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Embedding of Neutrosophic Graphs on Topological Surfaces

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تضمين الرسوم البيانية النيوتروسوفية على الأسطح الطوبولوجية

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ABSTRACT

Background:

A planar graph (PG) is a graph with no intersecting edges. Particular to both crisp and neutrosophic graphs (NG) is the planar graph, in contrast to crisp planar graphs. NPGs allow for the intersection of neutrosophic edge NEs, since the value of planarity in these graphs is the degree of planarity of the intersected NEs. The NPGs are often represented on a flat surface.

Materials and Methods:

This study discusses how to embed NGs on surfaces such as spheres and m-toruses by defining the degree of intersection of the neutrosophic edges of NGs with finding the faces on the given graph structures using Euler's theorems. Here, the proofs of Euler's theorems help us find, given the total NFV of G, the interval containing that value.

Result:

As result of this work obtained that for any two isomorphic planer graphs, they have the same planarity value. For any neutrosophic planer graph with f = (1,1,1) can be embedded in the plane if it can be embedded in the sphere and according to NPGs, for planar and spherical surfaces, equivalent theorems to Euler's formula are proved and shown.

Conclusion:

It concludes that by using neutrosophic sets and crisp graphs to construct neutrosophic graphs with the benefit of Euler’s theorem, it can provide the concept of embedding neutrosophic graphs in different topological surfaces such as a plane, sphere, and m-torus.

Keywords: Topological surface; Neutrosophic set; Planar graph; Single valued neutrosophic set; Embeddable graph.

INTRODUCTION

Graphs are sets containing vertices or points, along with a collection of ordered or nonordered pairs of nodes known as edges or lines[1]. Graph theory has numerous connections with various areas of mathematics like Topology, Numerical Analysis, Algebra and Probability. Furthermore, graph theory exhibits connections with other disciplines including Computer Science, Engineering, and Physics[2].

Smarandache provided the concept of the neutrosophic set in 1995[3,4]. It consists of components: T is truth ,I is indeterminacy, and F is falsehood, which are declared separately to tackle issues arising from inconsistent, imprecise, and indeterminate data[5]. The concept of a single-valued neutrosophic set (SVNS) was first suggested by Wang to facilitate application of set-theoretic operators to this particular type of neutrosophic set[6,7].Planar graphs are significant in both graph theory and graph drawing fields due to their numerous interesting properties[8]. These graphs are sparse, can be four-colored, enable more efficient operations compared to general graphs, and their inner structure can be described in a more concise and elegant manner. In the realm of information visualization, planar drawings of graphs are favored because they are clear and easy to comprehend, particularly since edge crossings, which often hinder readability, are minimized in such representations [9]. In 2015, a research paper was presented by Sovan Samanta and Madhumangal Pal to study the fuzzy planar graph [10]. in 2018, a Comprehensive Survey of Graph Embedding Techniques and Applications, studded by HongYun Cai, Vincent W. Zheng, and Kevin Chen-Chuan Chang [11]. In 2019, a research team consisting of A Galland and M Lelarge studied invariant embedding for graph classification [12]. They were followed in 2021 by Mengija Xu-SIAM to present a study entitled Understanding graph embedding methods and their applications [13], and recently in 2022. A method for graph embedding methodological survey was presented by Joseph R. Barr, Peter Shaw, and Faisal N. Abu-Khzam [14]. In this research outcome, the workflow is as follows:

sections 1and 2 were provided some introduction and material method

In section 3, the basic concept related of graph and planer graph were provided

In section 4, Neutrosophic graph embedding discussed

In section 5, Special cases on planer embedding studded

In section 6, the conclusion

MATERIAL AND METHODS

Finding the faces on the given crisp underline graph structures using Euler's theorems is easy. Nevertheless, it is difficult to determine the precise neutrosophic face values of NFVs in NGs. Here, the proofs of Euler's theorems help us find, given the total NFV of G, the interval containing that value to construct a planar graph and embed it in different surfaces.

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PRELIMINARIES

In order to fully profit from this article, it is required to grasp some basic principles, which we will discuss in this paper. An ordered pair (V, E) with an edge set (E) and non-empty vertex set (V) constitutes a graph G.[15] It is referred to as a multi-graph if there are numerous edges connecting any two vertices but no self-loops. A graph is referred to as planar when it is geometrically represented in a variety of ways so that edge crossings do not affect the plane surface.[16, 17] The planar representation of a graph divides the plane into multiple connected regions referred to as faces,[16] each of which is bound by the graph's edges. The infinite region is the area in the plane that is not part of the graph.

Theorem 1. [18], [19] On a planar surface, the overall count of faces present in the graph G is $f = 2 - |V| + |E|$ where V be number of vertices and E is the number of edges of the graph.

Theorem 2. [18], [20] On the m-torus surface, graph G has a total of facets of $f = 2 - 2g - |V| + |E|$ where $|V|, |E|$ and g stand for the graph's total count of the genus numbers, edges, and vertices, respectively.

Definition 1. A neutrosophic set A [2], [21] is arranged quadrilateral (X, T, I, F), considering X to be a universal set. and
$T: X \to [0,1], I: X \to [0,1], F: X \to [0,1]$ *where* (T, I, F) are membership values functions of A.

Definition 2. [6] A couple $G = (A, B)$ is a single-valued neutrosophic graph (SVNG), where $A: V \longrightarrow [0, 1]$ *and* $B: V \times V \longrightarrow [0, 1]$ are (SVN) set and relation on V respectively, furthermore the following satisfied

i) $T_B(xy) \leq \{T_A(x) \wedge T_A(y)\}$,
ii) $I_B(xy) \leq \{I_A(x) \wedge I_A(y)\}$,
iii) $F_B(xy) \geq \{F_A(x) \vee F_A(y)\}$, $\forall\, x, y \in V$.

Definition 3. Let $B = \{(xy, T_B(xy)_i, I_B(xy)_i, F_B(xy)_i), i = 1, 2, \ldots, m \mid xy \in V \times V\}$ be a SVN- multi-edge set in SVN- multi-graph G.
Then G is complete if

i) $\{T_A(x) \wedge T_A(y)\} = T_B(xy)_i$,
ii) $\{I_A(x) \wedge I_A(y)\} = I_B(xy)_i$
iii) $\{F_A(x) \vee F_A(y))\} = F_B(xy)_i$, $\forall\, x, y \in V$, $i = 1{:}m$.

Example 1. consider that $G^* = (V, E)$ is multigraph with $V = \{a, b, c, d\}$ and $E = \{ab, ab, ab, bc, bd\}$.
Let $A = (T_A, I_A, F_A)$ *be a (SVNS)* on V and $B = (T_B, I_B, F_B)$ be *SVNS* on $V \times V$

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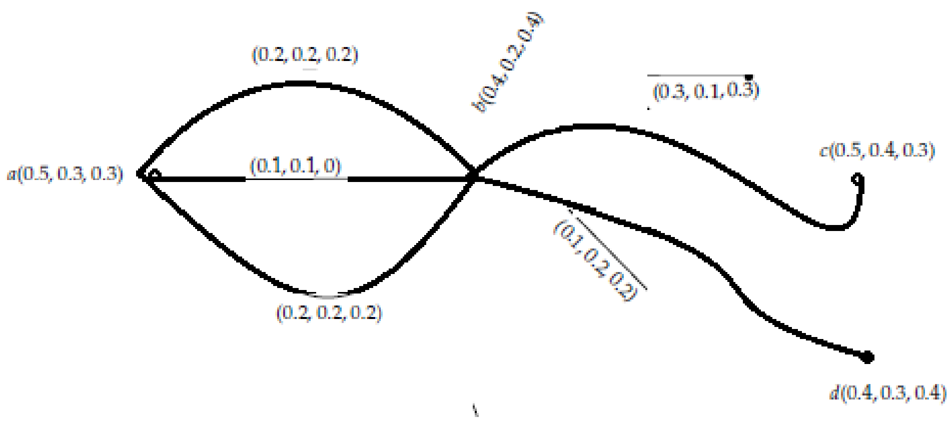

Figure 1: Single-valued neutrosophic multigraph.

Definition 4. [22] In NG G degree of any vertex x *be as follows:*

$$d(x) = \left(\sum_{i=1}^{n} T_B(x, y_i), \sum_{i=1}^{n} I_B(x, y_i), \sum_{i=1}^{n} F_B(x, y_i)\right)$$

And order of the graph n=$\sum_{i=1}^{n} d(x_i)$

Example 2. In the Previous example graph $d(a)$ = (.5, .5, .4), d(b) = (.9, .8, .9), $d(c)$ = (.3, .1, .3), and $d(d)$ = (.1, .2, .2).

the order of *G is O(G) = (1.8, 1.6, 1.8).*

Definition 5. Let $B = \{(xy, T_B(xy)i, I_B(xy)i, F_B(xy)i), i = 1, 2, \ldots m| xy \in V \times V\}$ be a SVN- multi-edges set in SVN-multigraph G.

A multi-edge xy of a graph G is said to be strong if

i) Min$\{T_A(x), T_A(y)\} \leq 2\, T_B(xy)_i$,

ii) Min $\{I_A(x), I_A(y)\} \leq 2\, I_B(xy)_i$

iii) Max $\{F_A(x), F_A(y)\} \geq 2\, F_B(xy)_i$, *for all i* = 1, 2, . . ., *m*, otherwise, it is weak.

Definition 6. *[9]* If a graph G is drawn on a plane surface without any edge intersections, it is referred to as a planar graph.

Definition 7. A NG is called a pure (NPG) if it is drawn on a plane surface without any edge intersections.

Definition 8. Assume that the NEs in G are (a, b) and (c, d), which are crossed together, and that p is the location where (c, d) and (a, b) overlap.

1) One may compute the strength of an edge (a, b) in G using

$$St_{(a,b)} = minS_{ti} = \left(\frac{min\, T_B(x,y)}{\min\{T_A(x), T_A(y)+\lambda}, \frac{min\, I_B(x,y)}{\min\{I_A(x), F_A(y)+\lambda}, \frac{min\, F_B(x,y)}{\max\{F_A(x), F_A(y)+\lambda}\right)$$

2) The ***intersecting value of p*** is computed as

$$v_p = \frac{St_{(a,b)} + St_{(c,d)}}{2}$$

$=\frac{1}{2}\left(\frac{T_B((a,b)}{\min\{T_A(a),T_A(b)+\lambda}+\frac{T_B(c,d)}{\min\{T_A(c),T_A(d)+\lambda}\right),\frac{I_B((a,b)}{\min\{I_A(a),I_A(b)+\lambda}+$
$\frac{I_B(c,d)}{\min\{I_A(c),I_A(d)+\lambda}),\frac{F_B((a,b)}{\max\{F_A(a),F_A(b)+\lambda}+\frac{F_B(c,d)}{\max\{F_A(c),F_A(d)+\lambda}))$
where the two NEs' strengths are denoted by $St_{(a,b)}$ and $St_{(c,d)}$.

3) The degree of planarity in (NPG) is indicated by its neutrosophic planarity value NPV in (NG)s. The symbol ***f*** represents the NPV. and

$$f=\frac{1}{1+\{v_{p1}+v_{p2}+\cdots+v_{pn}\}}=\frac{1}{1+\sum_{i=1}^{n}v_{pi}}=(f_T,f_I,f_F)$$

Planarity falls in a NG as the number of points of intersection rises.
As a result, the relationship between v_p and planarity is inverse. Thus, ***f*** is ranged in $0<f\leq 1$ i.e ($f=(f_T,f_I,f_F)$)is bounded and $0<f_T\leq 1$, $0<f_I\leq 1$, $0<f_F\leq 1$.)
In the NPG, any non-crossing NEs in G represent an NFV. When the NPV is (1,1,1), the boundaries of each area are determined by the NEs. The memberships value of the NFV is defined as
$\{\min\{\frac{T_B(x,y)}{\min\{T_A(x),T_A(y)+\lambda}\},\min\{\frac{I_B(x,y)}{\min\{I_A(x),I_A(y)+\lambda}\},\max\{\frac{F_B(x,y)}{\max\{F_A(x),F_A(y)+\lambda}\}\}/$
$(T_B(x,y),\ I_B(x,y),\ I_B(x,y))\in E_i{}',\lambda\in[0,1]$. where in NPG, $E_i{}'$ is the area enclosed by the NEs.
If a given geometrical representation of a single-valued NPG does not have a point of intersection, then its single-valued NPV is (1, 1, 1).

Remarks 1.

1) The underlying crisp graph (SVNG) of pure planarity graph is planar
2) If f_T and f_I decline and f_F increases, the quantity of intersection locations among the edges grows and declines correspondingly, and the characteristics of planarity changes.
3) Each SVNG is SVNPG and has specific single-valued neutrosophic planarity value.

The (NG)s are often defined and illustrated on a planar surface. Topology is composed of multiple surfaces; hence the NG can be embedded in any other surface. This section provides an explanation and associated $\tau's$ for the embedding of (NPG)s on various surfaces. We examine a particular kind of embedding on the flat surface.

EMBEDDING OF NEUTROSOPHIC GRAPH ON PLANE

It is impossible for NEs to intersect when the NPV is set to (1,1,1). Consequently, NG has neutrosophic faces. This section defines and illustrates the total NFV as well as internal and external neutrosophic faces, with appropriate examples. The illustration demonstrates that each neutrosophic face It could potentially be represented within an external neutrosophic aspect within a neutrosophic planar network.
Definition 9. The illustration shows that each neutrosophic expression can be represented as an external neutrosophic expression within a neutrosophic planar structure.
Theorem 3. The graphs $G=(A,B)$ and $G'=(A',B')$ are isomorphic graphs if and only if there exists a direct mapping between the neutrosophic face values (NFVs) of their vertices, and if the membership values assigned to vertices and edges in G match those in G'.
Proof. Consider that the (NG)s G and G' have an isomorphic planar representation. However, G and G' vertices and edges have distinct membership values. As a result, G and G's neutrosophic face values, which are made up of the vertex and edge membership values, differ from one

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another. Consequently, G and G', the (NG)s, are not isomorphic. This runs counter to the presumption. Should the two NPGs be isomorphic, then the vertex and edge membership values of G should match the vertex and edge membership values of G'.

However, let us assume that G and G' edges and vertices have the same membership value, meaning that their NFVs match one to one. In their planar form, the two (NG)s should have the same number of regions and the same amount of G and G'. As a result, G is isomorphic to G' if and only if the NFVs of G and G' correspond one to one and all of their edges and vertices have the same membership values.

Definition 10. Suppose G is the NPG with f = (1,1,1). The plane is divided into areas by the NEs. Internal neutrosophic faces are the inner regions that are bound by the NEs' membership value. The outside area, also known as the external neutrosophic faces, spans the outer surface of the (NG)s. Because of its embedding nature, every given neutrosophic face could possibly be transformed into an outer neutrosophic face. In accordance with the theorem provided below, since the isomorphic property is satisfied by the theorem's proof, it is evidently true.
Example 3. let G be in the figure bellow be neutrosophic graph with $f = (1,1,1)$,
Since $f = (1,1,1)$ then $v_{p_i} = 0\ \forall\ i$ i.e there exist no p as cross point of any pair of edge in the graph G which means that G is planer graph consist of three interior faces and one exterior face

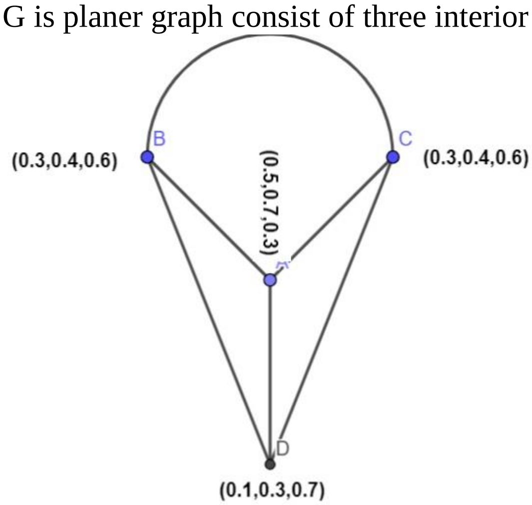

Figure 1.b Neutrosophic graph with f=(1,1,1)

Theorem 4. The planar embeddability of NPG G is open to alteration by transforming its graph structure in such a way that any face within it can be designated as the outer neutrosophic face.
Example 4. Consider the following: G is NPG, and its isomorphic planar graphs are G' and G". The external neutrosophic face of the G can be represented by any region in the NPG. It is evident from Fig. 2 that every region can be represented as an exterior neutrosophic face.

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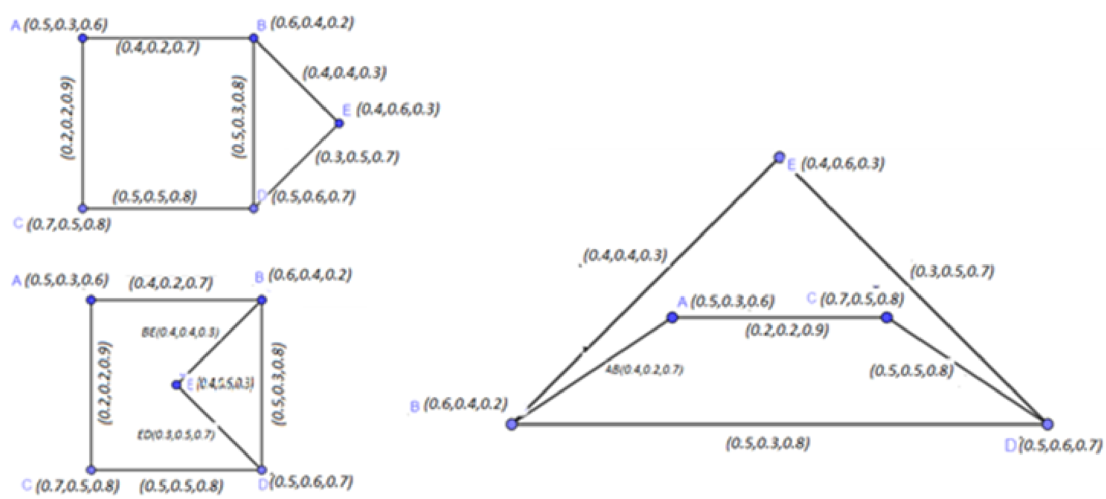

Figure 2: every region can be represented as an exterior neutrosophic face.

Remark 2. The areas that are represented by v are the finite number of regions found in every NPG, with f = (1, 1, 1). Consequently, only one exterior neutrosophic face and one internal neutrosophic face exist. V is an integer that is not negative.

Remark 3. According to Eq. 1, the total number of areas in the (NPG)is
$v = 2 - |V| + |E|$ where $|V|$ $and|E|$ are the NG's total number of edges and vertices.

Definition 11. The overall NFV is known as the total of the membership values of neutrosophic faces determined from each area., which is represented by.

$\tau = \{\sum_{i=1}^{v} \min_i\{\frac{T_B(x,y)}{\min\{T_A(x),T_A(y)+\lambda}\}, \sum_{i=1}^{v} \min_i\{\frac{I_B(x,y)}{\min\{I_A(x),I_A(y)+\lambda}\}, \sum_{i=1}^{v} \min_i\{\frac{F_B(x,y)}{\min\{F_A(x),F_A(y)+\lambda}\},\{/$
$(T_B(x,y),\ I_B(x,y),\ I_B(x,y)) \in E_i{}', \lambda \in [0,1]$. Where $E_i{}', \forall\ i = 1,2, \dots v$ is the face surrounded by the memberships value of the NEs.

Example 5. Let G represent NPG depicted in Figure 2. The presented NPG has a total neutrosophic value of (0.92307692, 1.02097902, 1.89285714).

Theorem 5. If G and G' are isomorphic (NPG)s, then. $\tau = \tau'$

Proof. The theorem's proof is definitely correct according to Theorem 3.

Corollary 5.1. The theorem's converse does not necessarily hold.

Proof. The corollary will be proved by the subsequent counterexample.

Example 6. Let τ and τ' represent the respective total NFVs of NPGs G and G'. The (NG)s do not have to be the same if the total NFVs are identical. The overall NFV of the two (NG)s in Fig. 3 is the same $\tau = \tau' = (0.7272,07272,0.875)$ However, they are distinct.

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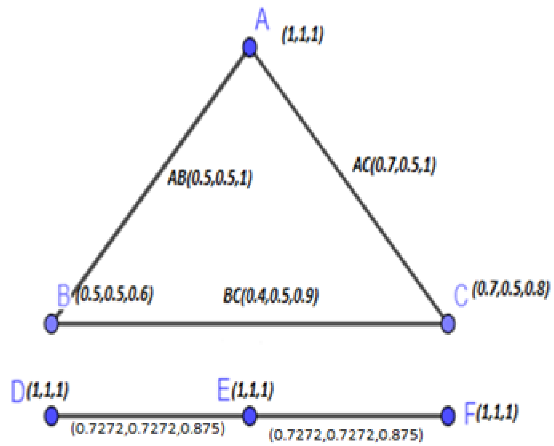

Figure 3: Two non-isiomorphic graph with same total neutrosophic face values

REPRESENTING NEUTROSOPHIC GRAPH ON SPHERE THROUGH EMBIDDING

Surfaces that can be extended into any geometric shape on both of these surfaces, for instance, spheres and planes, are simply embeddable in both crisp and (NG)s . It is possible to draw the (NG)s on both surfaces because of the embeddable characteristic.

Using the stereographic projection, we need to demonstrate that the (NPG)that may be formed on these two surfaces meets the embeddable property.

Consequently, the following theorem establishes that any NG can only be embedded in a plane or sphere's surface if its planarity value is (1,1).

Theorem 6. It is possible for (NPG) with f = (1,1,1) to be embedded in the plane only if it can also be embedded in the sphere.

Proof Considering NPG with (1, 1, 1) planarity value, to demonstrate its embeddability in the sphere's surface $S^2 = \{u, v, w / u^2 + v^2 + w^2 = 1\}$ to the surface of the plane $R^2 = \{(u, v) / \forall u, v \in R\}$

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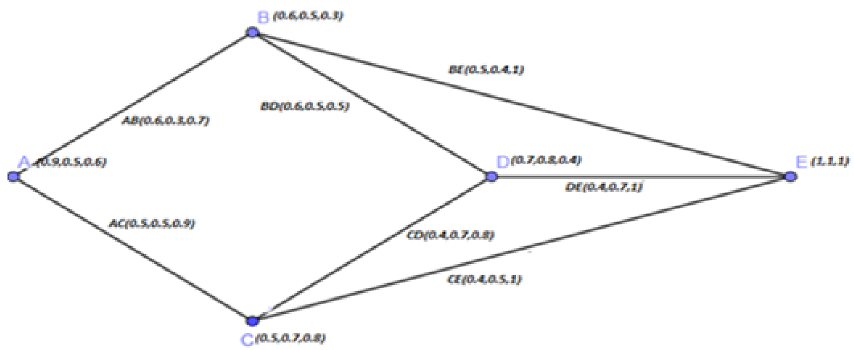

Figure 4: Total neutrosophic face value of neutrosophc graph

The function g maps points (u,v,w) on a sphere, where $u = \sqrt{(1-x^2)}\, Cos\theta,\ v = \sqrt{(1-x^2)}\, Sin\theta$ and $w{=}x$, with θ ranging from 0 to 2π and x ranging from -1 to 1, to points (u,v) in the plane R^2 using stereographic projection. Every point and line segment on a sphere is displayed on a plane in a manner that every point on the plane corresponds to a point on the sphere and also the same thing to a line segment, and vice versa, ensuring a correlation between the two. hence, g is onto and one-to-one. Since G is NPG with $f = (1,1,1)$, The membership values associated with the V(G) and E(G) are specified within the context of the sphere, while the planes serve as a contrasting aspect or reference point $(u,v,w) = \big(T_A(a), I_A(a), F_A(a)\big) = (u,v)$ and $(u,v,w)(u',v',w') = \big(T_B(a,b), I_B(a,b), F_B(a,b)\big) = (u,v)(u',v')$ a, where $T_A$and T_B are the couple of functions of (NPG)G. Except for the point (0, 0, 1) at the sphere's North Pole, each distinct point on the sphere that is identifiable corresponds uniquely to a location on the plane. By virtue of a planar surface possessing the external neutrosophic face, the point with coordinates (0, 0, 1) is designated as the outer neutrosophic aspect of the sphere. Think about projection mapping in stereography $sg: S^3 \backslash (0,0,1) \ \to R^2$ such that $sg(c) = (c')$ In this context, c and c' represent points or line segments on the sphere and plane of the NPG (Non-Euclidean Projective Geometry). Therefore, a mapping function sg of each point or line segment $c \in S^3$(specifically at coordinates (0,0,1) The mapping sg intersects the plane surface uniquely at a single point or line denoted as $c' \in R^2$,hence, the function sg is bijective. With a planarity value of 1, let G be any NG on a sphere, by excluding the embedding of the point (0, 0, 1), a stereographic projection can successfully achieve a planar embedding on the surface of the plane when transitioning from the sphere to plane. Conversely, the embedding on the sphere's surface is achieved by taking the reverse process of stereographic projection, given NPG on the plane's surface. Thus, NPG has NPV of 1 if and only if it can be embedded in both the sphere and the plane. Figure 4 demonstrates that NPG can indeed be embedded in both spheres and planes.

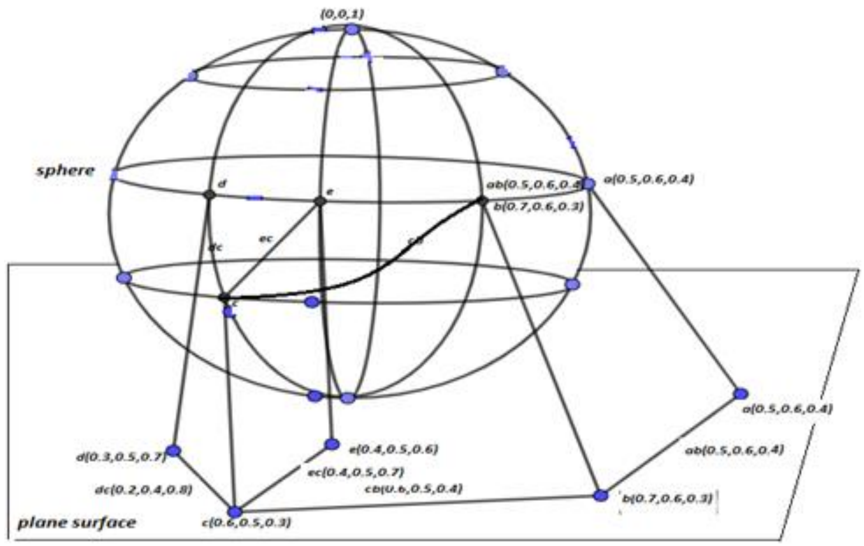

Figure. 5 A stereographic projection of neutrosophic graph

Note 1. For any NPG that can be easily embedded on both surfaces, meaning that the graph's total neutrosophic value stays constant.

REPRESNTING NEUTROSOPHIC GRAPH EMBIDDING ON M-TORUS SURFACE.

An m-torus surface is created by introducing a finite number of apertures to a sphere; the genus of the torus surface is denoted by the natural number m. For instance, a one-genus or one-torus surface represents a surface with one aperture or hole.

Note 2. At that point, the m-genus surface referred to as a sphere or 0-genus.

Note 3. The m-torus surface comprises a limited number of regions. The total number of regions on this surface, as described by Equation 2, is given by v = 2 - 2m - |V| + |E|, where |V| and |E| represent the total count of vertices and edges in NG, respectively.

Definition 12. The neutrosophic planar network, ensuring non-crossing of NEs, is possible constructed by incorporating the quantity of handles needed for a sphere. The neutrosophic m-torus value for the NPGs is computed and represented by h. It is defined as follows

$$h = \frac{\sum_{i=1}^{n} v_{Pi}}{1 + \sum_{i=1}^{n} v_{Pi}}$$

Corollary 5.2 Consider G as the NPG; then there exists a proportionality between f and h, where f and h denote the NPV and neutrosophic genus value, respectively.

Proof: G is NPG, crossings of the membership values of NEs may or may not occur. Assume that there are only a limited number of NE crossings in G, then to demonstrate that the NPV and m-torus value of (NPG)G is $f \propto h$.

Since $h = \frac{\sum_{i=1}^{n} v_{Pi}}{1+\sum_{i=1}^{n} v_{Pi}} = \sum_{i=1}^{n} v_{Pi} * \frac{1}{1+\sum_{i=1}^{n} v_{Pi}}$

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then $h = k * f$ where $k = \sum_{i=1}^{n} I_{Pi}$ and $f = \frac{1}{1+\sum_{i=1}^{n} v_{Pi}}$ .Therefore, $f \propto h$.

Theorem 7. It is possible to embed any NG onto the m-torus surface.
Proof: Assuming that the membership values of the NEs in the NG G are crossed together, we can proceed without losing generality. By decreasing the count of handles on the sphere, the occurrences of NE crossings diminish. Consequently, each handle includes at least one NE, leading to intersections of edges in G. Consequently, any NG can be transformed into an NPG with a planarity value of 1, specifically on the m-torus surface.
Theorem 8. On surface of the m-torus, the NPG drawn achieves infimum (Inf) and supremum (Sup) in [0, 1).
Proof: Assume G is NPG has a planarity of $0 < f_1$, moreover, there is at least one instance where the membership values of the NEs intersect. By putting a limited number of handles on the sphere, it is possible to decrease the number of crossings of NEs.
Consider two NEs, denoted by $\left(T_B(a,b), I_B(a,b), F_B(a,b)\right)$ and $\left(T_B(c,d), I_B(c,d), F_B(c,d)\right)$ which intersect in G. Assume that the membership values of these NEs are low, meaning that $(T,I,F)_{(a,b)}$ and $(T,I,F)_{(c,d)}$ are both less than or equal to 0.5. Hence, the neutrosophic genus value is at its minimum and falls within 0 < h < 1. Assume that the membership values of the NEs are high, such as $(T,I,F)_{(a,b)}$ and $(T,I,F)_{(c,d)}$ being both greater than or equal to 0.5, In this scenario, the neutrosophic m-torus value reaches its maximum and falls within $1 > h > 0$. Each neutrosophic m-torus value within the range (0, 1) indicates minimum m-genus value among all NEs in G, referred to as the infimum, and the maximum genus value among all NEs in G, referred to as the supremum. Let R and r denote the supremum and infimum of neutrosophic genus graph G, respectively. Thus, it can be expressed as $0 \le r \le h \le R < 1$.

SPECIAL CASES ON PLANAR EMBEDDING

Neutrosophic planar embedding encompasses several techniques that can be applied to any neutrosophic planar graph with ease. These methods include straight-line embedding, linear planar embedding, and neutrosophic planar triangulation, all of which facilitate the representation of such graphs.
Definition 13. When each neutrosophic face is enclosed by three edges or vertices with membership values, a neutrosophic planar or spherical embedding is termed as triangulation.
Example 7. According to Figure 6, the neutrosophic face contains a minimum of three membership-valued neutrosophic vertices within its interior. Hence, the depicted neutrosophic graph qualifies as a neutrosophic planar triangulation.
Definition 14. A piecewise-linear planar embedding is described as a straightforward polygonal path in neutrosophic terms, capable of accommodating any edge membership value.
Definition 15. For every simple neutrosophic PG with a planarity value of 1, there is a straight-line neutrosophic embedding, where each edge's membership value can be depicted as a single line segment.

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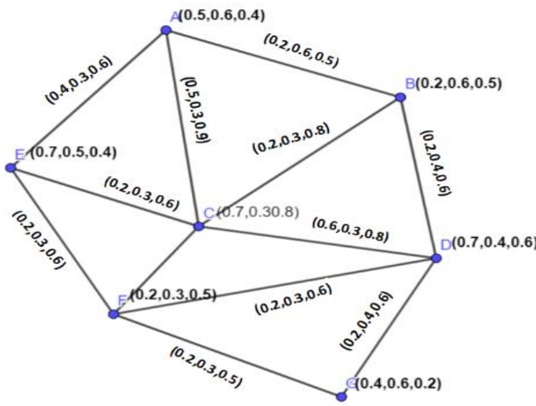

Figure 6:a neutrosophic planar triangulation

CONCLUSION

n this work, the process of embedding the neutrosophic graph on plane and spherical surfaces is presented, as well as embedding such graphs on surfaces with m-tours. Also, special cases of embedding on plane surfaces were studied. Additionally, we delved into further aspects of the distinctive properties of planar NGs, including their transformation from plane surfaces to spheres and 1-genus surfaces, and calculated the degree of planarity.

Conflict of interests.

There are non-conflicts of interest.

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الخلاصة

الخلفية

الرسم البياني المستوي هو رسم بياني بدون حواف متقاطعة. بخصوص كل من الرسوم البيانية الواضحة والنيوتروسوفيكية المستوية . على النقيض من الرسوم البيانية المستوية الاعتيادية، تسمح الرسوم البيانية النتروسوفيكية بتقاطع الحواف النيوتروسوفيكية، نظرًا لأن قيمة الاستواء في هذه الرسوم البيانية هي درجة استواء الحواف النتروسوفيكية المتقاطعة. غالبًا ما يتم تمثيل الرسوم البيانية النتروسوفيكية على سطح مستو.

المواد والطرق

تناقش هذه الدراسة كيفية تضمين الرسوم النتروسوفيكية على الأسطح مثل المستوي والكرات واشمال اخرئ من خلال تحديدها باستخدام درجة تقاطع الحواف النيوتروسوفية لـ NG مع إيجاد الوجوه على الهياكل البيانية المحددة باستخدام نظريات أويلر،حيث تساعدنا أدلة نظريات أويلر في العثور على الفترة التي تحتوي على تلك القيمة، بالنظر إلى إجمالي قيمة الوجوه النتروسوفيكية للرسم البياني G.

النتيجة

نتيجة لهذا العمل تم الحصول على أنه لأي رسمين بيانيين مستويين متماثلين، لهما نفس القيمة المستوية. بالنسبة لأي رسم بياني مستو نيوتروسوفي مع f = (1،1،1) يمكن تضمينه في المستوى إذا كان من الممكن تضمينه في المجال ووفقًا لـ NPGs، بالنسبة للأسطح المستوية والكروية، تم إثبات وعرض النظريات المكافئة لصيغة أويلر.

الاستنتاج

باستخدام مجموعات نيوتروسوفيكية مع الرسوم بيانية الاعتيادية لإنشاء رسوم بيانية نيوتروسوفكية مع الاستفادة من نظرية أويلر، فإنه يمكن أن يوفر مفهوم تضمين الرسوم البيانية النيوتروسوفكية في الأسطح الطوبولوجية المختلفة مثل المستوى، والكرة، وما شابه ذلك.

الكلمات المفتاحية: السطح الطوبولوجي، المجموعة النيوتروسوفكية، المجموعة النيوتروسوفكية ذات القيمة المفردة، الرسم البياني المستوي، الرسم البياني القابل للتضمين.