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# On Fuzzy S- Metric Space

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# حول الفضاء S الضبابي المتري

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## ABSTRACT

**Background:**

Fixed-point theory is an important area of study in pure and applied mathematics, and it is a flourishing area of research in the paperwork, the concept of fuzzy S-metric space is introduced, and The fixed point theorem's existence and uniqueness over fuzzy S-metric space are examined. We will go over The fixed point theorem's existence and uniqueness for k contractive mapping in fuzzy S-metric space. These theorems will be seen in fuzzy S-metric space. fixed point theory The transcendence is the main cause of the fixed point theory's recent resurgence. The significance of this theory and the questions it allows us to address regarding the dimensions of space. Proof of the existence of solutions to the physical equations is possible thanks to the notion of the fixed point.

**Materials and Methods:**

We will discuss the existence and uniqueness of the fixed point theorem by using k contractive mapping in fuzzy S-metric space.

**Results:**

This study proved the existence and uniqueness of new fixed point theorem in fuzzy S-metric space using k contractive mapping.

**Conclusions:**

According to this study, we were able to prove the existence and uniqueness of fixed points in s-metric space by using k-fuzzy contractive mapping.

**Key words:** Fuzzy S- metric space; k-fuzzy contractive mapping; Theorem of fixed point.

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## INTRODUCTION

Fixed point theory presented [1] According to Brower's fixed point theory, a conversion has at least one point for all continuous application that transforms the ball into the ball itself. Subsequently, numerous scholars expanded and generalized these well-known findings, enriching the idea of fixed points. proposed the idea of *S*-metric space and proved the fixed point type theorem for the k-contraction condition on S-metric space [2]. introduced fuzzy set is significant to analysis and topology. Numerous writers have since studied fuzzy sets and applications, first published in 1965 [3]. in particular introduced a novel idea of fuzzy metric space. Using continuous t-norm [4] reinterpreted the fuzzy metric space notion. Consequently, several fixed point theorems for diverse types of maps [5]. defined D-metric spaces as well as established numerous new fixed point theorems in them [6]. Dhage theory advanced significantly with the recent presentation of a new notion of G-metric space [7]. The main aim of this work is to study certain types of k contractive mapping in fuzzy S-metric space and also demonstrated the uniqueness and existence of fixed points in fuzzy s-metric space. We relied on the definition of t-norm)[8] and definition of the fuzzy metric space[4] , we define fuzzy S-metric space. A point x of a nonempty set X is said to be fixed point of T (self-function), if T(x) = x [9].

## MATERIAL AND METHODS

The study explains proved the existence and uniqueness of fixed point theorems where the fuzzy s- metric space is defined and depending on some concepts (k-fuzzy contractive mapping), the fixed point theorem has proven in this space.

## RESULTS AND DISCUSSION

Initially, we present the following idea of (S- metric space) and a k-fuzzy contractive.

**Definition(2.1)**

When X is a non-empty arbitrary set and $*$ is a continuous t-norm, and S is a fuzzy set on $X^3 \times (0,\infty) \to [0,\infty)$ that fulfills the requirements listed below, then $\forall\ u,v,r,a \in X$, and $t,s > 0$ such a triple $(X,S,*)$ is a fuzzy S-metric space.

1. $S(u,v,r,t) > 0$ and $S(u,u,v,t) \leq S(u,v,r,t)$ with $r \neq v$
2. $S(u,v,r,t) = 1$ iff $u = v = r$.
3. $S(u,v,r,t) = S(v,u,r,t) = S(v,r,u,t) = \cdots$
4. $S(u,v,a,t) * S(v,a,r,s) \leq S(u,v,r,t+s)$.
5. $\lim_{t\to\infty} S(u,v,r,t) = 1\ \forall\ u,v,r \in X$.
6. $S(u,v,r,.) : (0,\infty) \to [0,1]$ is non-decreasing and continuous.

It is said that a fuzzy S-metric space is symmetric if $S(u,u,v,t) = S(u,v,v,t)$.

**Definition(2.2)**

Let (X,S,*) be the fuzzy S-metric space. and $t > 0$ be a real number. we define an open ball and closed ball with center x and radius $\epsilon$as follows

Open ball

$B(x,\epsilon,t) = \{y \in X; S(x,y,y,t) > 1-\epsilon\}$

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Closed ball

$B[x,\epsilon,t] = \{y \in X; S(x,y,y,t) \geq 1-\epsilon\}$

**Definition(2.3)**

Let (X,S,*)be the fuzzy S-metric space and a sequence $\{x_n\}$ in X then

1. $\{x_n\}$ is claimed as converging to x if
$\forall \epsilon > 0, \exists k \in N$ such that $S(x_n,x_n,x,t) > 1-\epsilon \quad n \geq k$.
2. $\{x_n\}$ is claimed to be Cauchy if
$\forall \epsilon > 0, \exists k \in N$ such that $(x_n,x_m,x,t) > 1-\epsilon \quad n,m \geq k$.
3. If every Cauchy sequence in a fuzzy S-metric space is convergent, then it is considered to be complete fuzzy S-metric space.

**Definition (2.4)**

Assume that $(X,S,*)$ is a fuzzy S- metric space with $\epsilon \in (0,1)$. Accordingly, $B[x,\epsilon,t]$ is a closed ball with radius $\epsilon$ centered at x if each sequence $x_n$ in $B[x,\epsilon,t]$converges to a then $a \in B[x,\epsilon,t]$.

**Definition(2.5)**

Let (X,S,*)be the fuzzy S-metric space. The mapping f is called t-uniformly continuous if $\forall\epsilon$ with $0<\epsilon<1, \exists 0<r<1$ such that

$S(x,x,y,t) \geq 1-r$ implies $S(fx,fx,fy,t) \geq 1-\epsilon \ \forall x,y \in X, \ t>0$.

**Proposition(2.6)**

Let's say that the fuzzy S-metric space is (X,S,*) and $f: X \to X$ be a t-uniformly continuous on X, If $x_n \to x$ as $n \to \infty$ in $(X,S,*)$ then $fx_n \to fx$ as $n \to \infty$ in$(X,S,*)$

**Proof**

Let $x_n \to x$ as $n \to \infty$ in $(X,S,*)$. then by definition of convergent sequence $\forall\epsilon \in (0,1)$ and $t>0$ , $\exists n_o \in N$ such that

$S(x_n,x_n,x,t) \geq 1-\epsilon \qquad \forall n > n_o$

Since f is t-uniformly continuous on X then

$\exists r \in (0,1)$ (depends on $\epsilon$ )

$S(fx_n,fx_n,fx,t) \geq 1-r \qquad \forall n > n_o, \forall t > 0$

then $fx_n \to fx$ as $n \to \infty$ in $(X,S,*)$ .

**Definition(2.7)**

Let (X,S,*)be the fuzzy S-metric space and the mapping $f: X \to X$ is call k-fuzzy contractive mapping if $\exists k \in (0,1)$ such that

$S(h,j,l,t) \leq k\, S(f(h),f(j),f(l),t) \quad \forall h \neq j \neq l \in X$ and $t>0$, k is call contractive constant of f.

**Theorem(2.8)**

In fuzzy S-metric space $(X,S,*)$, every convergent sequence is Cauchy.

**Proof**

Consider a convergent sequence $\{x_n\}$ in the fuzzy S- metric space (X,S,*).

$\exists x \in X$ such that $x_n \to x$ . let $\alpha \in (0,1)$ , $t > 0$ .

Since $x_n \to x$ ,$\exists\, n \in Z^+ \ni S(x_n,x_n,x,t) \geq 1-\alpha \qquad \forall n \geq n_0$,

And $S(x_m,x_m,x,t) \geq 1-\alpha \qquad \forall m \geq n_0$,

By definition (2.1) property(4),Therefore

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$S(x_n,x_n,x_m,t) \geq S\left(x_n,x_n,x,\frac{t}{2}\right) * S\left(x,x,x_m,\frac{t}{2}\right)$

$> 1-\alpha \ * 1-\alpha$

$= \min\{1-\alpha \ ,1-\alpha\}$

$= 1-\alpha$

Then $S(x_n,x_n,x_m,t) \geq 1-\alpha$

Then $\{x_n\}$ is Cauchy .

**Theorem(2.9)**

The limit of convergent sequence in fuzzy S- metric space is unique

**Proof**

Let $\{x_n\}$ be a convergent sequence to two point such that $x$ *and* $y$ .

If $x \neq y$ .

$\lim_{n\to\infty} S(x_n,x_n,x,t) = 1$ (1)

$\lim_{n\to\infty} S(x_n,x_n,y,t) = 1$ (2)

Then By definition of convergent sequence, $\exists\, n_1,n_2 \in Z^+$ such that

$S(x_n,x_n,x,t) > 1-\epsilon$ $\forall n \geq n_1$, and $\epsilon \in (0,1]$
$S(x_n,x_n,y,t) > 1-\epsilon$ $\forall n \geq n_2$, and $\epsilon \in (0,1]$

Choose $n \geq n_0$ , $n_0 = min\{n_1 , n_2\}$

$S(x,x,y,t) \geq S\left(x,x,x_n,\frac{t}{2}\right) * S\left(x_n,x_n,y,\frac{t}{2}\right)$

$> 1-\epsilon * 1-\epsilon$

$= \min\{1-\epsilon, 1-\epsilon\}$

$= 1-\epsilon$

$S(x,x,y,t) \geq 1-\epsilon$

That implies

$\lim_{n\to\infty} S(x,x,y,t) = 1$

$S(x,x,y,t) = 1$ by definition (2.1),then

$x = y$

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**Proposition (2.10)**

Let (X,S,*) be the fuzzy S-metric space. If $\exists k \in (0,1)$ such that $S(u,u,w,kt) \geq S(u,u,w,t)$ $\forall u,w \in X$ then $u = w$.

**Proof**

Since $S(u,u,w,kt) \geq S(u,u,w,t)$

Then

$$S(u,u,w,t) \geq S(u,u,w,\tfrac{t}{k}) \geq S\left(u,u,w,\tfrac{t}{k^2}\right) \geq S\left(u,u,w,\tfrac{t}{k^3}\right) \geq \dots \geq S(u,u,w,\frac{t}{k^n})$$

Since $k \in (0,1)$, $n \in N$. Then as $n \to \infty$, and by definition (2.1)property(5) we have

$S(u,u,w,t) = 1$ as $n \to \infty$.

Then $u = w$.

**Theorem (2.11)**

Let (X,S,*)be the fuzzy S-metric space and Let $f: X \to X$ represent the k- contractive mapping providing a contractive constant k on the closed ball $B[x,\epsilon,t]$. Additionally, suppose that

$k\, S(x,x,fx,t) > (1-\epsilon)$ ……..(1)

Then $f$ has a unique fixed point in $B[x,\epsilon,t]$

**Proof**

Let $x_0 \in X$ and $x_n = f^n x_0$ for $n = 1,2,\dots$ such that

$x_1 = f x_0$

$x_2 = f^2 x_0 = f x_1$

:

:

$x_{n+1} = f x_n \quad \forall n$

From (1) , we have

$k\, S(x_0,x_0,fx_0,t) > (1-\epsilon)$

$S(x_0,x_0,fx_0,t) > \frac{(1-\epsilon)}{k}$

Since k ∈ (0,1)

$S(x_0,x_0,fx_0,t) > \frac{(1-\epsilon)}{k} > (1-\epsilon)$

$S(x_0,x_0,fx_0,t) > (1-\epsilon)$

$S(x_0,x_0,x_1,t) > (1-\epsilon)$

Then

We get $x_1 \in B[x,\epsilon,t]$

Assume that $\tilde{x}_{e1}\tilde{x}_{e2},\dots,\tilde{x}_{en-1} \in B[x,\epsilon,t]$ we show that $\tilde{x}_{en} \in B[x,\epsilon,t]$

$kS(x_1,x_1,fx_1,t) = kS(fx_0,fx_0\, ,fx_1\, ,t) \geq S(x_0,x_0,x_1,t)$

$S(x_1,x_1,x_2,t) \geq \frac{1}{k} S(x_0,x_0,x_1,t) > \frac{(1-\epsilon)}{k}$

Since $k \in (0,1)$, then

$S(x_1,x_1,x_2,t) > (1-\epsilon)$

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:
:
$S(x_{n-1}, x_{n-1}, x_n, t) > (1-\epsilon)$
Thus we see that

$$S(x_0, x_0, x_n, t) \geq S\left(x_0, x_0, x_1, \frac{t}{k}\right) * S\left(x_1, x_1, x_2, \frac{t}{k}\right) * \dots * S\left(x_{n-1}, x_{n-1}, x_n, \frac{t}{k}\right)$$
$$> (1-\epsilon) * (1-\epsilon) * \dots * (1-\epsilon)$$
$$= (1-\epsilon)$$

i.$e$
$S(x_0, x_0, x_n, t) > (1-\epsilon)$
Then , $x_n \in B[x, \epsilon, t]$
according to definition (3.4) , we have
$\exists\, x_1 \in B[x, \epsilon, t] \quad s.t \quad x_n \to x_1$ as $n \to \infty$ in $B[x, \epsilon, t]$

$$S(fx_n, fx_n, fx_1, t) \geq \frac{1}{k} S(x_n, x_n, x_1, t)$$
$$\lim_{n\to\infty} S(fx_n, fx_n, fx_1, t) \geq \lim_{n\to\infty} \frac{1}{k} S(x_n, x_n, x_1, t) = \frac{1}{k} > 1$$
$$1 \geq \lim_{n\to\infty} S(fx_n, fx_n, fx_1, t) > 1$$
$$S(fx_n, fx_n, fx_1, t) = 1$$
$$\lim_{n\to\infty} fx_n = fx_1 \qquad \forall t > 0$$

Then

$$\lim_{n\to\infty} x_{n+1} = fx_1$$
$$fx_1 = x_1$$

Therefore $x_1$ is fixed point of $(f, \varphi)$ in $B[x, \epsilon, t]$
For uniqueness
Assume $fx_2 = x_2$ for some $x_2$ in $B[x, \epsilon, t]$. Then we have

$$S(x_2, x_2, x_1,, t) = S(fx_2, fx_2, fx_1,, t) \geq \frac{1}{k} S(x_2, x_2, x_1,, t)$$
$$= \frac{1}{k} S(x_2, x_2, x_1,, t)$$
$$\geq \frac{1}{k^2} S(x_2, x_2, x_1,, t)$$
$$\geq \dots \geq \frac{1}{k^n} S(x_2, x_2, x_1,, t)$$

Since k ∈ (0,1), then

$$\frac{1}{k^n} S(x_2, x_2, x_1,, t) \to \infty \; as \; n \to \infty$$
$$1 \geq \lim_{n\to\infty} S(x_2, x_2, x_1,, t) \geq \frac{1}{k^n} S(x_2, x_2, x_1,, t) \geq 1$$
$$S(x_2, x_2, x_1,, t) = 1 \; \forall t > 0$$
$$x_2 = x_1.$$

**Proposition (2.12)**
Let (X,S,*) be the fuzzy S-metric space. Let {n}be a sequence in X such that $S(s_{n+1}, s_{n+1}, s_n, kt) \geq S(s_n, s_n, s_{n-1}, t)$ for $k \in (0,1)$. Then the sequence $\{s_n\}$ is Cauchy .

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**Proof**

Since $S(s_{n+1}, s_{n+1}, s_n, kt) \geq S(s_n, s_n, s_{n-1}, t)$

Then

$$S(s_{n+1}, s_{n+1}, s_n, t) \geq S(s_n, s_n, s_{n-1}, \frac{t}{k})$$
$$\geq S\left(s_{n-1}, s_{n-1}, s_{n-2}, \frac{t}{k^2}\right) \geq S\left(s_{n-2}, s_{n-2}, s_{n-3}, \frac{t}{k^3}\right) \geq$$
$$\dots \geq S(s_1, s_1, s_0, \frac{t}{k^n})$$

Since k ∈ (0,1), For n ∈ N. Then as n → ∞ we have

$S(s_{n+1}, s_{n+1}, s_n, t) = 1$ as n → ∞

Then $\{s_n\}$ is Cauchy sequence

**Proposition (2.13)**

Let (X,S,*) be the fuzzy S-metric space. If $x_n \to x$ $and$ $x_m \to y$ $as$ $n \to \infty$ in $(X, S,*)$ then $S(x_n, x_n, x_m, t) \to S(x, x, y, t)$ $as$ $n \to \infty$ in $(X, S,*)$ and $t > 0$.

**Proof**

Let $x_n \to x$ $and$ $x_m \to y$ $as$ $n \to \infty$ in $(X, S,*)$

i.e

$$\lim_{n\to\infty} S(x_n, x_n, x, t) = 1 \text{ and } \lim_{n\to\infty} S(x_m, x_m, y, t) = 1$$

Now, by definition(2.1)property(4), we have

$$S(x_n, x_n, x_m, t) \geq S(x_n, x_n, x, \frac{t}{2}) * S(x, x, x_m, \frac{t}{2})$$
$$\geq S\left(x_n, x_n, x, \frac{t}{2}\right) * S\left(x, x, y, \frac{t}{4}\right) * S(y, y, x_m, \frac{t}{4})$$

$$\lim_{n\to\infty} S(x_n, x_n, x_m, t) \geq \lim_{n\to\infty} S(x_n, x_n, x, \frac{t}{2}) * \lim_{n\to\infty} S\left(x, x, y, \frac{t}{4}\right)$$
$$* \lim_{n\to\infty} S(y, y, x_m, \frac{t}{4})$$

Since $x_n \to x$ $and$ $x_m \to y$ $as$ $n \to \infty$ in $(X, S,*)$. then

$$\lim_{n\to\infty} S(x_n, x_n, x_m, t) \geq \lim_{n\to\infty} S(x_n, x_n, x, \frac{t}{2}) * \lim_{n\to\infty} S\left(x, x, y, \frac{t}{4}\right)$$
$$* \lim_{n\to\infty} S(y, y, x_m, \frac{t}{4})$$
$$= 1 * \lim_{n\to\infty} S\left(x, x, y, \frac{t}{4}\right) * 1 = S\left(x, x, y, \frac{t}{4}\right)$$

$$\lim_{n\to\infty} S(x_n, x_n, x_m, t) \geq S\left(x, x, y, \frac{t}{4}\right)$$
$$\geq S(x, x, y, t) \qquad (1)$$

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$$S(x,x,y,t) \geq S(x,x,x_n,\frac{t}{2}) * S(x_n,x_n,y,\frac{t}{2})$$

$$\geq S\left(x,x,x_n,\frac{t}{2}\right) * S\left(x_n,x_n,x_m,\frac{t}{4}\right) * S(x_m,x_m,y,\frac{t}{4})$$

$$\lim_{n\to\infty} S(x,x,y,t) \geq \lim_{n\to\infty} S\left(x,x,x_n,\frac{t}{2}\right) * \lim_{n\to\infty} S\left(x_n,x_n,x_m,\frac{t}{4}\right) * \lim_{n\to\infty} S(x_m,x_m,y,\frac{t}{4})$$

Since $x_n \to x$ *and* $x_m \to y$ as $n \to \infty$ in $(X,S,*)$. then

$$\lim_{n\to\infty} S(x,x,y,t) \geq 1 * \lim_{n\to\infty} S\left(x_n,x_n,x_m,\frac{t}{4}\right) * 1 = \lim_{n\to\infty} S\left(x_n,x_n,x_m,\frac{t}{4}\right)$$

$$\lim_{n\to\infty} S(x,x,y,t) \geq \lim_{n\to\infty} S\left(x_n,x_n,x_m,\frac{t}{4}\right)$$

$$\geq \lim_{n\to\infty} S(x_n,x_n,x_m,t) \qquad (2)$$

From(1) and (2) we get, $\forall t > 0$

$S(x_n,x_n,x_m,t) \to S(x,x,y,t)$ *as* $n \to \infty$ in$(X,S,*)$.

## CONCLUSION

We examine The fixed point theorem's existence and uniqueness in this work in order to examine a novel class of fuzzy contractions called k contractions. A family like this enhanced the literature's several classical forms of fuzzy contractions and generalized, expanded, and united a number of findings. We can dive more and extract many implications from our primary findings. Even still, it makes sense to investigate the presence and uniqueness of a fixed point, therefore more study is required in this area.

## Conflict of interests:

There is no conflict of interest

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