}(a)}{\Gamma(k+\alpha_n-\alpha_l+1)} (t-a)^{k+\alpha_n-\alpha_l} \\ y_{r+1}(t) = y_0(t) - \sum_{l=1}^{n-1} z_l \left[{}_aI_t^{\alpha_n-\alpha_l} y_r(t) \right] - z_0\, {}_aI_t^{\alpha_n} y_r(t), \;\text{for all } \; r \in \{0\} \cup \mathbb{Z}^+ \end{array}\right\} (12)$$

7. ANALYTICAL RESULTS

In this section, we apply the proposed methods in this paper to obtain exact and approximate solutions of linear fractional differential equation of types (1) and (2). Also, apply the absolute error function $e(x_i) = |y(x_i) - \tilde{y}_N(x_i)|$, for all $i = 0,1,\ldots,\tilde{N}$ at the selected points for Sequential approximation method of the given interval. We provide tabular data along with graphs by using a Python program version 3.8.8 (2021).

Example 1 [7] Consider the linear fractional differential equation for Riemann-Liouville's fractional derivative

$${}^{R}D_t^{\frac{1}{2}} y(t) + y(t) = \frac{1}{2}t + \frac{\sqrt{t}}{\sqrt{\pi}} \text{ , with the initial conditions: } \left[{}^{R}D_t^{\frac{-1}{2}} (y(t)) \right]_{t=0} = 0.$$

First method: The Sawi transform method will be applied to solve our problem; as an example, we have: ${}^{R}D_t^{\frac{1}{2}} y(t) + y(t) = \frac{1}{2}t + \frac{\sqrt{t}}{\sqrt{\pi}}$, and $\left[{}^{R}D_t^{\frac{-1}{2}} (y(t)) \right]_{t=0} = 0$.

Apply Sawi transform for both sides of linear equation, we get:

$$S\left\{ {}^{R}D_t^{\frac{1}{2}} y(t) \right\} + S\{y(t)\} = S\left\{ \frac{1}{2}t + \frac{\sqrt{t}}{\sqrt{\pi}} \right\},$$

$$\left\{ (v)^{-\frac{1}{2}} Y(v) - \sum_{k=0}^{0} (v)^{k-2}\, {}^{R}D_t^{\frac{1}{2}-k-1} y(0) \right\} + Y(v) = \frac{1}{2} + \frac{\Gamma\left(\frac{3}{2}\right)}{\sqrt{\pi}}\, v^{-\frac{1}{2}},$$

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

$$\left\{(v)^{-\frac{1}{2}}Y(v)-(v)^{-2}\,{}^{R}D_t^{-\frac{1}{2}}y(0)\right\}+Y(v)=\frac{1}{2}+\frac{1}{2\sqrt{\pi}}\Gamma\left(\frac{1}{2}\right)\,v^{-\frac{1}{2}},$$

$$(v)^{-\frac{1}{2}}Y(v)+Y(v)=\frac{1}{2}+\frac{1}{2}\,v^{-\frac{1}{2}},$$

$$Y(v)=\frac{1}{2}.$$

Then, take inverse Sawi transform for both sides: $S^{-1}\{Y(v)\}=S^{-1}\left\{\frac{1}{2}\right\}$.

The exact solution is: $y(t)=\frac{1}{2}t$.

Remark: the above example solved by Kamal transform in [7] Exact solutions were obtained from both methods.

Second method: Our problem will be solved by using Sequential approximation method, from consider example we have:

$$\alpha=0.5\ \rightarrow\lceil\alpha\rceil=1,p=1$$

start with zeros approximation of equation (8) as follows:

$$y_0(t)=\sum_{j=1}^{1}\left[{}_0^RD_t^{-\frac{1}{2}}y(t)\right]_{t=0}\frac{(t-0)^{\frac{1}{2}-j}}{\Gamma\left(1+\frac{1}{2}-\mathrm{j}\right)}+{}_0I_t^{\frac{1}{2}}g(t)=\frac{1}{2}t+\frac{1}{2\Gamma(2.5)}t^{1.5}$$

Take $r=0$ yields: $y_1(t)=y_0(t)-p\,{}_0I_t^{\frac{1}{2}}y_0(t)=\frac{1}{2}t-\frac{1}{4}t^2$.

and for $r=1,2,3$ we obtain $y_2(t),y_3(t)$ and $y_4(t)$, as follows, respectively:

$$y_2(t)=\frac{1}{2}t+\frac{1}{2\Gamma(3.5)}t^{2.5},\ y_3(t)=\frac{1}{2}t-\frac{1}{2\Gamma(4)}t^3,\ y_4(t)=\frac{1}{2}t+\frac{1}{2\Gamma(4.5)}t^{3.5}.$$

Also, similarly this technique, for $r=9,10,11$ we obtain $y_{10}(t),y_{11}(t)$ and $y_{12}(t)$, as follows, respectively:

$$y_{10}(t)=\frac{1}{2}t+\frac{1}{2\Gamma(7.5)}t^{6.5},\ y_{11}(t)=\frac{1}{2}t-\frac{1}{2\Gamma(8)}t^7,\ y_{12}(t)=\frac{1}{2}t+\frac{1}{2\Gamma(8.5)}t^{7.5},$$

So, we can say: $y(t)\cong\lim_{r\to\infty}y_{r+1}(t)=\frac{1}{2}t$.

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

Table 1. Comparison between exact and approximate solution by using the (STM) and (SAM)

x	Exact solution	N_Approximate Solution			
		$y_1(t)$	$y_2(t)$	$y_{11}(t)$	$y_{12}(t)$
0.0	0.00	0.00	0.00	0.00	0.00
0.1	0.05	0.0475	0.05047577	0.05	0.05
0.2	0.10	0.09	0.10269134	0.10	0.10
0.3	0.15	0.1275	0.15741646	0.14999998	0.15
0.4	0.20	0.16	0.21522453	0.19999984	0.20000004
0.5	0.25	0.1875	0.27659615	0.24999922	0.25000020
0.6	0.30	0.21	0.34195386	0.29999722	0.30000077
0.7	0.35	0.2275	0.41167922	0.34999183	0.35000245
0.8	0.40	0.24	0.48612292	0.39997919	0.40000668
0.9	0.45	0.2475	0.56561124	0.44995255	0.45001617
1.0	0.50	0.25	0.65045056	0.49990079	0.50003563

Table 2. Comparison between the absolute errors (A.E.) of the example (1)

x	Absolute error			
	$N = 1$	$N = 2$	$N = 11$	$N = 12$
0.0	0.0	0.0	0.0	0.0
0.1	0.0025	$4.7576643097e-04$	$9.920633758e-12$	$1.12661269e-12$
0.2	0.01	$2.6913413568e-03$	$1.269841273e-09$	$2.039392821e-10$
0.3	0.0225	$7.416464678e-03$	$2.169642857e-08$	$4.267616038e-09$
0.4	0.04	$1.522452579e-02$	$1.625396825e-07$	$3.691695424e-08$
0.5	0.0625	$2.659615202e-02$	$7.750496031e-07$	$1.968117218e-07$
0.6	0.09	$4.195385973e-02$	$2.77714285e-06$	$7.725210215e-07$
0.7	0.1225	$6.167922326e-02$	$8.17006944e-06$	$2.454772381e-06$
0.8	0.16	$8.612292341e-02$	$2.08050793e-05$	$6.682682543e-06$
0.9	0.2025	$1.1561124272e-01$	$4.745008928e-05$	$1.616570871e-05$
1.0	0.25	$1.5045055562e-01$	$9.920634920e-05$	$3.562672719e-05$
Average A.E	$\mathbf{8.75e-02}$	$\mathbf{4.62020050e-02}$	$\mathbf{1.63062996e-05}$	$\mathbf{5.63096483e-06}$

Article
JOURNAL OF UNIVERSITY OF BABYLON
For Pure and Applied Sciences (JUBPAS)
Vol.32; No.1.| 2024

Figure 1. Exact and approximate solution of example (1)

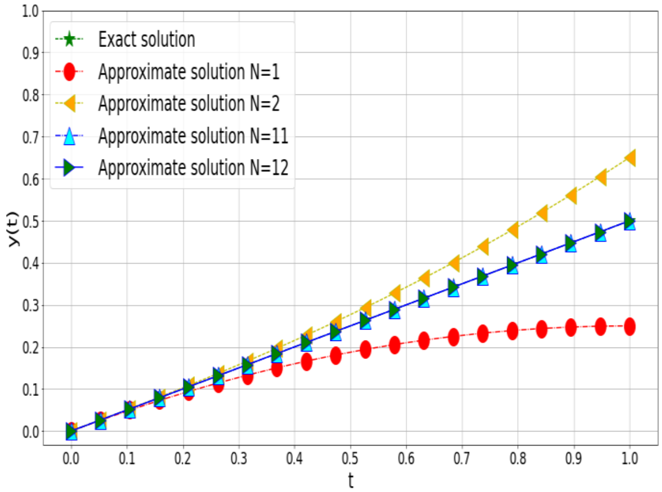
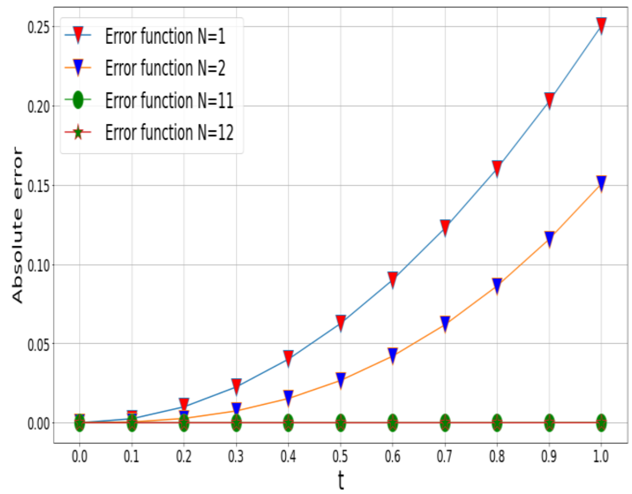


Figure 2. Absolute error of example (1)



Example 2: Consider the linear fractional differential equation given by

$${}_0^C D_t^{1.9} y(t) + y(t) = \frac{2}{\Gamma(1.1)} t^{0.1} + t^2 + 1 \,,$$

With the initial conditions: $y(0) = 1, y'(0) = 0$.

First method: Our problem will be solved by using Sawi transform method, from consider example, we have: ${}_0^C D_t^{1.9} y(t) + y(t) = \frac{2}{\Gamma(1.1)} t^{0.1} + t^2 + 1$

Take Sawi transform for both sides, we obtain:

$$S\{{}_0^C D_t^{1.9} y(t)\} + S\{y(t)\} = S\left\{\frac{2}{\Gamma(1.1)} t^{0.1} + t^2 + 1\right\},$$

$$v^{-1.9} Y(v) - \sum_{k=0}^{1} v^{k-2.9} y^{(k)}(0) + Y(v) = \frac{2}{\Gamma(1.1)} v^{-1.9} \Gamma(1.1) + v\Gamma(3) + \frac{1}{v}$$

$$Y(v)\left(\frac{1 + v^{1.9}}{v^{1.9}}\right) = 2v^{-0.9} + 2v + \frac{1}{v} + v^{-2.9}$$

$$Y(v) = \left(\frac{1}{1 + v^{1.9}}\right)\left(2v + 2v^{2.9} + v^{0.9} + \frac{1}{v}\right) = \left(\frac{1}{1 + v^{1.9}}\right)\left(2v(1 + v^{1.9}) + v^{-1}(1 + v^{1.9})\right)$$

$$Y(v) = \left(\frac{1}{1 + v^{1.9}}\right)(1 + v^{1.9})(2v + v^{-1})$$

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية

$Y(v) = v^{-1} + 2v$.

Taking Sawi inverse for both sides and we see that which the exact solution is $y(t) = 1 + t^2$.

Second method: Our problem will be solved by using Sequential approximation method, from consider example, we have:

$$\alpha_1 = 1.9 \rightarrow [\alpha_1] = 2, z_0 = 1$$

Start with zeros approximation of equation (12) as follows:

$y_0(t) = 1 + t^2 + \frac{1}{\Gamma(2.9)} t^{1.9} + \frac{2}{\Gamma(4.9)} t^{3.9}$,

And finding formulas equation (12), taking $r = 0$ yields:

$y_1(t) = y_0(t) - I_t^{1.9} y_0(t) = 1 + t^2 - \frac{1}{\Gamma(4.8)} t^{3.8} - \frac{2}{\Gamma(6.8)} t^{5.8}$,

for $r = 1,2,3,4$ we obtain $y_2(t), y_3(t), y_4(t)$ and $y_5(t)$, respectively:

$y_2(t) = y_0(t) - I_t^{1.9} y_1(t) = 1 + t^2 + \frac{1}{\Gamma(6.7)} t^{5.7} + \frac{2}{\Gamma(8.7)} t^{7.7}$,

$y_3(t) = y_0(t) - I_t^{1.9} y_2(t) = 1 + t^2 - \frac{1}{\Gamma(8.6)} t^{7.6} - \frac{2}{\Gamma(10.6)} t^{9.6}$,

$y_4(t) = y_0(t) - I_t^{1.9} y_3(t) = 1 + t^2 + \frac{1}{\Gamma(10.5)} t^{9.5} + \frac{2}{\Gamma(12.5)} t^{11.5}$,

$y_5(t) = y_0(t) - I_t^{1.9} y_4(t) = 1 + t^2 - \frac{1}{\Gamma(12.4)} t^{11.4} - \frac{2}{\Gamma(14.4)} t^{13.4}$,

And so on…

So, we can say: $y(t) \cong \lim_{r\to\infty} y_{r+1}(t) = 1 + t^2$.

مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية
ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية

info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com ISSN: 2312-8135 | Print ISSN: 1992-0652

Table 3. Comparison between the exact and approximate solution by using the (STM) and (SAM)

x	Exact solution	N_Approximate Solution			
		$y_1(t)$	$y_2(t)$	$y_5(t)$	$y_6(t)$
0.0	1.00	1.00	1.00	1.00	1.00
0.1	1.01	1.00999111	1.01	1.01	1.01
0.2	1.04	1.03987589	1.04000025	1.04	1.04
0.3	1.09	1.08941854	1.09000254	1.09	1.09
0.4	1.16	1.15825639	1.16001312	1.16	1.16
0.5	1.25	1.24590293	1.25004698	1.25	1.25
0.6	1.36	1.35174493	1.36013338	1.36	1.36
0.7	1.49	1.47503577	1.49032274	1.49	1.49
0.8	1.64	1.61488569	1.64069483	1.64	1.64
0.9	1.81	1.77024953	1.81136846	1.81	1.81
1.0	2.00	1.93991213	2.0025127	1.99999999	2.00

Table 4. Comparison of the absolute errors (A.E.) of the example (2)

x	Absolute error			
	$N = 1$	$N = 2$	$N = 5$	$N = 6$
0.0	0.00	0.00	0.00	0.00
0.1	$8.891378163e-06$	$4.828252464e-09$	0.00	0.00
0.2	$1.241130268e-04$	$2.512844070e-07$	$2.220446049e-16$	0.00
0.3	$5.814560164e-04$	$2.539366699e-06$	$1.043609643e-14$	0.00
0.4	$1.743606933e-03$	$1.312348208e-05$	$2.728928194e-13$	$2.220446049e-16$
0.5	$4.0970742267e-03$	$4.698286380e-05$	$3.476108290e-12$	$7.327471962e-15$
0.6	$8.2550731181e-03$	$1.333837583e-04$	$2.781797014e-11$	$8.237854842e-14$
0.7	$1.4964234259e-02$	$3.227400844e-04$	$1.615092504e-10$	$6.408207298e-13$
0.8	$2.5114312329e-02$	$6.948296451e-04$	$7.414540093e-10$	$3.791189584e-12$
0.9	$3.9750468180e-02$	$1.368458608e-03$	$2.845141455e-09$	$1.818789563e-11$
1.0	$6.0087867119e-02$	$2.512695490e-03$	$9.478110962e-09$	$7.397815693e-11$
Average A.E	$\mathbf{1.40660996e-02}$	$\mathbf{4.6318267e-04}$	$\mathbf{1.2052539e-09}$	$\mathbf{8.78981735e-12}$

مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com ISSN: 2312-8135 | Print ISSN: 1992-0652

Figures 1, 3, and 5 shows a comparison of the exact and approximate solutions of linear fractional differential equations (LFDEs) from examples (1), (2), and (3), respectively. Tables (1), (3), and (5) show the outcome of the proposed method to an exact solution. Figures 2, 4, and 6 also show a comparison of the absolute error of examples (1), (2), and (3), respectively. Each plot was created with our Python program version 3.8.8 (2021).

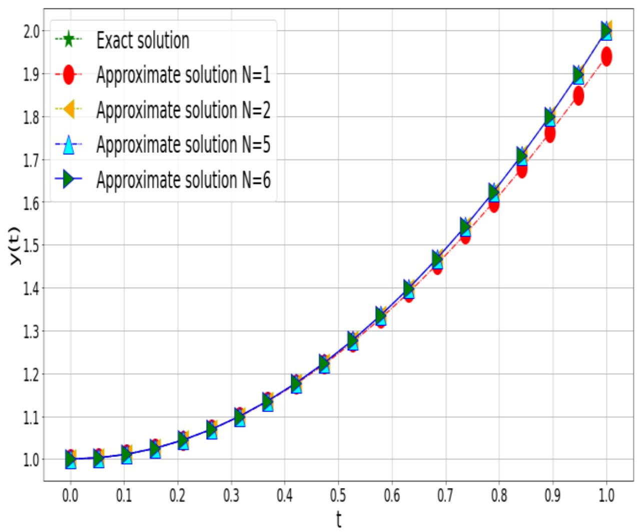


Figure 3. Exact and approximate solution of example (2)

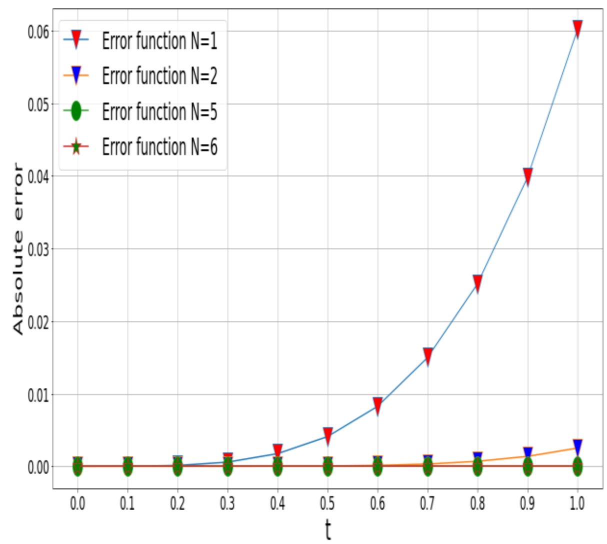


Figure 4. Absolute error of example (2)

Example 3: Consider the linear fractional differential equation

$$ {}_{0}^{C}D_t^{1.2}y(t) + {}_{0}^{C}D_t^{0.2}y(t) + y(t) = \frac{\Gamma(2.5)}{\Gamma(1.3)}t^{0.3} + \frac{\Gamma(2.5)}{\Gamma(2.3)}t^{1.3} + t^{1.5} + 1 \, , $$

With the initial conditions: $y(0) = 1, y'(0) = 0$

First method: Our problem will be solved by using Sawi transform method**,** the given example is

$$ {}_{0}^{C}D_t^{1.2}y(t) + {}_{0}^{C}D_t^{0.2}y(t) + y(t) = \frac{\Gamma(2.5)}{\Gamma(1.3)}t^{0.3} + \frac{\Gamma(2.5)}{\Gamma(2.3)}t^{1.3} + t^{1.5} + 1, $$

With the initial conditions: $y(0) = 1, y'(0) = 0$.

Take Sawi transform for both sides, we obtain:

$$ S\{{}_{0}^{C}D_t^{1.2}y(t) + {}_{0}^{C}D_t^{0.2}y(t)\} + S\{y(t)\} = S\left\{\frac{\Gamma(2.5)}{\Gamma(1.3)}t^{0.3} + \frac{\Gamma(2.5)}{\Gamma(2.3)}t^{1.3} + t^{1.5} + 1\right\} $$

$$ v^{-1.2}Y(v) - \sum_{k=0}^{1} v^{k-2.2}y^{(k)}(0) + v^{-0.2}Y(v) - \sum_{k=0}^{0} v^{k-1.2}y^{(k)}(0) + Y(v) = \Gamma(2.5)v^{-0.7} + \Gamma(2.5)v^{0.3} + \Gamma(2.5)v^{0.5} + \frac{1}{v} $$

$$Y(v)(v^{-1.2}+v^{-0.2}+1)=\Gamma(2.5)v^{-0.7}+\Gamma(2.5)v^{0.3}+\Gamma(2.5)v^{0.5}+\frac{1}{v}+v^{-2.2}+v^{-1.2}$$

$$Y(v)(1+v+v^{1.2})=\Gamma(2.5)v^{0.5}+\Gamma(2.5)v^{1.5}+\Gamma(2.5)v^{1.7}+v^{0.2}+\frac{1}{v}+1$$

$$Y(v)=\left(\frac{1}{1+v+v^{1.2}}\right)\left(\Gamma(2.5)v^{0.5}(1+v+v^{1.2})+\frac{1}{v}(1+v+v^{1.2})\right)$$

$$Y(v)=\ \Gamma(2.5)v^{0.5}+\frac{1}{v}.$$

Taking Sawi inverse for both sides and we see that which the exact solution is: $y(t)=1+t\sqrt{t}$.

Second method: The sequential approximation method will be used to solve our problem. As an example, we have $\alpha_1=0.2,\alpha_2=1.2$

start with zeros approximation of equation (12) as follows:

$$y_0(t)=1+t^{1.5}+t+\frac{\Gamma(2.5)}{\Gamma(3.5)}t^{2.5}+\frac{\Gamma(2.5)}{\Gamma(3.7)}t^{2.7}+\frac{1}{\Gamma(2.2)}t^{1.2}\ ,$$

and finding formulas equation (12), taking $r=0$ yields:

$$y_1(t)=1+t^{1.5}-\frac{t^2}{2}-\frac{\Gamma(2.5)}{\Gamma(4.5)}t^{3.5}-\frac{2\Gamma(2.5)}{\Gamma(4.7)}t^{3.7}-\frac{3}{\Gamma(3.2)}t^{2.2}-\frac{\Gamma(2.5)}{\Gamma(4.9)}t^{3.9}\ ,$$

and for $r=1,2,3,4$ we obtain $y_2(t),y_3(t),y_4(t)$ and $y_5(t)$, as follows, respectively:

$$y_2(t)=1+t^{1.5}+\frac{t^3}{6}+\frac{\Gamma(2.5)}{\Gamma(5.5)}t^{4.5}+\frac{3\Gamma(2.5)}{\Gamma(5.7)}t^{4.7}+\frac{4}{\Gamma(4.2)}t^{3.2}+\frac{3\Gamma(2.5)}{\Gamma(5.9)}t^{4.9}+\frac{3}{\Gamma(4.4)}t^{3.4}+\frac{\Gamma(2.5)}{\Gamma(6.1)}t^{5.1}.$$

$$y_3(t)=1+t^{1.5}-\frac{t^4}{24}-\frac{\Gamma(2.5)}{\Gamma(6.5)}t^{5.5}-\frac{4\Gamma(2.5)}{\Gamma(6.7)}t^{5.7}-\frac{10}{\Gamma(5.2)}t^{4.2}-\frac{6\Gamma(2.5)}{\Gamma(6.9)}t^{5.9}-\frac{7}{\Gamma(5.4)}t^{4.4}-\frac{4\Gamma(2.5)}{\Gamma(7.1)}t^{6.1}-\frac{3}{\Gamma(5.6)}t^{4.6}-\frac{\Gamma(2.5)}{\Gamma(7.3)}t^{6.3}.$$

$$y_4(t)=1+t^{1.5}+\frac{t^5}{120}+\frac{\Gamma(2.5)}{\Gamma(7.5)}t^{6.5}+\frac{5\Gamma(2.5)}{\Gamma(7.7)}t^{6.7}+\frac{11}{\Gamma(6.2)}t^{5.2}+\frac{10\Gamma(2.5)}{\Gamma(7.9)}t^{6.9}+\frac{17}{\Gamma(6.4)}t^{5.4}+\frac{10\Gamma(2.5)}{\Gamma(8.1)}t^{7.1}+\frac{5\Gamma(2.5)}{\Gamma(8.3)}t^{7.3}+\frac{\Gamma(2.5)}{\Gamma(8.5)}t^{7.5}+\frac{10}{\Gamma(6.6)}t^{5.6}+\frac{3}{\Gamma(6.8)}t^{5.8}.$$

$$y_5(t)=1+t^{1.5}-\frac{t^6}{720}-\frac{\Gamma(2.5)}{\Gamma(8.5)}t^{7.5}-\frac{6\Gamma(2.5)}{\Gamma(8.7)}t^{7.7}-\frac{12}{\Gamma(7.2)}t^{6.2}-\frac{15\Gamma(2.5)}{\Gamma(8.9)}t^{7.9}-\frac{28}{\Gamma(7.4)}t^{6.4}-\frac{27}{\Gamma(7.6)}t^{6.6}-\frac{13}{\Gamma(7.8)}t^{6.8}-\frac{3}{\Gamma(8)}t^{7}-\frac{20\Gamma(2.5)}{\Gamma(9.1)}t^{8.1}-\frac{15\Gamma(2.5)}{\Gamma(9.3)}t^{8.3}-\frac{6\Gamma(2.5)}{\Gamma(9.5)}t^{8.5}-\frac{\Gamma(2.5)}{\Gamma(9.7)}t^{8.7}.$$

and so on…

So, we can say: $y(t)\cong\lim_{r\to\infty}y_{r+1}(t)=1+t^{1.5}=1+t\sqrt{t}$.

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية

مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية مجلة جامعة بابل للعلوم الصرفة والتطبيقية

ISSN: 2312-8135 | Print ISSN: 1992-0652

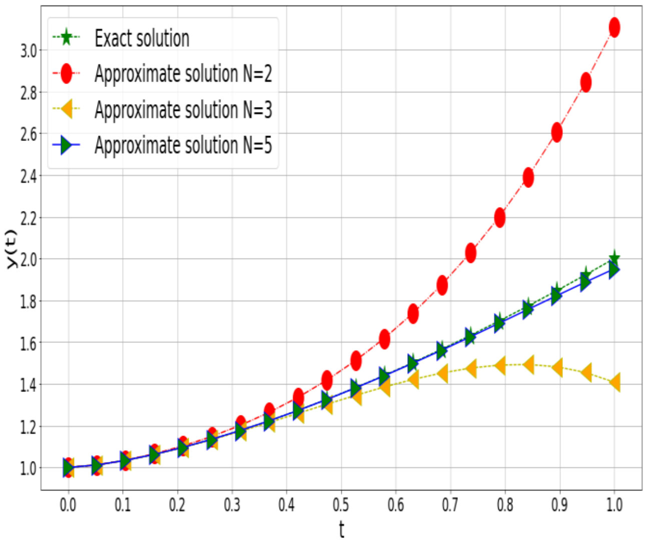
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

Table 5. Comparison between the exact and approximate solution by using the (STM) and (SAM)

x	Exact solution	N_Approximate Solution		
		$y_2(t)$	$y_3(t)$	$y_5(t)$
0.0	1.00	1.00	1.00	1.00
0.1	1.03162278	1.03223512	1.03159171	1.03162276
0.2	1.08944272	1.09507395	1.08885511	1.08944112
0.3	1.16431677	1.18513011	1.16101281	1.16429549
0.4	1.25298221	1.30593749	1.24168290	1.25284805
0.5	1.35355339	1.46336986	1.32412848	1.35299178
0.6	1.46475800	1.66488858	1.40026287	1.46294406
0.7	1.58566202	1.91930994	1.46014430	1.58076355
0.8	1.71554175	2.23673403	1.49161171	1.70393876
0.9	1.85381497	2.62854057	1.47997015	1.82894976
1.0	2.00	3.10741726	1.40769230	1.95076127

Table 6. Comparison of the absolute errors (A.E.) of the example (3)

x	Absolute error		
	$N = 2$	$N = 3$	$N = 5$
0.0	0.00	0.00	0.00
0.1	$6.12339829880737e-04$	$3.10693729514977e-05$	$1.9504349912935e-08$
0.2	$5.631229209922983e-03$	$5.87607317605431e-04$	$1.5999634381813e-06$
0.3	$2.081334076930696e-02$	$3.30396120286646e-03$	$2.1280418223801e-05$
0.4	$5.295528041129871e-02$	$1.12993083763124e-02$	$1.3416704099200e-04$
0.5	$1.098164647811144e-01$	$2.94249079359765e-02$	$5.6161163177748e-04$
0.6	$2.001305791368957e-01$	$6.44951286975010e-02$	$1.8139388458160e-03$
0.7	$3.336479230224571e-01$	$1.25517722333997e-01$	$4.8984647723884e-03$
0.8	$5.211922797608144e-01$	$2.23930042052625e-01$	$1.1602994264518e-02$
0.9	$7.747256053359330e-01$	$3.73844820516722e-01$	$2.4865207764611e-02$
1.0	1.107417264302744	$5.92307699247562e-01$	$4.9238732627032e-02$
Average A.E.	$\mathbf{2.8426748241457e-01}$	$\mathbf{1.295220242776e-01}$	$\mathbf{8.467092439377e-03}$



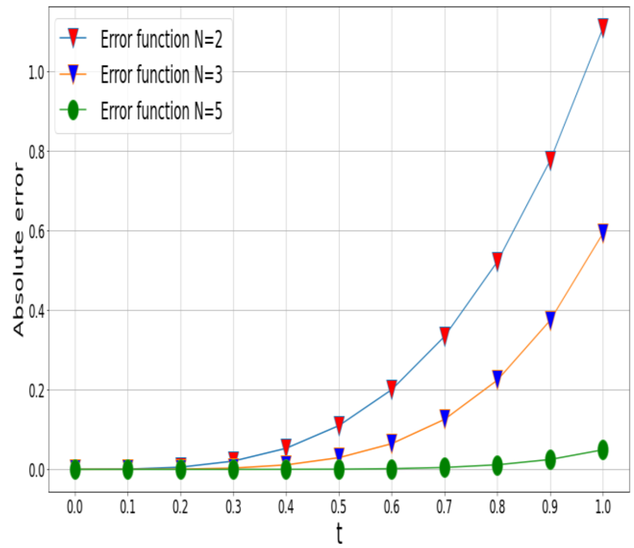


Figure 5. Exact and approximate solution of Example (3) **Figure 6**. Absolute error of Example (3)

Example 4 [25]**:** Consider the fractional differential equation

$$-{}_{0}^{C}D_{x}^{\frac{3}{2}}y(u)+y(u)=\lambda^{2}y(u);\ y(0)=0\,,y'(0)=1.$$

First method: Our problem will be solved by using Sawi transform method; if we have $\lambda=1$

$$-{}_{0}^{C}D_{x}^{\frac{3}{2}}y(u)+y(u)=-y(u)\rightarrow {}_{0}^{C}D_{x}^{\frac{3}{2}}y(u)=0.$$

Take Sawi transform for both sides, we get:

$$S\left\{{}_{0}^{C}D_{x}^{\frac{3}{2}}y(u)\right\}=S\{0\}, \text{and } S\{0\}=0$$

$$S\left\{{}_{0}^{C}D_{x}^{\frac{3}{2}}y(u)\right\}=(v)^{-\frac{3}{2}}Y(v)-\sum_{k=0}^{1}\ (v)^{k-\frac{5}{2}}y^{(k)}(0),$$

$$=(v)^{-\frac{3}{2}}Y(v)-(v)^{-\frac{3}{2}}y'(0)-(v)^{-\frac{5}{2}}y(0)=(v)^{-\frac{3}{2}}Y(v)-(v)^{-\frac{3}{2}}.$$

So, we get to the bottom of this form and we can say:

$$(v)^{-\frac{3}{2}}Y(v)-(v)^{-\frac{3}{2}}=0,$$

$$(v)^{-\frac{3}{2}}Y(v)=(v)^{-\frac{3}{2}}\rightarrow Y(v)=1.$$

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

Take inverse Sawi transform for both sides the exact solution is: $y(u) = u$.

Second method: Our problem will be solved by using Sequential approximation method, if we have $\lambda = 1$, we can write above equation of the form:

$$ {}_0^C D_x^{\frac{3}{2}} y(u) = 0, \alpha_1 = \frac{3}{2} \to \lceil \alpha_1 \rceil = 2, g(t) = 0, a = 0 $$

start with zeros approximation of equation (12) as follows:

$$ y_0(u) = \sum_{k=0}^{\lceil \alpha_1 \rceil - 1} \frac{y^{(k)}(0)}{k!} (t-0)^k + {}_0I_u^{\alpha_1} g(u) $$

$$ y_0(u) = \sum_{k=0}^{1} \frac{y^{(k)}(0)}{k!} u^k = \frac{y(0)}{0!} + \frac{y'(0)}{1!} u = u $$

take $r = 0$ yields: $y_1(u) = y_0(u) = u$,

So, we can say: $y_{r+1}(u) = y_0(u) = u$, for all $r \in \{0\} \cup \mathbb{Z}^+$

Hence, the exact solution is: $y(u) = \lim_{r \to \infty} y_{r+1}(u) = u$.

CONCLUSION

It is evident from reading the literature on fractional differential equations (FDEs) that a viable approach for handling these issues is the Sawi transformation method. The FDEs must be simplified in order to employ this method, which also perfectly solved fractional differential equations (we displayed the results). With the Sawi transformation method, FDEs may be solved with constant coefficients, and the order and complexity of the equation's structure are both reduced.

With our successful fractional derivation, we were able to conclude Sawi transform formulae that we may use in future research. It can be used to investigate a wide variety of fractional differential equations. Numerous studies have shown the Sawi transformation strategy to be an effective tool for tackling different FDE types. However, it does have certain shortcomings, much like other strategies. The technique, however, provides a different approach from the FDE techniques now in use and has the potential to make considerable progress in the field of fractional calculus.

The sequential approximation method is a powerful approximation technique used to iteratively solve complex problems by breaking them down into simpler, manageable steps. Through repeated refinements, this method converges towards a more accurate solution. Its effectiveness lies in its ability to handle various types of problems.

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

The Sawi transformation technique and the sequential approximation method are a promising method for solving FDEs, and with more research and development, it may become a tool that is even more effective and efficient for solving FDEs that get more complicated.

Future directions:

We have come a long way. in advancing the Sawi transform and Sequential approximation method. As a result, we can now efficiently solve complex system multi-high order fractional differential equations for both types (1) and (2), even when dealing with equations that include delays. This approach, in particular, allows us to achieve precise error estimation, which is critical in improving the overall accuracy and reliability of our obtained results.

Acknowledgments:

We appreciate the revisionary helpful remarks and suggestions for improving our manuscript.

Conflict of Interest:

The authors declare that they have no conflict of interest.

References

[1] J. T. Machado, V. Kiryakova, and F. Mainardi, "Recent history of fractional calculus," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 16, no. 3, pp. 1140–1153, 2011, doi: 10.1016/j.cnsns.2010.05.027.

[2] K. B. Oldham and Jerome Spanier, *The Fractional Calculus Theory and Applications of Differentiation and Integration to Arbitrary Order*. New York: Academic press ,INC, 1974. [Online]. Available: https://www.elsevier.com/books/the-fractional-calculus-theory-and-applications-of-differentiation-and-integration-to-arbitrary-order/oldham/978-0-12-525550-9

[3] R. Hilfer, "Applications of Fractional Calculus in Physics," *Applications of Fractional Calculus in Physics*. World Scientificpublishing company, Singapore, p. 90, 2000. doi: 10.1142/3779.

[4] R. Gorenflo and F. Mainardi, "Essentials of Fractional Calculus," *MaPhySto Cent.*, p. 33, 2000.

[5] K. S. Miller and B. Ross, "An introduction to the fractional calculus and fractional differential equations," *John-Wily and Sons*. Wiley-Inter Science, New York, p. 9144, 1993.

[6] S. A. Ahmad, S. K. Rafiq, H. D. M. Hilmi, and H. U. Ahmed, "Mathematical modeling techniques to predict the compressive strength of pervious concrete modified with waste glass powders," *Asian J. Civ. Eng.*, no. 0123456789, 2023, doi: 10.1007/s42107-023-00811-1.

[7] R. Khandelwal, P. Choudhary, and Y. Khandelwal, "Solution of fractional ordinary differential equation by Kamal transform," *Int. J. Stat. Appl. Math.*, vol. 3, no. 2, pp. 279–284, 2018.

[8] K. S. Aboodh, "The new integral transform 'Aboodh transform,'" *Glob. J. Pure Appl. Math.*, vol. 9, no. 1, pp. 35–43, 2013.

[9] V. Daftardar-Gejji, *Fractional Calculus and Fractional Differential Equations*, 1st ed. Birkhauser, 2019. doi: 10.1142/9789814667050_0002.

[10] M. Higazy and S. Aggarwal, "Sawi transformation for system of ordinary differential equations with application," *Ain Shams Eng. J.*, vol. 12, no. 3, pp. 3173–3182, 2021, doi: 10.1016/j.asej.2021.01.027.

[11] M. E. H. Attaweel, H. A. A. Almassry, and S. F. Khyar, "A New Application of Sawi Transform for Solving Volterra Integral Equations and Volterra Integro-differential Equations," vol. 2, pp. 65–

77, 2019.

[12] R.kumar, J.Chandel, and S.Aggarwal, "A New Integral Transform 'Rishi Transform' with Application," *J. Sci. Res.*, vol. 14, no. 2, pp. 521–532, 2022, doi: 10.3329/jsr.v2i3.4899.

[13] A. Turab, H. Hilmi, J. L. G. Guirao, S. Jalil, and N. Chorfi, "The Rishi Transform method for solving multi-high order fractional differential equations with constant coefficients," *AIMS Mathwmatics*, vol. 9, no. November 2023, pp. 3798–3809, 2024, doi: 10.3934/math.2024187.

[14] H. Jafari, "A comparison between the variational iteration method and the successive approximations method," *Appl. Math. Lett.*, vol. 32, no. 1, pp. 1–5, 2014, doi: 10.1016/j.aml.2014.02.004.

[15] M. A. Jafari and A. Aminataei, "Method of Successive Approximations for Solving the Multi-Pantograph Delay Equations," *Gen. Math. Notes*, vol. 8, no. 1, pp. 23–28, 2012, [Online]. Available: www.i-csrs.orgavailablefreeonlineathttp://www.geman.in

[16] S. S. Ahmed and S. J. Mohammedfaeq, "Bessel collocation method for solving fredholm–volterra integro-fractional differential equations of multi-high order in the caputo sense," *Symmetry (Basel).*, vol. 13, no. 12, 2021, doi: 10.3390/sym13122354.

[17] H. Hilmi, "Study of spectral characteristics of the T.Regge fractional order problem with smooth coeficients," *univesrsity Sulaimani site*, 2022, [Online]. Available: https://drive.google.com/file/d/1rjdpLi5EgsdICfHUF-rOzBGxprUqbOx3/view

[18] B. M. Faraj, S. K. Rahman, D. A. Mohammed, H. D. Hilmi, and A. Akgul, "Efficient Finite Difference Approaches for Solving Initial Boundary Value Problems in Helmholtz Partial Differential Equations," vol. 4, no. 3, pp. 569–580, 2023.

[19] S. S. Ahmed, "On system of linear volterra integro-fractional differential equations," 2009.

[20] S. S. Ahmed and shabaz Jalil, "Operational matrix of generalized block Pulse function for solving fractional Volterra - Fredholm integro differential equations," *J. SOUTHWEST JIAOTONG Univ.*, vol. 57, no. June, 2022.

[21] M. Dalir and M. Bashour, "Applications of fractional calculus," *Appl. Math. Sci.*, vol. 4, no. 21–24, pp. 1021–1032, 2010.

[22] K. Diethelm, *The Analysis of Fractional Differential Equations*. Springer, 2004. doi: 10.1007/978-3-642-14574-2.

[23] I. Podlubny, *Fractional Differential Equations*. San Diego: Elsevier, 1999. doi: 10.1007/978-3-030-00895-6_4.

[24] Shadi Ahmed Al-Taraweneh, "Solving Fractional Differential Equations by Using Conformable Fractional Derivatives Definition," University of ZARQA, 2016. doi: 10.1155/2021/5589905.

[25] H. Hilmi and K. H. F. Jwamer, "Existence and Uniqueness Solution of Fractional Order Regge Problem," *J. Univ. BABYLON*, vol. 30, no. 2, pp. 80–96, 2022.

[26] K. Jwamer and H. Dlshad, "Asymptotic behavior of Eigenvalues and Eigenfunctions of T . Regge Fractional Problem," *J. Al-Qadisiyah Comput. Sci. Math.*, vol. 14, no. 3, pp. 89–100, 2022.

[27] M. F. Kazem and A. Al-Fayadh, "Solving Fredholm Integro-Differential Equation of Fractional Order by Using Sawi Homotopy Perturbation Method," *J. Phys. Conf. Ser.*, vol. 2322, no. 1, 2022, doi: 10.1088/1742-6596/2322/1/012056.

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

ISSN: 2312-8135 | Print ISSN: 1992-0652
info@journalofbabylon.com | jub@itnet.uobabylon.edu.iq | www.journalofbabylon.com

الخلاصة

مقدمة:

في هذا البحث، نقترح طريقتين جديدتين تسمى تحويلات الصاوي وطريقة التقريبات المتسلسلة، والتي يتم تطبيقها لحل المعادلات التفاضلية الكسرية الخطية متعددة الرتب العالية بمعاملات ثابتة. حيث يقوم ريمان ليوفيل وكابوتو بتعريف المشتقات الكسرية، تم اشتقاق الصيغة الكسرية لجميع الأنواع، قمنا أولاً بتطوير تحويل الصاوي للدوال الرياضية الأساسية لهذا الغرض ثم وصفنا الخصائص المهمة للتحويل الصاوي، والتي يمكن تطبيقها لحل المعادلات التفاضلية العادية والمعادلات التفاضلية الكسرية. بعد ذلك، وجد المؤلفون حلاً دقيقًا لمثال معين للمعادلات التفاضلية الكسرية.

طرق العمل:

باستخدام هذه الطرق، يمكن الحصول على حلول دقيقة وتقريبية جيدة من خلال تكرارات قليلة فقط لطريقة التقريبات التسلسلية، وتضمن الحلول التقريبية الدقة المطلوبة. لمزيد من التحقق من صحة الأساليب، والصيغة الكسرية لطريقة تحويل الصاوي تعمل مثل التحويلات الأخرى.

النتائج:

تم الحصول على الحلول الدقيقة والتقريبية لبعض المعادلات التفاضلية الكسرية، وشرح عدة أمثلة لبيان كفاءة الطرق المقترحة وتنفيذها.

الاستنتاجات:

يتضح من قراءة الأدبيات المتعلقة بالمعادلات التفاضلية الكسرية (FDEs) أن طريقة تحويل الصاوي هي حل عملي لهذه المشكلات. لاستخدام هذا النهج، والذي يحل أيضًا المعادلات التفاضلية الكسرية بشكل كامل، يجب تبسيط FDEs.

الكلمات المفتاحية: تحويل الصاوي، التقريب المتسلسل، المعادلات التفاضلية الكسرية الخطية، مشتقة ريمان ليوفيل الكسرية، مشتقة كابوتو الكسرية.