



# Comparing the Useful Life of Two Types of Energy Devices using Mathematical Time Equations

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**Accepted: 24/11/2024**

**Published: 31/12/2024**

## ABSTRACT

### Background:

Solar energy devices, after a time has passed since their use, become older, their exposure to failure increases, their maintenance costs increase, and their production decreases. Thus, one solution is to replace them with a new device; this is referred to as preventive device replacement.

### Materials and Methods:

When a solar panel fails, its life is over, and it is replaced. Alternatively, when it reaches the age  $T_1$ , a preventative solar panel replacement is implemented.

### Results:

This work aims to identify the optimal time for the preventive replacement of solar energy devices, as the continued operation of an old device increases costs and necessitates a replacement that reduces maintenance costs and maintains the device's production continuity. The current research identifies the optimal time for preventive replacement of solar energy devices synthesized from  $n$  similar tubes, as their failure and replacement costs increase as they age. As for devices consisting of large solar panel, the cost of panel failure is expensive and replacing them is expensive.

### Conclusions:

The researcher studied the best time to replace panel with new panel, and that is before the panels fail. At the end of the paper, we identified some industrial and economic decisions that affect the life of replacing these two types of devices.

**Keywords:** Preventive replacement, Tubes, Solar energy devices, Solar panel, Weibull distribution.



## INTRODUCTION

One of the aims of manufacturing companies is to manufacture devices that can live as long as possible at the lowest costs. Working on plans to protect and maintain a device that is profitable for companies [1]. When the device gets tired and ages, its maintenance costs increase and become more prone to failure [2]. When an appliance gets bigger, the maintenance costs can grow, and the maintenance total costs can reach more than the cost of installing a new device [3]. So a preventive replacement policy (i.e. replacing the device with a new device) is a solution to get rid of the high maintenance costs at the end of the life of the device [4]. It remains for companies to determine the best time to implement preventive replacement of devices, by setting a time in which costs are minimal. Therefore, this article identifies the optimal time for a preventive device replacement [5–7].

Over time, a device's tube count increases, raising the cost of replacement [8]. The best time to replace the preventive panels was investigated because the device, which is made up of large solar panels, can be used before being remanufactured [9]. After that, two studies of the two devices were compared using the mathematical theory of replacement, from which some industrial and economic decisions were taken [10].

The research attempted to find the ideal period for preventative device replacement. The number of tubes in a device rises with time, increasing the cost of replacement. The device, consisting of large solar panels, can be used before re-manufacturing, so the optimal time for preventive panel replacement was studied. Then a comparison was made between two studies of the two devices using theory of replacement in math and some economic and industrial decisions were extracted.

## Theoretical Basis of Representative Curve of Failure of Devices

There are random operations that should be investigated in math in renewal theory during investigating a damaged model for replacement before failure [11], but the system that was shocked has a fixed time for a preventive replacement [12]. There is a policy of preventive replacement of traumatized systems [13]. The significant factor represented by weather enhanced the impact of extreme weather and climate change on the resilience of power systems [14]. The geographical areas impact the optimal time to replace the same device in different geographical areas and the practical life of a power device composed of tubes [15].

Solar energy devices age, leading to increased failure rates, decreased production, and higher maintenance costs, necessitating the replacement of old devices as a maintenance solution. This is a preventive replacement that is preferable to be performed before the beginning of the final

stage of the end of the device's life. We drew a mathematical curve for the economic life of replacing solar panels, to determine the best age for a solar energy device as illustrated in [Fig. 1].

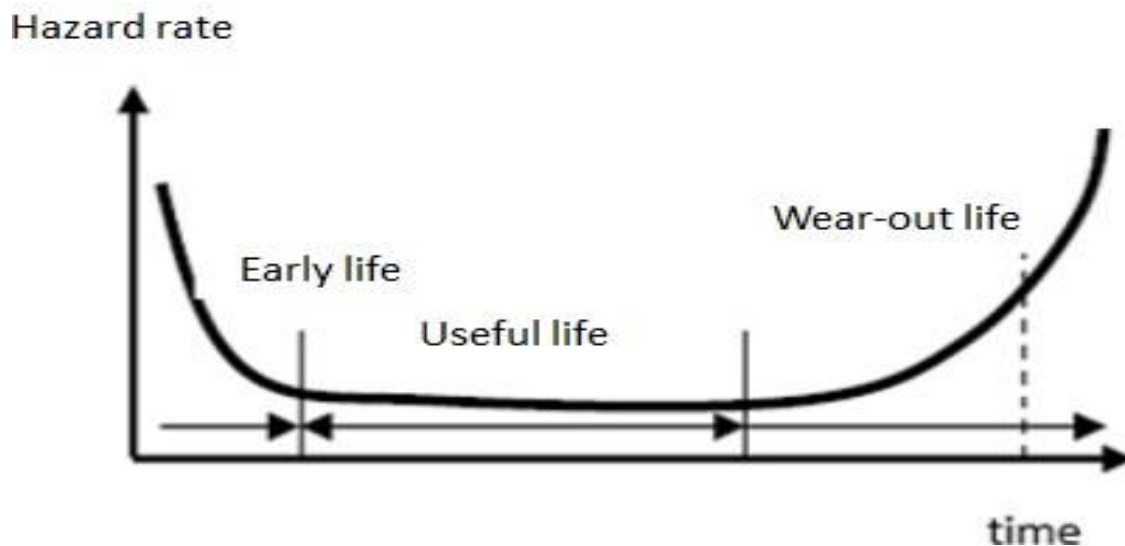


Figure 1: Risk rate of the device over time in the current study.

### Model of solar energy devices consisting of large panels

In this model we study the age of the athlete for each plate, to implement the preventive plate replacement policy, let  $T_1$  be the planned time for preventive plate replacement as seen in [Fig. 2], let the random variable  $X$  be the plate failure time, and it has the distribution  $F(t) = P(X \leq t)$ .



Figure 2: Solar energy devices consisting of large panels

There are two probabilities for replacing panel with new panel, the first probability is that the panel will fail before reaching the time of  $T_1$  and the probability of failure of panel,

$$F(T_1) = P(X \leq T_1).$$

The second probability is that we will reach the time  $T_1$  for preventive replacement for panel before the panel fail and their probability,

$$P(T_1 \leq X) = 1 - P(X \leq T_1) = 1 - F(T_1)$$

### Model of solar energy device consisting number (n) of tubes



Figure 3: solar energy device consisting number (n) of tubes in the current study.

as seen in [Fig. 3], The device is susceptible to tube failure as it ages. Suppose  $T_2$  is the planned time for preventive replacement for device. Let the random variable  $Y$  is the time of the tube failure, has distribution

$$F(t) = P(Y \leq t).$$

The tube's potential failure before the time  $T_2$  for preventive device replacement is

$$F(T_2) = P(Y \leq T_2).$$



## RESULTS AND DISCUSSION

### The costs associated with the solar panel device model:

The costs of replacing panel after their sudden failure is high. As for the costs of replacing them before their failure (i.e. preventive replacement), this reduces the costs for companies and factories because they benefit from the panel used in recycling.  $C_{FP}$ : is the cost of replace panel after they fail,  $C_{PP}$ : Is the cost of preventive replacement of panel. Its logic  $C_{FP} > C_{PP}$ .

### The costs associated with the device model consisting of number (n) oftubes:

As the device ages, tube failure increases, necessitating preventive replacement. The device's life cycle costs include:

$C_{FT}$ : is the cost is to replace the tube after its failure

$C_{FD}$ : is the cost of preventive replacement of device.

### Expected cost of life cycle

### The costs we are expected to place on solar panel during its life cycle are:

The life of the solar panel ends when it fails, so we replace it or when it reaches the age  $T_1$  we implement the preventive replacement for solar panel,

$$\left\{ \begin{array}{l} C_{FP} \text{ With probability } P(X \leq T_1) \\ C_{PP} \text{ With probability } P(T_1 \leq X) \end{array} \right.$$

$$C_{PP} \text{ With probability } P(T_1 \leq X)$$

The expected cost of the solar panel life cycle are equal with

$$C_{FP} P(X \leq T_1) + C_{PP} P(T_1 \leq X)$$

$$= C_{FP} F(T_1) + C_{PP} (1 - F(T_1))$$

$$= C_{PP} + (C_{FP} - C_{PP})F(T_1)$$



### The costs that we put on the device consisting of the number of $n$ tubes during its life cycle:

The device life starts from 0 and ends when it reaches the time  $T_2$  to carry out the preventive replacement of the device. Each tube fails before reaching age  $T_2$ , we replace it, we add the cost of preventive replacement for device in end its life cycle.

The expected cost for life cycle of device consisting  $n$  of tubes is equal to:

$$\begin{aligned} C_{PD} + nC_{FT} P(Y \leq T_2) \\ = C_{PD} + nC_{FT} G(T_2) \end{aligned}$$

### The mean life cycle

The mean solar panel life cycle

The mean solar panel life cycle is equal to

$$\int_0^{T_1} 1 - F(t) dt \quad (1)$$

See (1) in appendix.

The life cycle of a device consisting of  $n$  tubes is denoted as  $T_2$

### Expected cost per unit time

The expected cost per unit time represents the expected cost movement over time.

The expected cost per unit time for solar panel  $C(T_1)$  is equal to

### expected cost of cycle of solar panel

mean time of cycle of solar panel

$$= \frac{C_{PP} + (C_{FP} - C_{PP}) F(T_1)}{\int_0^{T_1} 1 - F(t) dt}$$





Expected cost per unit time for device consisting **cycle of solar panel**  $n$  tubes ( $T_2$ ) is equal to

$$= \frac{\text{expected cost of cycle of device}}{\text{time of cycle of device}} = \frac{C_{PD} + nC_{FTG}(T_2)}{T_2}$$

### Optimal time

The optimal time for a solar panel preventive replacement is determined by calculating  $C(T_1)$  and comparing it to zero.

$$\frac{dC(T_1)}{dT_1} = 0 \quad \text{Note: } f(t) = \frac{dF(t)}{dt}$$

$$\frac{f(T_1)}{1-F(T_1)} \int_0^{T_1} 1-F(t) dt - (T_1) = \frac{C_{PP}}{(C_{FP}-C_{PP})} \quad (2)$$



See (2) in appendix.

The optimal time for a preventive replacement of a device with  $n$  tubes is determined by calculating  $C(T_2)$  and comparing it with zero.

$$\frac{C(T_2)}{dT_2} = 0 \quad \text{Note: } g(t) = \frac{dG(t)}{dt}$$

$$T_2 = \frac{nC_{PD} + G(T_2)}{g(T_2)} \quad (3)$$

See (3) in appendix.

To get the best age  $T^*_1$  for replacing solar panels with new panels, we use Equation (2).

To get the best life  $T^*$  for replacing the device consisting of  $n$  tubes with a new device, we use Equation (3).

### Numerical example

In this example, we took two solar energy devices, solar panels device and solar tubes device. We calculated the optimal time to replace the solar panels with new ones using the formula (2). The optimal time to replace a solar tube device was determined using a specific formula (3).

The Weibull distribution, with scale parameter  $\lambda$  and shape parameter  $k$ , describes the solar panel failure time distribution.

$$F(t) = 1 - e^{-\left(\frac{t}{\lambda}\right)^k}$$

The distribution of tube failure times follows a Weibull distribution characterized by a scale parameter  $\theta$  and a shape parameter  $\beta$ .

$$G(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^\beta}$$

We have a device consisting of 40 tubes and each tube costs 25\$ to replace it after failure and the cost for preventive replacement of device is 500\$ with  $\beta=1.7$ . We calculated the best time to replace the device with a new device using Equation (3).





| $\theta = 8 \text{ years}$ | $\theta = 7 \text{ years}$ | $\theta = 6 \text{ years}$ | $\theta = 5 \text{ years}$ |
|----------------------------|----------------------------|----------------------------|----------------------------|
| $T^* = 7.58 \text{ years}$ | $T^* = 6.63 \text{ years}$ | $T^* = 5.69 \text{ years}$ | $T^* = 4.7 \text{ years}$  |
| 2                          | 2                          | 2                          | 2                          |

Table 1: the optimal age of preventive replacement for device after change  $\theta$ .

We note when  $\theta$  increase, the lifespan of replacing the device with a new device increase.

In Table 2, we took a solar panel, the cost of a preventive replacement 70\$, as for the cost of a forced replacement after failure 200\$, with  $k = 1.7$  we changed in  $\lambda$ .

| $\lambda = 8 \text{ years}$ | $\lambda = 7 \text{ years}$ | $\lambda = 6 \text{ years}$ | $\lambda = 5 \text{ years}$ |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $T^* = 7.2 \text{ years}$   | $T^* = 6.7 \text{ years}$   | $T^* = 5.2 \text{ years}$   | $T^* = 4.9 \text{ years}$   |
| 1                           | 1                           | 1                           | 1                           |

Table 2: the optimal age of preventive replacement for panel after change  $\lambda$ .

We note when  $\lambda$  increase, the lifespan of replacing panel with a new panel is increased. The higher the failure time of tubes, the longer the lifespan of replacing them with a new device, and the longer the lifespan of solar panels, both of which are influenced by their industrial specifications.

### Economic decisions

In this paragraph, we changed costs to see how they affected the time for a preventive replacement.

In Table 3 and 4 we took a solar panel with  $\lambda = 6$  and  $k = 1.7$ .

In Table 1,  $CFP = 200\$$ , we changed the cost of the preventive replacement of panel

| $C_{PP} = 50\$$            | $C_{PP} = 60\$$            | $C_{PP} = 70\$$            | $C_{PP} = 80\$$            |
|----------------------------|----------------------------|----------------------------|----------------------------|
| $T^* = 4.11 \text{ years}$ | $T^* = 4.55 \text{ years}$ | $T^* = 5.76 \text{ years}$ | $T^* = 6.52 \text{ years}$ |
| 1                          | 1                          | 1                          | 1                          |

Table 3: the optimal age of preventive replacement for panel after change  $C_{PP}$ .

The optimal period for preventative solar panel replacement increases with the cost of the replacement.



In Table 4 the cost of a preventive replacement for panel is  $CPP=70\$$ . Then we changed the cost of replacing the panel after it failed.

| $C_{FP} = 150\$$           | $C_{FP} = 180\$$           | $C_{FP} = 200\$$           | $C_{FP} = 250\$$           |
|----------------------------|----------------------------|----------------------------|----------------------------|
| $T^* = 8.18 \text{ years}$ | $T^* = 6.37 \text{ years}$ | $T^* = 5.76 \text{ years}$ | $T^* = 4.47 \text{ years}$ |
| 1                          | 1                          | 1                          | 1                          |

Table 4: the optimal age of preventive replacement for panel after change  $CFP$ .

The ideal period for preventative solar panel replacement decreases with the cost of replacing the solar panel after failure. We used a device with 40 tubes with  $\theta = 5$  and  $\beta = 1.7$  in tables 5 and 6. In Table 5, the cost of replacing the tube is changed once it failed with  $C_{PD}=500\$$ .

| $C_{FT} = 25\$$           | $C_{FT} = 35\$$           | $C_{FT} = 45\$$           | $C_{FT} = 50\$$            |
|---------------------------|---------------------------|---------------------------|----------------------------|
| $T^* = 4.7 \text{ years}$ | $T^* = 3.7 \text{ years}$ | $T^* = 3.1 \text{ years}$ | $T^* = 2.88 \text{ years}$ |
| 2                         | 2                         | 2                         | 2                          |

Table 5: the optimal age of preventive replacement for device after change  $C_{FT}$

Thus, the optimal time for preventive device replacement decreases as the cost of replacing the tube increases after it fails.

In Table 6 We changed the cost of preventive replacement for device with

$C_{FT}=25\$$ .

| $C_{PD} = 400\$$        | $C_{PD} = 425\$$          | $C_{PD} = 450\$$           | $C_{PD} = 500\$$          |
|-------------------------|---------------------------|----------------------------|---------------------------|
| $T^* = 4 \text{ years}$ | $T^* = 4.2 \text{ years}$ | $T^* = 4.38 \text{ years}$ | $T^* = 4.7 \text{ years}$ |
| 2                       | 2                         | 2                          | 2                         |

Table 6: the optimal age of preventive replacement for device after change  $C_{PD}$

The optimal time for device preventive replacement increases as the cost of such replacement increases.

How to calculate (1)







### Conflict of interests:

There are non-conflicts of interest.

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## الخلاصة

### مقدمة:

أجهزة الطاقة الشمسية، بعد مرور وقت على استخدامها، تصبح أكبر سناً، ويزداد تعرضها للفشل، وتزداد تكاليف صيانتها، وينخفض إنتاجها. وبالتالي، فإن أحد الحلول هو استبدالها بجهاز جديد؛ يشار إلى هذا باسم استبدال الجهاز الوقائي.

### طرق العمل:

عندما تفشل لوحة شمسية، تنتهي حياتها ويتم استبدالها. بدلاً من ذلك، عندما تصل إلى سن  $T1$ ، يتم تنفيذ استبدال الألواح الشمسية الوقائية..

### النتائج:

يهدف هذا العمل إلى تحديد الوقت الأمثل لاستبدال الوقائي لأجهزة الطاقة الشمسية، حيث أن استمرار تشغيل جهاز قديم يزيد من التكاليف ويتطلب استبدالاً يقلل من تكاليف الصيانة ويحافظ على استمرارية إنتاج الجهاز. يحدد البحث الحالي الوقت الأمثل للاستبدال الوقائي لأجهزة الطاقة الشمسية المركبة من أنابيب مماثلة، مع زيادة تكاليف فشلها واستبدالها مع تقدم العمر. أما بالنسبة للأجهزة التي تتكون من لوحة شمسية كبيرة، فإن تكلفة فشل اللوحة باهظة الثمن واستبدالها باهظ الثمن.

### الاستنتاجات:

درس الباحث أفضل وقت لاستبدال اللوحة بلوحة جديدة، وذلك قبل فشل اللوحات. في نهاية الورقة، حددنا بعض القرارات الصناعية والاقتصادية التي تؤثر على عمر استبدال هذين النوعين من الأجهزة.

**الكلمات المفتاحية:** الاستبدال الوقائي، الأنابيب، أجهزة الطاقة الشمسية، الألواح الشمسية، توزيع ويبول.