Case Study

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Fuzzy Socle-Ɉacobson–Two-Absorbiang Submodules

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المقاسات الجزئية سوكل-جاكوبسون الضبابية المستحوذة على-2

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ABSTRACT

Background:

The concept has been studied fuzzy- Socle-Ɉacobson–Two-Absorbiang submodules and some fuzzy relationships According to the concept above. Where the new definition was named fuzzy Socle-Ɉacobson–Two-Absorbiang, we used a few algebraic techniques to determine the rapport between fuzzy Two-Absorbing and fuzzy Socle-Ɉacobson–Two-Absorbiang. In addition to studying functions and some other related concepts. Our study's ramifications can be utilized to identify novel ideas that rely on hazy Socle-Ɉacobson–Two-Absorbiang concepts. In addition, we used some of the basics of modules and partial modules to prove that the intersection process in the new concept does not necessarily have to be achieved. In this study, the functions in modules and their relationship with the same concept and investigated some of the properties of those functions. The properties of the distributive module were also used in this research to achieve some theorems. In this research, it was also proven that the process of combining does not necessarily occur between two sub modules that bear the characteristics of the studied concept.

Results:

Using previous properties to find a new fuzzy sub module and a new definition.

Conclusions:

In this work, we presented a new type of fuzzy sub modules using the properties of addition and intersection between them and some properties of the new fuzzy submodule.

Keywords: Two-Absorbing submodule; F-SɈ-2Absorbiang; Fuzzy module; Fuzzy submodule; distributive Ғ-module .

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INTRODUCTION

In 1965, L. A. A fuzzy subset of a set is a concept that Zadeh presented as a way to describe uncertainty in the real physical world[1]. In algebra, one of the first fields of pure mathematics, the idea of fuzzy sets was initially used. Rosenfeld supplied the fuzzy subgroups in 1971[2]. The idea of fuzzy modules and fuzzy submodules were first introduced by Negoita and Ralescu in 1975 [3]. In this article, every modules is unitary and every ring is commutative with identity. An essential turn in module theory over a commutative ring is played by prime submodules [4]. Aldorri [5] formulated the idea of Strongly Pseudo Nearly 2 Absorbing submodule: a proper sub module V of an Ʀ-modul Ɯ is named to be Strongly Pseudo Nearly 2 Absorbing submodule of Ɯ if $ɑɓҳ \in V$, where $ɑ, ɓ \in Ʀ$, $ҳ \in Ɯ$ implies that $ɑҳ \in V + (soc(Ɯ) \cap J(Ɯ))$ or $ɓҳ \in V + (soc(Ɯ) \cap J(Ɯ))$ or $ɑɓƜ \subseteq V + (soc(Ɯ) \cap J(Ɯ))$. Where Soc(Ɯ) is the total of every simple submodules of M and the Jacobson of an R-module Ɯ shortened by $J(Ɯ)$ is defined as the intersection of every max submodule of Ɯ. Wafaa presented the concept of T-Absorbiang fuzzy submodule[6]. The idea of 2-Absorbiang fuzzy submodule is expanded upon in this piece to fuzzy Socle-jacobson 2-Absorbiang submodule. The research consists of the basics of work in the first part, while the second part consists of the definition of fuzzy Socle-jacobson T-Absorbing and some characteristics and examples that pertain to this definition.

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We denote fuzzy Socle-jacobson T-Absorbiang submodule by F-SJ-2Absorbiang subm.

PRELIMINARIES

In this section, there is a set of definitions about Ƒ-sets, Ƒ-moduls & 2Absorbiang Ƒ-subms.

Definition 1.[1] If a_h:$W \to [0,1]$ be Ƒ-set in W, where $a \in W$, t∈ [0,1] defined by

$a_h(k) = \begin{cases} h & if\ a = k \\ 0 & if\ a \neq k \end{cases}$ $\forall\ k \in W$. a_h is named Ƒ-singleton.

Definition 2.[7] If W & V be two Ƒ-moduls of R-modul $\mathcal{M}$. V is named to be Ƒ-subm of W if $V \subseteq W$.

Definition 3.[8] $\mathcal{M}$ be an R-module. An Ƒ-set W of $\mathcal{M}$ is named Ƒ-module of an R-module $\mathcal{M}$ if :

- $W(a-b) \geq min\ \{W(a), W(b), \forall\ a, b \in\ \mathcal{M}\}$
- $W(ra) \geq W(a), \forall\ a \in \mathcal{M}\ \&\ r \in R$.
- $W(0) = 1$.

Proposition 4.[9] If V be an Ƒ-subm of an Ƒ-module W ∴ $f(V)$ is an Ƒ-subm of W'.

Definition 5.[8] If W & V be 2-Ƒ-subms of an R-module $\mathcal{M}$. The addition $W + V$ is defined by :$(W + V)(a) = sup\{inf\ \{W(b), V(c)\}$ such that $a = b + c, \forall\ a, b, c \in \mathcal{M}\}$. In addition, W & V be an Ƒ-subm of an R-modul $\mathcal{M}$.

Lemma 6.[10] If V be an Ƒ-subm of an Ƒ-module $\mathcal{M}$, $(V_t: \mathcal{M}_t) \geq (V: \mathcal{M})_t$,∀ t ∈ [0, 1].

Proposition 7.[11]

If $f: \mathcal{M} \longrightarrow W$ be an $\mathcal{R}$-homomorphism, then $f(Soc(\mathcal{M})) \subseteq Soc(W)$.

Proposition 8.[11]

If $f: \mathcal{M} \longrightarrow \mathcal{M}'$ be an $\mathcal{R}$-epiomorphism & $ker\ f$ is a small subm of $\mathcal{M}$, then

$f(J(\mathcal{M})) = J(\mathcal{M}')$ & $f^{-1}(J(\mathcal{M}')) = J(\mathcal{M})$.

Proposition 9.[11]

Given $f: \mathcal{M} \longrightarrow \mathcal{M}'$ be an $\mathcal{R}$-homomorephism & $\{\mathcal{A}_i': i \in I\}$ are famalies of subms of $\mathcal{M}$ & $\mathcal{M}'$ respectiviely. Then:

a) $f(\sum_{i \in I} \mathcal{A}_i) = \sum_{i \in I} f(\mathcal{A}_i)$; $f^{-1}(\cap_{i \in I} \mathcal{A}_i') = \cap_{i \in I} f^{-1}(\mathcal{A}_i')$.

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b) "$f^{-1}(\sum_{i\in I} \mathcal{A}_i') \supseteq \sum_{i\in I} f^{-1}(\mathcal{A}_i')$" and "$f(\bigcap_{i\in I} \mathcal{A}_i) \subseteq \bigcap_{i\in I} f(\mathcal{A}_i)$". If $\mathcal{A}_i' \subseteq Im\, f$, $\forall\, i \in I$, then $f^{-1}(\sum_{i\in I} \mathcal{A}_i') = \sum_{i\in I} f^{-1}(\mathcal{A}_i')$ & if$Ker\, f \subseteq \mathcal{A}_i$, $\forall\, i \in I$, then $f(\bigcap_{i\in I} \mathcal{A}_i) = \bigcap_{i\in I} f(\mathcal{A}_i)$.

Definition 10.[12] The module $\mathcal{M}$ is named distributive if $\mathcal{A} \cap (\beta + C) = (\mathcal{A} \cap \beta) + (\mathcal{A} \cap C)$ or $\mathcal{A} + (\beta \cap C) = (\mathcal{A} + \beta) \cap (\mathcal{A} + C)$, $\forall$ subms $\mathcal{A}, \beta$ & C.

Fuzzy Socle-*J*acobson–Two-Absorbiang Submoduls.

In this part, we presented F- Socle-Jacobson–Two-Absorbiang Subms as a generalisation of ordinary concept.

Definition 11. A proper Ƒ-subm Ɏ of a Ƒ-module $\mathscr{M}$ of an R-module Ɯ is named Ƒ-Socle-Jacobson–Two-Absorbiang (simply Ƒ-SJ-2Absorbiang) submodule of $\mathscr{M}$ if $a_h b_q c_t \subseteq$ Ɏ,where $a_h b_q$ are Ƒ-singletons of R & c_t is an Ƒ-singleton of $\mathscr{M}$ implies that $a_h b_q \subseteq$ [Ɏ + Ƒ-($Soc(\mathcal{M}) \cap J(\mathcal{M})$) $:_R \mathscr{M}$] or $b_q c_t \subseteq$ Ɏ + Ƒ-($Soc(\mathcal{M}) \cap J(\mathcal{M})$) or $a_h c_t \subseteq$ Ɏ + Ƒ-($Soc(\mathcal{M}) \cap J(\mathcal{M})$), $\forall\, h, q, t \in [0,1]$.

In addition, a proper Ƒ-ideal K of R is named an Ƒ-SJ-2Absorbiang ideal of R if $a_h b_q c_t \subseteq K$,where $a_h b_q$ & c_t are Ƒ-singletons of R implies that $a_h b_q \subseteq K$ + Ƒ-($Soc(\mathcal{M}) \cap J(\mathcal{M})$) or $b_q c_t \subseteq K$ + Ƒ-($Soc(\mathcal{M}) \cap J(\mathcal{M})$) or $a_h c_t \subseteq K$ + Ƒ-($Soc(\mathcal{M}) \cap J(\mathcal{M})$), $\forall\, h, q, t \in [0,1]$.

like that, subm K of R-module $\mathscr{M}$ is named Socle-Jacobson–Two-Absorbiang (simply SJ-2Absorbiang) submodule of $\mathscr{M}$ if $abc \in K$, where $a, b \in$ R & $c \in K$ implies that $ac \in K$ + ($Soc(\mathcal{M}) \cap J(\mathcal{M})$) or $bc \in K$ + ($Soc(\mathcal{M}) \cap J(\mathcal{M})$) or $ab \in$ [K +($Soc(\mathcal{M}) \cap J(\mathcal{M})$) $:_R \mathscr{M}$], our definition will be in form:

Ƒ-($Soc(\mathcal{M}) \cap J(\mathcal{M})$):$\mathscr{M} \to [0,1]$

$$\text{Ƒ-}(Soc(\mathcal{M}) \cap J(\mathcal{M}))(c) = \begin{cases} 1 & if\ c \in (Soc(\mathcal{M}) \cap J(\mathcal{M})) \\ k & if\ c \notin (Soc(\mathcal{M}) \cap J(\mathcal{M})) \end{cases} \qquad k \in [0,1]$$

Lemma 12. Any Ƒ-module $\mathscr{M}$, $\forall\, k \in [0,1]$ with $((\text{Ƒ} - (Soc(\mathcal{M}) \cap J(\mathcal{M})))_k \neq (\mathcal{M})_k((\text{Ƒ} - (Soc(\mathcal{M}) \cap J(\mathcal{M})))_k = (Soc(\mathcal{M}_k) \cap J(\mathcal{M}_k))$.

Proposition 13. Let F be a subm of Ɯ & $\mathscr{M}$ be a distributive Ƒ-module of an R-module Ɯ with $\mathscr{M}(k)$=1 $\forall\, k \in$ Ɯ, if K be an Ƒ-subm of $\mathscr{M}$ defined by K:Ɯ $\to$ [0,1] such that :

$$K(k) = \begin{cases} 1 & if\ k \in \text{F} \\ h & if\ k \notin \text{F} \end{cases} \qquad h \in [0,1]$$

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Then K is a Ƒ-SJ-2Absorbiang of $\mathscr{M}$ iff W is Ƒ-SJ-2Absorbiang of Ꟃ.

Proof:

$\Longrightarrow$ (K+Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$))($k$)=$sup\{min(K$(y), Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$)(z), $y+z=k\}$. So, we have (K + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$))($k$)=

$$\begin{cases} 1 & if\ k \in K\ +\ Ƒ-(Soc(\mathcal{M})\cap J(\mathcal{M})) \\ d & if\ k \notin K\ +\ Ƒ-(Soc(\mathcal{M})\cap J(\mathcal{M})) \end{cases} \quad d = max[v_1, v_2]$$

Now, let K 2-Absorbiang subm of Ꟃ. Assume $a_h b_q c_t \subseteq K$, where $a_h b_q \subseteq$ R & $c_t \subseteq \mathscr{M}$ where $h, q, t \in [0,1]$ that is either $abc \in K$ or

$abc \notin K$ either $a_h c_t \subseteq$ (K + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$)) or $b_q c_t \subseteq$ (K + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$)) or $a_h b_q \subseteq [K$ + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$)$:_R \mathscr{M}]$, we have two cases:

Case (1) $abc \in K$, then $ac \in K$ + ($Soc(\mathcal{M})\cap J(\mathcal{M})$) or $bc \in K$ + ($Soc(\mathcal{M})\cap J(\mathcal{M})$) or $ab \in [K$ +($Soc(\mathcal{M})\cap J(\mathcal{M})$) $:_R \mathscr{M}]$. We have three cases

1. If $ac \in K$ + ($Soc(\mathcal{M})\cap J(\mathcal{M})$) ∴ ($K$ + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$))($ac$) =1 ∴ $a_h c_t = (ac)_n \subseteq (ac)_1 \subseteq$ (K + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$)).
2. If $bc \in K$ + ($Soc(\mathcal{M})\cap J(\mathcal{M})$) ∴ ($K$ + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$))($bc$) =1 ∴ $b_q c_t = (bc)_n \subseteq (bc)_1 \subseteq$ (K + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$)).
3. If $ab \in [K$ +($Soc(\mathcal{M})\cap J(\mathcal{M})$) $:_R \mathscr{M}]$, let $k_p \subseteq \mathscr{M}$ with $p\in$[0, 1], then $a_h b_q k_p = (abk)_z$ where $z = inf\{h, q, p\}$ but $abk \in K$ + ($Soc(\mathcal{M})\cap J(\mathcal{M})$), that is $(abk)_z \subseteq (abc)_1 \subseteq$ (K + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$)), therefore $a_h b_q c_t \subseteq$ (K + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$)) that's mean $a_h b_q \subseteq [K$ + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$) $:_R \mathscr{M}]$

Case (2) If $abc \notin K$, then $K(abc) = p$ with $ac \notin K$, thus $K(ac) = p$. Since $a_h b_q c_t \subseteq K$, then $(abc)_z \subseteq K$ where $z = inf\{h, q, t\}$ that is $K(abc) \geq z$, thus $p \geq z$. Now, if $z = t$ this implies $(ac)_t \subseteq (ac)_p \subseteq K \subseteq K$ + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$)). If $z = q$, K(j) ≥ t for any j ∈ Ꟃ, &

$$(a_h b_q \mathcal{M}_{Ꟃ})(j) = \begin{cases} q & if\ j = abs\ \ for\ some\ \ s \in Ꟃ \\ 0 & \quad j \neq abs \end{cases}$$

then $(a_h b_q \mathcal{M}_{Ꟃ})(j) \leq K\ (j)$, hence $(a_h b_q \mathcal{M}_{Ꟃ})(j) \subseteq K \subseteq K$ + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$)) then $a_h b_q \subseteq [K$ + Ƒ-($Soc(\mathcal{M})\cap J(\mathcal{M})$) $:_R \mathscr{M}]$. Therefore K be an Ƒ-SJ-2Absorbiang of $\mathscr{M}$.

$\Longleftarrow$ Let K be an Ƒ-SJ-2Absorbiang of $\mathscr{M}$. Let $x_1 x_2 x_3 \in (K)_t$, with $x_1, x_2 \in$ R & $x_3 \in (\mathcal{M})_t$ it follows that $(x_1 x_2 x_3)_t \subseteq K$, that is $x_{1t} x_{2t} x_{3t} \subseteq K$. But K is a Ƒ-SJ-2Absorbiang, then

$x_{1t}x_{3t}$ ⊆ K + Ғ-($Soc(\mathcal{M})$∩ $J(\mathcal{M})$) or $x_{2t}x_{3t}$⊆ K + Ғ-($Soc(\mathcal{M})$∩ $J(\mathcal{M})$) or $x_{1t}x_{2t}$ ⊆ [K + Ғ-($Soc(\mathcal{M})$∩ $J(\mathcal{M})$) $:_R \mathcal{M}$]. Thus

(K + Ғ-($Soc(\mathcal{M})$∩ $J(\mathcal{M})$)($x_{1t}x_{3t}$) ≥ t or (K + Ғ-($Soc(\mathcal{M})$∩ $J(\mathcal{M})$) ($x_{2t}x_{3t}$) ≥ t or (K + Ғ-($Soc(\mathcal{M})$∩ $J(\mathcal{M})$)($x_{1t}x_{2t}$) ≥ t, hence by (Lemma 2.6) & (Lemma 3.2) $x_{1t}x_{3t} \in (K + Ғ - \left(Soc(\mathcal{M}) \cap J(\mathcal{M})\right)_t = K_t + (Ғ - \left(Soc(\mathcal{M}) \cap J(\mathcal{M})\right)_t$ or $x_{2t}x_{3t} \in (K + (Ғ - \left(Soc(\mathcal{M}) \cap J(\mathcal{M})\right)_t = K_t + (Ғ - \left(Soc(\mathcal{M}) \cap J(\mathcal{M})\right)_t$ or $x_{1t}x_{2t} \in [K + (Ғ - (Soc(\mathcal{M}) \cap J(\mathcal{M})) :_R \mathcal{M}]_t \subseteq [K_t + (Ғ - \left(Soc(\mathcal{M}) \cap J(\mathcal{M})\right)_t :_R \mathcal{M}_t]$. Now, $\mathcal{M}$ is distributive Ғ-modul, then $x_{1t}x_{3t} \in K_t + (Soc(\mathcal{M}))_t \cap (J(\mathcal{M}))_t)$ or $x_{2t}x_{3t} \in K_t + (Soc(\mathcal{M}))_t \cap (J(\mathcal{M}))_t)$ or $x_{1t}x_{2t} \in [K_t + ((Soc(\mathcal{M}))_t \cap (J(\mathcal{M}))_t) :_R \mathcal{M}_t]$. Therefore

$K_t = W$ is SJ-2Absorbiang of $\mathcal{M}_t$= Ⱳ. ∀ t > 0.

Remark 14. Let $\mathcal{M}$ be an Ғ-module & K, L are SJ-2Absorbiang subms of $\mathcal{M}$. $K + L$ need not to be SJ-2Absorbiang subms of $\mathcal{M}$.

Proof.

If $\mathscr{M}$: Ⱳ⟶[0,1], $\mathscr{M}(h)$=1, ∀ $h \in \mathscr{M}$,

Let $K: Z \longrightarrow [0,1]$ where $K(h)=\begin{cases}1 & if\ h \in 2Z \\ k & if\ h \notin 2Z\end{cases}$, 0< $h < 1$ like that

$L: Z \longrightarrow [0,1]$ where $L(h)=\begin{cases}1 & if\ h \in 3Z \\ v & if\ h \notin 3Z\end{cases}$, 0< $v < 1$

suppose K_1=2Z, L_1 =3Z where K_1 & $L_1$are SJ-2Absorbiang subms of $\mathcal{M}$ & $\mathcal{M}$=Z. But $K_1 + L_1$= 2Z+3Z = Z = $\mathcal{M}_1$ is not SJ-2Absorbiang subms.

Remark 15. Let $\mathcal{M}$ be an Ғ-module & K, L are Ғ-SJ-2Absorbiang subms of $\mathcal{M}$. $K \cap L$ need not to be Ғ-SJ-2Absorbiang subms of $\mathcal{M}$.

Proof.

If $\mathscr{M}$: Ⱳ⟶[0,1], $\mathscr{M}(h)$=1, ∀ $h \in \mathscr{M}$,

Let $K: Z \longrightarrow [0,1]$ where $K(h)=\begin{cases}1 & if\ h \in 6Z \\ k & if\ h \notin 6Z\end{cases}$, 0< $h < 1$ like that

$L: Z \longrightarrow [0,1]$ where $L(h)=\begin{cases}1 & if\ h \in 5Z \\ v & if\ h \notin 5Z\end{cases}$, 0< $v < 1$

suppose K_1=6Z, L_1 =5Z where K_1 & $L_1$are SJ-2Absorbiang subms of $\mathcal{M}$ & $\mathcal{M}$=Z. We know $Soc(Z) = 0$ & $J(Z) = 0$ then

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$Soc(Z) \cap J(Z) = 0$. But $K_1 \cap L_1 = 6Z \cap 5Z = 30Z$ is not SJ-2Absorbiang subms. Since $2.3.5 = 30 \in 30Z$, but $2.5 \notin 30Z + (Soc(Z) \cap J(Z))$ & $3.5 \notin 30Z + (Soc(Z) \cap J(Z))$ & $2.3 \notin 30Z + (Soc(Z) \cap J(Z))$.

Lemma 16. If $f: \mathcal{M} \to \mathcal{M}'$ be isomorphism mapping from an R-module $\mathcal{M}$ in to R-module $\mathcal{M}'$. If V & V' are Ƒ-moduls of $\mathcal{M}$ & $\mathcal{M}'$ respectiavely. Then

$f(\text{Ƒ} - SJ(\mathcal{M}) - 2\text{Absorbiang}) \subseteq (\text{Ƒ} - SJ(\mathcal{M}') - 2\text{Absorbiang})$.

Proposition 17. If $f: \mathcal{M} \to \mathcal{M}'$ be an Ƒ-isomorphism from Ƒ-module $\mathcal{M}$ in to Ƒ-module $\mathcal{M}'$, with V is an Ƒ-SJ-2Absorbiang subm of $\mathcal{M}$, with $ker\, f \subseteq V$ & $ker\, f$ is a small subm. Then $f(V)$ is an Ƒ-SJ-2Absorbiang subm of $\mathcal{M}'$.

Proof.

$f(V)$ ↳ of $\mathcal{M}'$. If the previous statement is incorrect, that is $f(V) = \mathcal{M}'$. Let $a_t \subseteq \mathcal{M}$, then $f(a_t) \subseteq \mathcal{M}' = f(V)$, so $\exists\ b_q \subseteq V$ where $t, q \in [0,1]$ such that $f(a_t) = f(b_q)$, $\Longrightarrow f(a_t) - f(b_q) = 0_1 \Longrightarrow f(a_t - b_q) = 0_1 \Longrightarrow a_t - b_q \subseteq Ker\, f \subseteq V$ it follows that $a_t \subseteq V$. Thus $V = \mathcal{M}$ contradiction. Now assuume that $a_h b_q c_t \subseteq f(V)$, where $a_h b_q$ are Ƒ-singletons of R & c_t is an Ƒ-singleton of $\mathcal{M}'$, but f is an Ƒ-isomorphism then f is onto $\Longrightarrow f(d_t) = c_t$ for some $d_t \subseteq V$, that is $a_h b_q c_t = a_h b_q f(d_t) = f(a_h b_q d_t) \subseteq f(V) \Longrightarrow f(a_h b_q d_t) = f(wz)$ for some $wz \subseteq V$. Thus $f(a_h b_q d_t) - f(wz) = 0_1$, then $f(a_h b_q d_t - wz) = 0_1$ that means $a_h b_q d_t - wz \subseteq ker\, f \subseteq V$, so $a_h b_q d_t \subseteq V$ but V is V is an Ƒ-SJ-2Absorbiang, thus $a_h b_q \subseteq [V +$ Ƒ-$(Soc(\mathcal{M}) \cap J(\mathcal{M})) :_R \mathcal{M}]$ or $b_q c_t \subseteq V +$ Ƒ-$(Soc(\mathcal{M}) \cap J(\mathcal{M}))$ or $a_h c_t \subseteq V +$ Ƒ-$(Soc(\mathcal{M}) \cap J(\mathcal{M}))$, $\forall\ h, q, t \in [0,1]$. Now, we get $f(a_h b_q \mathcal{M}) = a_h b_q \mathcal{M}' \subseteq f(V +$ Ƒ-$(Soc(\mathcal{M}) \cap J(\mathcal{M})))$ or $f(b_q d_t) = b_q c_t \subseteq f(V +$ Ƒ-$(Soc(\mathcal{M}) \cap J(\mathcal{M})))$ or $f(a_h d_t) = a_h c_t \subseteq f(V +$ Ƒ-$(Soc(\mathcal{M}) \cap J(\mathcal{M})))$. By above lemma $a_h b_q \mathcal{M}' \subseteq f(V) + f($Ƒ-$(Soc(\mathcal{M}) \cap J(\mathcal{M}))) \subseteq f(V) +$ Ƒ-$(Soc(\mathcal{M}') \cap J(\mathcal{M}'))$ or $b_q c_t \subseteq f(V) + f($Ƒ-$(Soc(\mathcal{M}) \cap J(\mathcal{M}))) \subseteq f(V) +$ Ƒ-$(Soc(\mathcal{M}') \cap J(\mathcal{M}'))$ or $a_h c_t \subseteq f(V) + f($Ƒ-$(Soc(\mathcal{M}) \cap J(\mathcal{M}))) \subseteq f(V) +$ Ƒ-$(Soc(\mathcal{M}') \cap J(\mathcal{M}'))$.

Thus $f(V)$ is an Ƒ-SJ-2Absorbiang subm of $\mathcal{M}'$.

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CONCLUSIONS

In this work, we presented a new type of fuzzy sub modules using the properties of addition and intersection between them and some properties of the new fuzzy submodule.

Conflict of interests.

There are no conflicts to declare.

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الخلاصة

تمت دراسة مفهوم الوحدات الفرعية الضبابية – سوكل – جاكوبسون – ثنائية الامتصاص وبعض العلاقات الضبابية وفقاً للمفهوم أعلاه. حيث تم تسمية التعريف الجديد بـ Fuzzy Socle–Jacobson–Two–Absortiang. استخدمنا بعض التقنيات الجبرية لتحديد العلاقة بين الامتصاص الثنائي الغامض وسوكل–جاكوبسون–الامتصاصين الغامضين. بالإضافة إلى دراسة الوظائف وبعض المفاهيم الأخرى ذات الصلة. يمكن الاستفادة من تداعيات دراستنا لتحديد الأفكار الجديدة التي تعتمد على مفاهيم سوكل–جاكوبسون–تو–أبسوربنج الضبابية. بالاضافة الى ذلك استخدمنا بعض من اساسيات الموديولات والموديولات الجزئية لاثبات ان عملية التقاطع في المفهوم الجديد ليس بالضرورة ان تكون متحققة , ودرسنا الدوال في الموديولات وعلاقتها مع نفس المفهوم وحققنا بعض من خواص تلك الدوال. كما استخدم في هذا البحث خواص الموديول التوزيعي لتحقيق بعض المبرهنات. وفي هذا البحث ايضا تم اثبات ان عملية الجمع ليس بالضرورة ان تتحقق بين موديوليين جزئيين تحمل خواص المفهوم المدروس.

النتائج:

استخدام خصائص سابقة لانتاج موديول جزئي ضبابي جديد من تعريفة.

الاستنتاجات:

في هذا العمل قدمنا نوع جديد من الموديولات الجزئية الضبابية بإستخدام خاصيتي الجمع والتقاطع فيما بينها وبعض الخواص للموديول الجزئي الضبابي الجديد.

الكلمات المفتاحية: المقاسات الجزئية المستحوذة على–2; المقاسات الجزئية سوكل–جاكوبسون الضبابية المستحوذة على–2; المقاسات الضبابية; المقاسات الجزئية الضبابية; المقاسات الجزئية الضبابية التوزيعية.