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On New Generalizations of Nano-Open Sets in Nano Topological Spaces

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حول التعميمات الجديدة للمجموعات النانوية المفتوحة في الفضاءات الطوبولوجية النانوية

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Abstract

This paper focuses on a novel class of Nano $\mathcal{B}$-open sets are known as Nano $\mathcal{B}_c$-open ,sets which are related to Nano closed sets in Nano topological spaces. In this paper, we find all forms of the family of Nano $\mathcal{B}_c$-open setsin terms of upper and lower Approximations of sets and we can easily find Nano $\mathcal{B}_c$-open sets and they are a gate to more study. We also examine the fundamental properties of Nano $\mathcal{B}_c$-open sets and their relationships with other known classes of Nano -open sets in Nano topological spaces, and introduce related operators such as the Nano $\mathcal{B}_c$-limit point and Nano $\mathcal{B}_c$-boundary point and investigate the properties of the operators. We aim to refine the understanding of Nano topological spaces and their potential applications.

Keywords: Nano topological spaces, Nano $\mathcal{B}_c$-open sets, Nano $\mathcal{B}_c$-limit points, nano $\mathcal{B}_c$-boundary point.

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1 INTRODUCTION

Nano topology is a generalized branch of topology developed to deal with uncertainty and approximations, often using lower and upper approximations of sets. It serves as an extension of classical topology and rough set theory, making it particularly useful for applications involving imprecise or granular data. A key concept in Nano topology is the Nano-open set, which forms the basis for constructing Nano topological space. This area has attracted growing attention due to its flexibility and potential in modeling complex systems. This paper aims to contribute to Nano topology by introducing a new class of Nano –open sets. Offering a broader framework for studying generalized topological structure.

The concept of b-open sets was first introduced by [1], followed by [2] in the proposal of new $\mathcal{B}_c$open collections in topological spaces in 2013. In the same year. By [3] introduced the notion of Nano topology based on lower and upper approximations of subsets in a universe. They further defined Nano -open sets, Nano-closed, Nano-interior, and Nano-closure of a set, and Nano $\mathcal{B}$ -open sets. By[4] also explored the concept of Nano s_c-open sets, enriching the theoretical framework. More recently, [5] applied Nano topology to real- world problem, such as Decision-Making during the COVID-19 pandemic illustrating the practical value of Nano topological models. This paper introduces the concept of Nano $\mathcal{B}_c$-open sets as a strong version of nano $\mathcal{B}$-open sets and investigates their relationship with other established classes of Nano-open sets. It aims to develop a more refined classification under various approximations scenarios, contributing to the structural understanding of Nano topological spaces. The study also presents corresponding Nano $\mathcal{B}_c$-operators such as Nano $\mathcal{B}_c$-limit and Nano $\mathcal{B}_c$-boundary points. The methodology is based on formal definitions, set-theoretic analysis, and logical deductions supported by examples and comparisons to existing Nano topological concepts.

2 PRELIMINARIES

Definition 2.1[6]

Let $\mathcal{V}$ denote a non-empty finite set . Let Ω be an equivalence relation on $\mathcal{V}$ named as the indiscernibility relation. Then $\mathcal{V}$ is divided into disjoint equivalence class are. Elements belonging to the same equivalent class said to be indiscernible from one another. The pair $(\mathcal{V},\Omega)$ is called the approximation space. Let $\mathcal{Y} \subseteq \mathcal{V}$.

- The lower approximation of $\mathcal{Y}$ with respect to Ω is the set of all objects ,which can be possibly classified as $\mathcal{Y}$ with respect to Ω and it is denoted by $L_\Omega(\mathcal{Y})$. That is, $L_\Omega(\mathcal{Y}) = \cup_{\mathcal{Y}\in\mathcal{V}} \{\Omega(\mathcal{Y}); \Omega(\mathcal{Y}) \subseteq \mathcal{Y}\}$, where $\Omega(\mathcal{Y})$ denoted equivalence class determined by Ω .
- The upper approximation of $\mathcal{Y}$ with respect to Ω is the set of all objects that can be possibly classified as $\mathcal{Y}$ with respect to Ω and it is denoted by $u_\Omega(\mathcal{Y})$. That is $U_\Omega(\mathcal{Y}) = \cup_{\mathcal{Y}\in\mathcal{V}} \{\Omega(\mathcal{Y}); \Omega(\mathcal{Y}) \cap \mathcal{Y} \neq \emptyset\}$.

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- The boundary region of $\mathcal{Y}$ with respect to Ω is the set of all objects that can be neither nor, as, not $\mathcal{Y}$ classified neither the form of $\mathcal{Y}$ nor in the form not- $\mathcal{Y}$ with respect to Ω and is denoted by $B_\Omega(\mathcal{Y})$. That is, $B_\Omega(\mathcal{Y}) = U_\Omega(\mathcal{Y}) - L_\Omega(\mathcal{Y})$.

Proposition 2.1 [3]

If $(\mathcal{V}, \Omega)$ is an approximation space and $\mathcal{Y}, \mathcal{M} \subseteq \mathcal{V}$,afterward

- $L_\Omega(\mathcal{Y}) \subseteq \mathcal{Y} \subseteq U_\Omega(\mathcal{Y})$;
- $L_\Omega(\emptyset) = U_\Omega(\emptyset) = \emptyset$;
- $L_\Omega(\mathcal{V}) = U_\Omega(\mathcal{V}) = \mathcal{V}$;
- $U_\Omega(\mathcal{Y} \cup \mathcal{M}) = U_\Omega(\mathcal{Y}) \cup U_\Omega(\mathcal{M})$;
- $U_\Omega(\mathcal{Y} \cap \mathcal{M}) \subseteq U_\Omega(\mathcal{Y}) \cap U_\Omega(\mathcal{M})$;
- $L_\Omega(\mathcal{Y} \cup \mathcal{M}) \supseteq L_\Omega(\mathcal{Y}) \cup L_\Omega(\mathcal{M})$;
- $(\mathcal{Y} \cap \mathcal{M}) \subseteq L_\Omega(\mathcal{Y}) \cap L_\Omega(\mathcal{M})$;
- $L_\Omega(\mathcal{Y}) \subseteq L_\Omega(\mathcal{M})$and $U_\Omega(\mathcal{Y}) \subseteq U_\Omega(\mathcal{M})$ whenever $\mathcal{Y} \subseteq \mathcal{M}$;
- $U_\Omega(\mathcal{Y}^C) = [L_\Omega(\mathcal{Y})]^C$ and $L_\Omega(\mathcal{Y}^C) = [U_\Omega(\mathcal{Y})]^C$;
- $U_\Omega U_\Omega(\mathcal{Y}) = L_\Omega U_\Omega(\mathcal{Y}) = U_\Omega(\mathcal{Y})$;
- $L_\Omega L_\Omega(\mathcal{Y}) = U_\Omega L_\Omega(\mathcal{Y}) = L_\Omega(\mathcal{Y})$;

Definition 2.2 [3]

Suppose $\mathcal{V}$ be the universe, Ω be equivalence relation on $\mathcal{V}$ and $\mathbb{T}_\Omega(\mathcal{Y}) = \{\mathcal{V}, \emptyset, L_\Omega(\mathcal{Y}), U_\Omega(\mathcal{Y}), B_\Omega(\mathcal{Y})\}$, where $\mathcal{Y} \subseteq U$. Then $\mathbb{T}_\Omega(\mathcal{Y})$ meets the following requirements

- $\mathcal{V}$ and $\emptyset \in \mathbb{T}_\Omega(\mathcal{Y})$,
- The union of elements of any sub-collection of $\mathbb{T}_\Omega(\mathcal{Y})$ is in $\mathbb{T}_\Omega(\mathcal{Y})$,
- The intersection of the elements of any finite sub-collection of $\mathbb{T}_\Omega(\mathcal{Y})$ is in $\mathbb{T}_\Omega(\mathcal{Y})$.

The Nano topology on set $\mathcal{V}$ with respect to $\mathcal{Y}$ is define as the topological $\mathbb{T}_\Omega(\mathcal{Y})$ the context of short $\mathcal{NT}$. This creates a $\mathcal{NTS}$ $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$, and the dual $\mathcal{NT}$ of $\mathbb{T}_\Omega(\mathcal{Y})$ can be written as $[\mathbb{T}_\Omega(\mathcal{Y})]^C$.Sets within $\mathbb{T}_\Omega(\mathcal{Y})$ have the name Nano-open sets(the context of short $\mathcal{NO}$-sets, while their complements are refer as Nano-closed sets(the context of short $\mathcal{NC}$-sets).

Definition 2.3 [3]

Assume $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ be a $\mathcal{NTS}$ with respect to $\mathcal{Y}$, where $\mathcal{Y}$ is a subset of $\mathcal{V}$. If $\mathfrak{H}$ is a subset of $\mathcal{V}$, afterward

The union of all $\mathcal{NO}$-sets included in the set $\mathfrak{H}$ is known as the Nano interior of the set, or $\mathcal{N}int(\mathfrak{H})$.. Stated differently, the greatest $\mathcal{NO}$-set contained in $\mathfrak{H}$ is represented by $\mathcal{N}int(\mathfrak{H})$.

The intersection of all $\mathcal{NC}$-sets that contain the set $\mathfrak{H}$ is the set's nano closure, or $\mathcal{N}cl(\mathfrak{H})$. Accordingly, the smallest $\mathcal{NC}$-set which includes $\mathfrak{H}$ is $\mathcal{N}cl(\mathfrak{H})$.

Definition 2.4 [3]

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Suppose $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ be a $\mathcal{NTS}$ and $\mathfrak{H}$ be a subset of $\mathcal{V}$. Then $\mathfrak{H}$ is claimed to be:

- Nano semi-open if, $\mathfrak{H} \subseteq \mathcal{N}cl(\mathcal{N}int(\mathfrak{H}))$.
- Nano ∝-open if, $\mathfrak{H} \subseteq \mathcal{N}int(\mathcal{N}cl(\mathcal{N}int(\mathfrak{H})))$.
- Nano pre-open if, $\mathfrak{H} \subseteq \mathcal{N}int(\mathcal{N}cl(\mathfrak{H}))$.
- Nano $\mathcal{B}$ -open if, $\mathfrak{H} \subseteq \mathcal{N}cl(\mathcal{N}int(\mathfrak{H})) \cup \mathcal{N}int(\mathcal{N}cl(\mathfrak{H}))$.

The complements of the $\mathcal{NO}$-sets sets described above are known as their corresponding $\mathcal{NC}$-sets sets. The family of all Nano semi-open (resp. Nano ∝-open , Nano pre-open and Nano $\mathcal{B}$-open) sets in a Nano topological space $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ is signified by $\mathcal{NSO}(\mathcal{V}, \mathcal{Y})$ (resp. $\mathcal{N} \propto \mathcal{O}(\mathcal{V}, \mathcal{Y})$,$\mathcal{NPO}(\mathcal{V}, \mathcal{Y})$ and $\mathcal{NBO}(\mathcal{V}, \mathcal{Y})$).The family of all Nano semi-closed (resp. Nano ∝-closed, nano pre-closed and nano $\mathcal{B}$-closed) sets in a Nano topological space$(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y})))$ is signified by $\mathcal{NSC}(\mathcal{V}, \mathcal{Y})$(resp. $\mathcal{N} \propto \mathcal{C}(\mathcal{V}, \mathcal{Y})$, $\mathcal{NPC}(\mathcal{V}, \mathcal{Y})$ and $\mathcal{NBC}(\mathcal{V}, \mathcal{Y})$).

Definition 2.5 [7]

Suppose $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ be a $\mathcal{NTS}$ and $\mathfrak{H}$ be a subset of $\mathcal{V}$. Then $\mathfrak{H}$ is claimed to be Nano β-open sets if $\mathfrak{H} \subseteq \mathcal{N}cl(\mathcal{N}int(\mathcal{N}cl(\mathfrak{H})))$. The family of all Nano β-open sets is signified by $\mathcal{N}\beta\mathcal{O}(\mathcal{V}, \mathcal{Y})$). The complement of the Nano β-open sets is known as nano β-closed sets and the family of all nano β-closed is signified by $\mathcal{N}\beta\mathcal{C}(\mathcal{V}, \mathcal{Y})$).

Definition 2.6 [8]

In $\mathcal{NTS}$ $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$a space $\mathcal{V}$is called $\mathcal{N}\mathbb{T}_1$ -space for $\mathfrak{i}, \mathfrak{j} \in \mathcal{V}$ and $\mathfrak{i} \neq \mathfrak{j}$, there exist a $\mathcal{NO}$-sets $\mathfrak{E}$ and $\mathfrak{H}$ such that $\mathfrak{i} \in \mathfrak{E}, \mathfrak{j} \notin \mathfrak{E}$ and $\mathfrak{j} \in \mathfrak{H}, \mathfrak{i} \notin \mathfrak{H}$.

Definition 2.7 [4]

A subset $\mathfrak{A} \in \mathcal{NSO}(\mathcal{V}, \mathcal{Y})$is said to be Nano $\mathcal{S}_c$-open sets (the context of short $\mathcal{NS}_c\mathcal{O}$-sets) in $\mathcal{NTS}$ $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ if for each $\mathfrak{x} \in \mathfrak{A}$ there a $\mathcal{NC}$-set $\mathfrak{F}$ such that $\mathfrak{x} \in \mathfrak{F} \subseteq \mathfrak{A}$. The entire family. $\mathcal{NS}_c\mathcal{O}$-sets of a $\mathcal{NTS}$ $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ is signified by $\mathcal{NS}_c\mathcal{O}(\mathcal{V}, \mathcal{Y})$.

Proposition 2.2 [4]

A subset $\mathfrak{H}$ of $\mathcal{NTS}$ $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ is $\mathcal{NS}_c\mathcal{O}$-sets if and only if $\mathfrak{H}$ is $\mathcal{NSO}$-sets and it is a union of $\mathcal{NC}$- sets.That is $\mathfrak{H} = \cup \mathfrak{F}_\propto$ where $\mathfrak{H}$ is $\mathcal{NSO}$-set and $\mathfrak{F}_\propto$is $\mathcal{NC}$-sets for each ∝ .

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3 Nano $\mathcal{B}_c$ – open sets

Definition 3.1

A subset $\mathfrak{A} \in \mathcal{NBO}(\mathcal{V},\mathcal{Y})$is said to be Nano $\mathcal{B}_c$-open sets (simply $\mathcal{NB}_c\mathcal{O}$-sets) in $\mathcal{NTS}$ $(\mathcal{V},\mathbb{T}_\Omega(\mathcal{Y}))$ if for each $\mathfrak{x} \in \mathfrak{A}$, ther exist a $\mathcal{NC}$-set $\mathfrak{F}$ such that $\mathfrak{x} \in \mathfrak{F} \subseteq \mathfrak{A}$. The entire family $\mathcal{NB}_c\mathcal{O}$-sets of a $\mathcal{NTS}$ $(\mathcal{V},\mathbb{T}_\Omega(\mathcal{Y}))$ is signified by $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$.

Remark 3.1

The complement of $\mathcal{NB}_c\mathcal{O}$-sets in $\mathcal{NTS}$ $(\mathcal{V},\mathbb{T}_\Omega(\mathcal{Y}))$ is claimed to be Nano $\mathcal{B}_c$-closed sets (the context of short $\mathcal{NB}_c\mathcal{C}$-sets), and $\mathcal{NB}_c\mathcal{C}(\mathcal{V},\mathcal{Y})$) signifies The entire family $\mathcal{NB}_c\mathcal{C}$-sets .

Example 3.1

Let's get started $\mathcal{V} = \{\mathfrak{i},\mathfrak{j},\mathfrak{l},\mathfrak{k}\}$, $\mathcal{V}/\Omega = \{\{\mathfrak{i},\mathfrak{j}\},\{\mathfrak{l}\},\{\mathfrak{k}\}\}$ and $\mathcal{Y} = \{\mathfrak{l},\mathfrak{j}\}$.
Then $L_\Omega(\mathcal{Y}) = \{\mathfrak{l}\}$, $U_\Omega(\mathcal{Y}) == \{\mathfrak{i},\mathfrak{j},\mathfrak{l}\}$ and $B_\Omega(\mathcal{Y}) = \{\mathfrak{i},\mathfrak{j}\}$. Consequently $\mathbb{T}_\Omega(\mathcal{Y}) = \{\mathcal{V},\emptyset,\{\mathfrak{i},\mathfrak{j}\},\{\mathfrak{i},\mathfrak{j},\mathfrak{l}\},\{\mathfrak{l}\}\}$ and $[\mathbb{T}_\Omega(\mathcal{Y})]^C = \{\mathcal{V},\emptyset,\{\mathfrak{k},\mathfrak{l}\},\{\mathfrak{k}\},\{\mathfrak{i},\mathfrak{j},\mathfrak{k}\}\}$. The $\mathcal{NBO}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V},\emptyset,\{\mathfrak{l}\},\{\mathfrak{i}\},\{\mathfrak{j}\},\{\mathfrak{i},\mathfrak{j}\},\{\mathfrak{i},\mathfrak{l}\},\{\mathfrak{j},\mathfrak{l}\},\{\mathfrak{k},\mathfrak{l}\},\{\{\mathfrak{i},\mathfrak{j},\mathfrak{l}\},\{\mathfrak{i},\mathfrak{j},\mathfrak{k}\},\{\mathfrak{j},\mathfrak{k},\mathfrak{l}\},\{\mathfrak{i},\mathfrak{k},\mathfrak{l}\}\}$ and $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V},\emptyset,\{\mathfrak{k},\mathfrak{l}\},\{\mathfrak{i},\mathfrak{j},\mathfrak{k}\}\}$.

Remarks 3.2

1) Every $\mathcal{NB}_c\mathcal{O}$-set is a $\mathcal{NBO}$-set, however as the example above illustrates, this may not always be correct The subset $\{\mathfrak{l}\}$ is a $\mathcal{NBO}$-set, but not a $\mathcal{NB}_c\mathcal{O}$-set in $\mathcal{V}$, as can be shown.
2) Give $\{\mathfrak{A}_\propto : \propto \in \triangle\}$ be a collection of $\mathcal{NB}_c\mathcal{O}$-set in a $\mathcal{NTS}$ $(\mathcal{V},\mathbb{T}_\Omega(\mathcal{Y}))$. Then $\cup\{\mathfrak{A}_\propto : \propto \in \triangle\}$ is $\mathcal{NB}_c\mathcal{O}$- set .
3) Intersection of two $\mathcal{NB}_c\mathcal{O}$-sets may not be $\mathcal{NB}_c\mathcal{O}$-set The subsets $\{\mathfrak{k},\mathfrak{l}\}$ and $\{\mathfrak{i},\mathfrak{j},\mathfrak{k}\}$ in the example above are $\mathcal{NB}_c\mathcal{O}$-sets, but $\{\mathfrak{k},\mathfrak{l}\}\cap\{\mathfrak{i},\mathfrak{j},\mathfrak{k}\} = \{\mathfrak{k}\}$, which is not a $\mathcal{NB}_c\mathcal{O}$-set in $\mathcal{V}$. Consequently, a supra topology is generated by the family of $\mathcal{NB}_c\mathcal{O}$-sets.

Proposition 3.1

A subset $\mathfrak{A}$ of $\mathcal{NTS}(\mathcal{V},\mathbb{T}_\Omega(\mathcal{Y}))$ is $\mathcal{NB}_c\mathcal{O}$-set if and only if $\mathfrak{A}$ is $\mathcal{NBO}$-set and it is a union of $\mathcal{NC}$-sets That is $\mathfrak{A} = \cup\mathfrak{F}_\propto$, where $\mathfrak{A}$ is $\mathcal{NBO}$-set and $\mathfrak{F}_\propto$ is $\mathcal{NC}$-sets for each $\propto$
Proof**:** Clear

We examine every possible kind of $\mathcal{NB}_c\mathcal{O}$-sets in terms of upper and lower approximations in $\mathcal{NTS}$ $(\mathcal{V},\mathbb{T}_\Omega(\mathcal{Y}))$ and the results are as follows.

Theorem 3.1

Let's take $(\mathcal{V},\mathbb{T}_\Omega(\mathcal{Y}))$ be a $\mathcal{NTS}$, and then

(1) If $U_\Omega(\mathcal{Y}) = \mathcal{V}$ and $L_\Omega(\mathcal{Y}) = \emptyset$,then .$\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V},\emptyset\}$.
(2) If $U_\Omega(\mathcal{Y}) = \mathcal{V}$ and $L_\Omega(\mathcal{Y}) \neq \emptyset$, then $\mathbb{T}_\Omega(\mathcal{Y}) = \mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$.
(3) If $U_\Omega(\mathcal{Y}) = L_R(X) \neq \mathcal{V}$ then $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V},\emptyset\}$.
(4) If $U_\Omega(\mathcal{Y}) \neq \mathcal{V}$ and $L_\Omega(\mathcal{Y}) = \emptyset$, then $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V},\emptyset\}$.
(5) If $U_\Omega(\mathcal{Y}) \neq L_R(X)$ where $U_\Omega(\mathcal{Y}) \neq \mathcal{V}$and $L_\Omega(\mathcal{Y}) \neq \emptyset$,then

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$\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V},\emptyset,\mathrm{B}_\Omega(\mathcal{Y})\cup[\mathrm{U}_\Omega(\mathcal{Y})]^C,\mathrm{L}_\Omega(\mathcal{Y})\cup[\mathrm{U}_\Omega(\mathcal{Y})]^C\}$.

Proof:

(1) presume $\mathrm{U}_\Omega(\mathcal{Y}) = \mathcal{V}$ and $\mathrm{L}_\Omega(\mathcal{Y}) = \emptyset$, then $\mathrm{B}_\Omega(\mathcal{Y}) = \mathcal{V}$. And Then $\mathrm{T}_\Omega(\mathcal{Y}) = [\mathrm{T}_\Omega(\mathcal{Y})]^C = \{\mathcal{V},\emptyset\}$. Thus for any subset $\mathfrak{A}$ of $\mathcal{V}$, $\mathcal{N}int(\mathcal{N}cl(\mathfrak{A}) = \mathcal{V}$ and $\mathcal{N}cl(\mathcal{N}int(\mathfrak{A}) = \emptyset$. As a result $\mathfrak{A} \subset \mathcal{N}int(\mathcal{N}cl(\mathfrak{A}) \cup \mathcal{N}cl(\mathcal{N}int(\mathfrak{A})$. Consequently $\mathfrak{A}$ is $\mathcal{NBO}$-set. In $\mathcal{V}$. As a result $\mathcal{NBO}(\mathcal{V},\mathcal{Y}))$ is $\mathrm{P}(\mathcal{V})$. Consequently by definition of $\mathcal{NB}_c\mathcal{O}$-set we get $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V},\emptyset\}$. □

1) Suppose $\mathrm{U}_\Omega(\mathcal{Y}) = \mathcal{V}$ and $\mathrm{L}_\Omega(\mathcal{Y}) \neq \emptyset$ then $\mathbb{T}_\Omega(\mathcal{Y}) = \{\mathcal{V},\emptyset,\mathrm{L}_\Omega(\mathcal{Y}),\mathrm{B}_\Omega(\mathcal{Y})\} = [\mathbb{T}_\Omega(\mathcal{Y})]^C$. Now each element in $\mathcal{V}$ is either in $\mathrm{L}_\Omega(\mathcal{Y})$ or in $\mathrm{B}_\Omega(\mathcal{Y})$ due to $\mathrm{B}_\Omega(\mathcal{Y}) = \mathrm{U}_\Omega(\mathcal{Y}) - \mathrm{L}_\Omega(\mathcal{Y})$.

(a) If $\mathfrak{A} \subset \mathrm{L}_\Omega(\mathcal{Y})$,afterward $\mathcal{N}nt(\mathcal{N}cl(\mathfrak{A})) = \mathrm{L}_\Omega(\mathcal{Y})$ and $\mathcal{N}cl(\mathcal{N}int(\mathfrak{A}) = \emptyset$.

(b) If $\mathfrak{A} \subset \mathrm{B}_\Omega(\mathcal{Y})$, afterward $\mathcal{N}int\big(\mathcal{N}cl(\mathfrak{A})\big) = \mathrm{B}_\Omega(\mathcal{Y})$ and $\mathcal{N}cl(\mathcal{N}int(\mathfrak{A}) = \emptyset$.

© If $\mathfrak{A}$ intersects both $\mathrm{L}_\Omega(\mathcal{Y})$ and $\mathrm{B}_\Omega(\mathcal{Y})$, $\mathcal{N}int\big(\mathcal{N}cl(\mathfrak{A})\big) = \mathcal{V}$ and $\mathcal{N}cl(\mathcal{N}int(\mathfrak{A}) = \emptyset$ in all cases we get that the $\mathfrak{A}$ is $\mathcal{NBO}$-set . As a result $\mathcal{NBO}(\mathcal{V},\mathcal{Y})$ is $\mathrm{P}(\mathcal{V})$. Consequently by definition of $\mathcal{NB}_c\mathcal{O}$-set we get

$\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V},\emptyset,\mathrm{L}_\Omega(\mathcal{Y}),\mathrm{B}_\Omega(\mathcal{Y})\}$. □

2) Assuming $\mathrm{U}_\Omega(\mathcal{Y}) = \mathrm{L}_\Omega(\mathcal{Y}) \neq \mathcal{V}$, afterward $\mathrm{B}_\Omega(\mathcal{Y}) = \emptyset$.And then $\mathbb{T}_\Omega(\mathcal{Y}) = \{\mathcal{V},\emptyset,\mathrm{U}_\Omega(\mathcal{Y})\}$.So $[\mathbb{T}_\Omega(\mathcal{Y})]^C = \{\mathcal{V},\emptyset,[\mathrm{U}_\Omega(\mathcal{Y})]^C$. Each element in $\mathcal{V}$ is either in $\mathrm{U}_\Omega(\mathcal{Y})$or in $[\mathrm{U}_\Omega(\mathcal{Y})]^C$. If $\mathfrak{A} \subset \mathrm{U}_\Omega(\mathcal{Y})$, afterward $\mathcal{N}int(\mathcal{N}cl(\mathfrak{A})) = \mathcal{V}$ as well $\mathcal{N}cl(\mathcal{N}int(\mathfrak{A}) = \emptyset$.

Consequently $\mathfrak{A}$ is $\mathcal{NBO}$-set in $\mathcal{V}$. But if $\mathfrak{A} \subset [\mathrm{U}_\Omega(\mathcal{Y})]^C$, afterward $\mathcal{N}int\big(\mathcal{N}cl(\mathfrak{A})\big) = \emptyset$ as well $\mathcal{N}cl(\mathcal{N}int(\mathfrak{A}) = \emptyset$. Consequently $\mathfrak{A}$ is not $\mathcal{NBO}$-set in $\mathcal{V}$. As a result $\mathcal{V},\emptyset$ And any set that the intersects $\mathrm{U}_\Omega(\mathcal{Y})$ are $\mathcal{NBO}$-set in $\mathcal{V}$. As a result $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V},\emptyset\}$, since for each element $\mathfrak{x}$ in $\mathfrak{A}$ there is no $\mathcal{NC}$- set $\mathfrak{F}$ such that $\mathfrak{x} \in \mathfrak{F} \subseteq \mathfrak{A}$. □

3) The proof is analogous to **(3)**

4) presume $\mathrm{U}_\Omega(\mathcal{Y}) \neq \mathrm{L}_\Omega(\mathcal{Y})$ where $\mathrm{U}_\Omega(\mathcal{Y}) \neq \mathcal{V}$ and $\mathrm{L}_\Omega(\mathcal{Y}) \neq \emptyset$, afterward $\mathbb{T}_\Omega(\mathcal{Y}) = \{\mathcal{V},\emptyset,\mathrm{U}_\Omega(\mathcal{Y}),\mathrm{L}_\Omega(\mathcal{Y}),\mathrm{B}_\Omega(\mathcal{Y})\}$ and

$[\mathbb{T}_\Omega(\mathcal{Y})]^C = \{\mathcal{V},\emptyset,[\mathrm{U}_\Omega(\mathcal{Y})]^C,\mathrm{B}_\Omega(\mathcal{Y})\cup[\mathrm{U}_\Omega(\mathcal{Y})]^C,\mathrm{L}_\Omega(\mathcal{Y})\cup[\mathrm{U}_\Omega(\mathcal{Y})]^C\}$.

a) If $\mathfrak{A} \subseteq \mathrm{U}_\Omega(\mathcal{Y})$

presume $\mathfrak{A} \subseteq \mathrm{L}_\Omega(\mathcal{Y})$, afterward $\mathcal{N}int(\mathcal{N}cl(\mathfrak{A})) = \mathrm{L}_\Omega(\mathcal{Y})$ and $\mathcal{N}cl(\mathcal{N}int(\mathfrak{A}) = \emptyset$ and As a result $\mathfrak{A} \subseteq \mathrm{L}_\Omega(\mathcal{Y})\cup\emptyset$. Consequently $\mathfrak{A}$ is $\mathcal{NBO}$- set in $\mathcal{V}$.

Assuming $\mathfrak{A} \subset \mathrm{B}_\Omega(\mathcal{Y})$, afterward $\mathcal{N}int(\mathcal{N}cl(\mathfrak{A})) = \mathrm{B}_\Omega(\mathcal{Y})$ and $\mathcal{N}cl(\mathcal{N}int(\mathfrak{A}) = \emptyset$ and As a result $\mathfrak{A} \subseteq \mathrm{B}_\Omega(\mathcal{Y})\cup\emptyset$. Consequently $\mathfrak{A}$ is $\mathcal{NBO}$-set in $\mathcal{V}$. Assuming $\mathfrak{A}$ intersect both $\mathrm{L}_\Omega(\mathcal{Y})$ and $\mathrm{B}_\Omega(\mathcal{Y})$, afterward $\mathcal{N}int(\mathcal{N}cl(\mathfrak{A})) = \mathcal{V}$ as well $\mathcal{N}cl(\mathcal{N}int(\mathfrak{A}) = \emptyset$ and As a result $\mathfrak{A} \subseteq \mathrm{L}_\Omega(\mathcal{Y})\cup\emptyset$. Consequently $\mathfrak{A}$ is $\mathcal{NBO}$-set in $\mathcal{V}$.If $\mathfrak{A} \subseteq [\mathrm{U}_\Omega(\mathcal{Y})]^C$, afterward $\mathcal{N}int\big(\mathcal{N}cl(\mathfrak{A})\big) = \emptyset$ and $\mathcal{N}cl(\mathcal{N}int(\mathfrak{A}) = \emptyset$. Consequently $\mathfrak{A}$ is not $\mathcal{NBO}$-set in $\mathcal{V}$. □

b) If $\mathfrak{A}$ intersect both $\mathrm{U}_\Omega(\mathcal{Y})$ and $[\mathrm{U}_\Omega(\mathcal{Y})]^C$.

Suppose $\mathfrak{A}$ Contain elements of $\mathrm{L}_\Omega(\mathcal{Y})$ and $[\mathrm{U}_\Omega(\mathcal{Y})]^C$, afterward $\mathcal{N}int(\mathcal{N}cl(\mathfrak{A}) = \mathrm{L}_\Omega(\mathcal{Y})$. Consequently $\mathfrak{A}$ Is $\mathcal{NBO}$-set in $\mathcal{V}$. Assuming $\mathfrak{A}$ contain elements of $\mathrm{B}_\Omega(\mathcal{Y})$ and $[\mathrm{U}_\Omega(\mathcal{Y})]^C$, afterward $\mathcal{N}int(\mathcal{N}cl(\mathfrak{A})) = B_\Omega(\mathcal{Y})$ .Consequently $\mathfrak{A}$ is $\mathcal{NBO}$-set in $\mathcal{V}$. Assuming $\mathfrak{A}$ intersect $L_\Omega(\mathcal{Y})$, $B_\Omega(\mathcal{Y})$ and $[U_\Omega(\mathcal{Y})]^C$, $\mathcal{N}int(\mathcal{N}cl\mathfrak{A})) = \mathcal{V}$. Consequently $\mathfrak{A}$ is $\mathcal{NBO}$-set in $\mathcal{V}$. As a result the $\mathcal{NBO}$-sets are $\mathcal{V}, \emptyset, U_\Omega(\mathcal{Y}), L_\Omega(\mathcal{Y}), B_\Omega(\mathcal{Y}), B_\Omega(\mathcal{Y})\cup[U_\Omega(\mathcal{Y})]^C$ and $L_\Omega(\mathcal{Y})\cup[U_\Omega(\mathcal{Y})]^C$ Consequently

$\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V}, \emptyset, B_\Omega(\mathcal{Y})\cup[U_\Omega(\mathcal{Y})]^C, L_\Omega(\mathcal{Y})\cup[U_\Omega(\mathcal{Y})]^C\}$. □

Example 3.2

Let's take $\mathcal{V} = \{\mathfrak{i}, \mathfrak{j}, \mathfrak{l}, \mathfrak{k}\}$, , $\mathcal{V}/\Omega = \{\{\mathfrak{i}\}, \{\mathfrak{k}\}, \{\mathfrak{j}, \mathfrak{l}\}\}$ and $\mathcal{Y} = \{\mathfrak{i}, \mathfrak{j}\}$. Then $L_\Omega(\mathcal{Y}) = \{\mathfrak{i}\}$, $U_\Omega(\mathcal{Y}) = \{\mathfrak{i}, \mathfrak{j}, \mathfrak{l}\}$ and $B_\Omega(\mathcal{Y}) = \{\mathfrak{j}, \mathfrak{l}\}$. Consequently $\mathbb{T}_\Omega(\mathcal{Y}) = \{\mathcal{V}, \emptyset, \{\mathfrak{i}\}, \{\mathfrak{i}, \mathfrak{j}, \mathfrak{l}\}, \{\mathfrak{j}, \mathfrak{l}\}\}$ and $[\mathbb{T}_\Omega(\mathcal{Y})]^C = \{\mathcal{V}, \emptyset, \{\mathfrak{j}, \mathfrak{k}, \mathfrak{l}\}, \{\mathfrak{k}\}, \{\mathfrak{i}, \mathfrak{k}\}\}$.

Then, we can easily find the following families of sets: $\mathcal{NBO}(\mathcal{V},\mathcal{Y}) =$
$\{\mathcal{V}, \emptyset, \{\mathfrak{i}\}, \{\mathfrak{j}\}, \{\mathfrak{l}\}, \{\mathfrak{i}, \mathfrak{j}\}, \{\mathfrak{i}, \mathfrak{k}\}, \{\mathfrak{i}, \mathfrak{l}\}, \{\mathfrak{j}, \mathfrak{l}\}, \{\mathfrak{i}, \mathfrak{j}, \mathfrak{k}\}, \{\mathfrak{i}, \mathfrak{j}, \mathfrak{l}\}, \{\mathfrak{i}, \mathfrak{k}, \mathfrak{l}\}, \{\mathfrak{j}, \mathfrak{k}, \mathfrak{l}\}\}$,
$\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V}, \emptyset, \{\mathfrak{i}, \mathfrak{k}\}, \{\mathfrak{j}, \mathfrak{k}, \mathfrak{l}\}\}$,
$\mathcal{NSO}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V}, \emptyset, \{\mathfrak{i}\}, \{\mathfrak{i}, \mathfrak{k}\}, \{\mathfrak{j}, \mathfrak{l}\}, \{\mathfrak{i}, \mathfrak{j}, \mathfrak{l}\}, \{\mathfrak{j}, \mathfrak{k}, \mathfrak{l}\}\}$,
$\mathcal{NPO}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V}, \emptyset, \{\mathfrak{i}\}, \{\mathfrak{j}\}, \{\mathfrak{l}\}, \{\mathfrak{i}, \mathfrak{j}\}, \{\mathfrak{i}, \mathfrak{l}\}, \{\mathfrak{j}, \mathfrak{l}\}, \{\mathfrak{i}, , \mathfrak{k}\}, \{\mathfrak{i}, \mathfrak{j}, \mathfrak{l}\}, \{\mathfrak{i}, \mathfrak{k}, \mathfrak{l}\}\}$,
$\mathcal{N} \propto \mathcal{O}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V}, \emptyset, \{\mathfrak{i}\}, \{\mathfrak{j}, \mathfrak{l}\}, \{\mathfrak{i}, \mathfrak{j}, \mathfrak{l}\}\}$and
$\mathcal{N}\beta\mathcal{O}(\mathcal{V},\mathcal{Y})) = \{\mathcal{V}, \emptyset, \{\mathfrak{i}\}, \{\mathfrak{j}\}, \{\mathfrak{l}\}, \{\mathfrak{i}, \mathfrak{j}\}, \{\mathfrak{i}, \mathfrak{k}\}, \{\mathfrak{i}, \mathfrak{l}\}, \{\mathfrak{j}, \mathfrak{k}\}, \{\mathfrak{j}, \mathfrak{l}\}, \{\mathfrak{k}, \mathfrak{l}\}, \{\mathfrak{i}, \mathfrak{j}, \mathfrak{k}\}$,
$\{\mathfrak{i}, \mathfrak{j}, \mathfrak{l}\}, \{\mathfrak{i}, \mathfrak{k}, \mathfrak{l}\}, \{\mathfrak{j}, \mathfrak{k}, \mathfrak{l}\}\}$.

From the above example we notice the following results:

(1) The family of $\mathcal{NO}$-sets in $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ is incomparable with $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$. The subset $\{\mathfrak{i}, \mathfrak{k}\}$ is $\mathcal{NB}_c\mathcal{O}$-set but not $\mathcal{NO}$-set in $\mathcal{V}$,also the subset $\{\mathfrak{i}\}$ is $\mathcal{NO}$-set while not $\mathcal{NB}_c\mathcal{O}$-set in $\mathcal{V}$.

(2) $\mathcal{NPO}(\mathcal{V},\mathcal{Y})$ is incomparable with $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$.The subset $\{\mathfrak{i}, \mathfrak{k}\}$ is $\mathcal{NB}_c\mathcal{O}$- set while not $\mathcal{NPO}$-set in $\mathcal{V}$,also the subset $\{\mathfrak{j}\}$ is $\mathcal{NPO}$-set but not $\mathcal{NB}_c\mathcal{O}$- set in $\mathcal{V}$.

(3) $\mathcal{N} \propto \mathcal{O}(\mathcal{V},\mathcal{Y})$ is incomparable with $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$.The subset $\{\mathfrak{j}, \mathfrak{k}, \mathfrak{l}\}$ is $\mathcal{NB}_c\mathcal{O}$-set while not $\mathcal{N} \propto \mathcal{O}$-set in $\mathcal{V}$,also the subset $\{\mathfrak{j}, \mathfrak{l}\}$ is $\mathcal{N} \propto \mathcal{O}$-set but not $\mathcal{NB}_c\mathcal{O}$-set in $\mathcal{V}$.

Proposition 3.2

Each of the $\mathcal{NB}_c\mathcal{O}$-set is $\mathcal{N}\beta\mathcal{O}$-set

Proof **:** Assume $\mathfrak{A}$ is $\mathcal{NB}_c\mathcal{O}$-set in $\mathcal{V}$. If $\mathfrak{A} = \emptyset$, afterward $\mathfrak{A}$ is a $\mathcal{N}\beta\mathcal{O}$-set . If $\mathfrak{A} \neq \emptyset$, afterward it is a $\mathcal{NBO}$-set. According to the notion of a $\mathcal{NBO}$-set $\mathfrak{A} \subseteq \mathcal{N}cl(\mathcal{N}int(\mathfrak{A}))\cup\mathcal{N}int(\mathcal{N}cl(\mathfrak{A}))$. As a result,

$\mathfrak{A} \subseteq \mathcal{N}cl(\mathcal{N}int(\mathfrak{A}))\cup\mathcal{N}int(\mathcal{N}cl(\mathfrak{A})) \subseteq \mathcal{N}cl(\mathcal{N}int(\mathcal{N}cl(\mathfrak{A})))$. Consequently, $\mathfrak{A}$ is a $\mathcal{N}\beta\mathcal{O}$-set in $\mathcal{V}$. □

The inverse of the preceding proposition does not have to be correct, as demonstrated in example (3.2)

Although the subset $\{\mathfrak{i}, \mathfrak{j}\}$ is not a $\mathcal{NB}_c\mathcal{O}$-set , it is a $\mathcal{N}\beta\mathcal{O}$-set .

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Proposition 3.3

Every $\mathcal{NB}_c\mathcal{O}$-set is $\mathcal{NSO}$-sets

Proof: Take $\mathfrak{A}$ be the set in $\mathcal{V}$ of $\mathcal{NB}_c\mathcal{O}$-set . $\mathfrak{A}$ is a $\mathcal{NSO}$-set if $\mathfrak{A} = \emptyset$. $\mathfrak{A}$ is a $\mathcal{NBO}$- set if $\mathfrak{A} \neq \emptyset$, and a $\mathcal{NC}$-set if $\mathfrak{A} = \cup \mathfrak{F}_\propto$. According to the proposition "3.2", $\mathfrak{A}$ is a $\mathcal{N}\beta\mathcal{O}$-set since it is a $\mathcal{NB}_c\mathcal{O}$-open set. For this reason, $\mathfrak{A} \subseteq \mathcal{N}\mathrm{cl}(\mathcal{N}\mathrm{int}(\mathcal{N}\mathrm{cl}(\mathfrak{A}))) \subseteq \mathcal{N}\mathrm{cl}(\mathcal{N}\mathrm{int}(\mathfrak{A}))$. Consequently, in $\mathcal{V}$, $\mathfrak{A}$ is a $\mathcal{NSO}$-set. □

As seen in example "3.2", the opposite of the above proposition does not have to be correct. Although the subset $\{\mathfrak{i}\}$ is not a $\mathcal{NB}_c\mathcal{O}$-set, it is a $\mathcal{NSO}$-set.

Proposition 3.4

The family of all $\mathcal{NB}_c\mathcal{O}$-sets is likewise topological on $\mathcal{V}$ if the family of all $\mathcal{NBO}$-sets of a $\mathcal{NTS}$ $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ is a topology on $\mathcal{V}$.

Proof: It suffices to demonstrate that the finite intersection of $\mathcal{NB}_c\mathcal{O}$-sets is itself a $\mathcal{NB}_c\mathcal{O}$-set. If $\mathfrak{A}$ and $\mathfrak{B}$ are both $\mathcal{NB}_c\mathcal{O}$-sets, then they are also $\mathcal{NBO}$-sets. Since $\mathcal{NBO}(\mathcal{V}, \mathcal{Y})$ is a topology on $\mathcal{V}$, $\mathfrak{A} \cap \mathfrak{B}$ is a $\mathcal{NBO}$-set. Let $\mathfrak{x} \in \mathfrak{A} \cap \mathfrak{B}$, afterward $\mathfrak{x} \in \mathfrak{A}$ and $\mathfrak{x} \in \mathfrak{B}$. There are $\mathcal{NC}$-sets $\mathfrak{F}$ and $\mathfrak{E}$ such that $\mathfrak{x} \in \mathfrak{F} \subseteq \mathfrak{A}$ as well $\mathfrak{x} \in \mathfrak{E} \subseteq \mathfrak{B}$, which implies that $\mathfrak{x} \in \mathfrak{F} \cap \mathfrak{E} \subseteq \mathfrak{A} \cap \mathfrak{B}$. Since every intersection of $\mathcal{NC}$-sets is $\mathcal{NC}$-set , $\mathfrak{F} \cap \mathfrak{E}$ is a $\mathcal{NC}$-set. Consequently,$\mathfrak{A} \cap \mathfrak{B}$ is a $\mathcal{NB}_c\mathcal{O}$-se .As a result, the $\mathcal{NB}_c\mathcal{O}$- sets family has a topology on $\mathcal{V}$. □

Proposition 3.5

A subset $\mathfrak{A}$ of $\mathcal{NTS}(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ is $\mathcal{NB}_c\mathcal{O}$-set if and only if for each $\mathfrak{x} \in \mathfrak{A}$, there is a $\mathcal{NB}_c\mathcal{O}$ set $\mathfrak{B}$ such that $\mathfrak{x} \in \mathfrak{B} \subseteq \mathfrak{A}$.

Proof: Put $\mathfrak{B} = \mathfrak{A}$ is a $\mathcal{NB}_c\mathcal{O}$-set such that $\mathfrak{x} \in \mathfrak{B} \subseteq \mathfrak{A}$ for any $\mathfrak{x}$ in $\mathfrak{A}$, assuming that $\mathfrak{A}$ is $\mathcal{NB}_c\mathcal{O}$-set in $\mathcal{NTS}(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$.

On the other hand, let's say that for every $\mathfrak{x}$ in $\mathfrak{A}$, there is a $\mathcal{NB}_c\mathcal{O}$-set $\mathfrak{B}$ such that $\mathfrak{x} \in \mathfrak{B} \subseteq \mathfrak{A}$. In this case, $\mathfrak{A} = \cup \mathfrak{B}_\mathfrak{x}$, where $\mathfrak{B}_\mathfrak{x} \in \mathcal{NB}_c\mathcal{O}(\mathcal{V}, \mathcal{Y})$ for every $\mathfrak{x}$. Consequently, $\mathfrak{A}$ is a $\mathcal{NB}_c\mathcal{O}$-set . □

Theorem 3.2

A $\mathcal{NTS}$ $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ is $\mathcal{N}\mathbb{T}_1$ -space if and solely if for any point $\mathfrak{x} \in \mathcal{V}$, the singleton set $\{\mathfrak{x}\}$ is $\mathcal{NC}$-set .

Proof: Suppose $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ be $\mathcal{N}\mathbb{T}_1$ -space and suppose $\mathfrak{x}$ be any point of $\mathcal{V}$. Now, we want to show that $\{\mathfrak{x}\}$ is $\mathcal{NC}$- set , that is , to show that $\mathcal{V} \setminus \{\mathfrak{x}\}$ is $\mathcal{NO}$-set . Consider $\mathfrak{y}$ belongs to $\mathcal{V} \setminus \{\mathfrak{x}\}$. Afterward $\mathfrak{y}$ does not equal $\mathfrak{x}$. Since $\mathcal{V}$ is $\mathcal{N}\mathbb{T}_1$ -space, there is a $\mathcal{NO}$-set $\mathfrak{G}$ such that $\mathfrak{y}$ belongs to $\mathfrak{G}$ and $\mathfrak{x}$ does not belongs to $\mathfrak{G}$. It follows that $\mathfrak{y} \in \mathfrak{G} \subseteq \mathcal{V} \setminus \{\mathfrak{x}\}$. Hence $\{\mathfrak{x}\}$ is $\mathcal{NC}$-set .

Conversely, suppose every singleton set $\{\mathfrak{x}\}$ of $\mathcal{V}$ be $\mathcal{NC}$-set. Now we show that $\mathcal{V}$ is $\mathcal{N}\mathbb{T}_1$ -space. Suppose $\mathfrak{x}$ and $\mathfrak{y}$ be two separate spots on $\mathcal{V}$. Afterward $\mathcal{V} \setminus \{\mathfrak{x}\}$ is a $\mathcal{NO}$-set which includes $\mathfrak{y}$ but does not include $\mathfrak{x}$. Similary $\mathcal{V} \setminus \{\mathfrak{y}\}$ is $\mathcal{NO}$-set which includes $\mathfrak{x}$ but does not include $\mathfrak{y}$. As a result the $\mathcal{NTS}$ $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ is $\mathcal{N}\mathbb{T}_1$ -space . □

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Proposition 3.6

If a $\mathcal{NTS}\ (\mathcal{V},\mathbb{T}_{\Omega}(\mathcal{Y}))$ is $\mathcal{N}\mathbb{T}_1$-space , afterward the families $\mathcal{NBO}(\mathcal{V},\mathcal{Y})=\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$.

Proof: Suppose $\mathfrak{A}$ be any subset of $\mathcal{V}$ and $\mathfrak{A}$ belongs to $\mathcal{NBO}(\mathcal{V},\mathcal{Y})$. If $\mathfrak{A}=\emptyset$, then $\mathfrak{A}$ belongs to $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$. If $\mathfrak{A}$ does not equal $\emptyset$ afterward for each $\mathfrak{x}$ in $\mathfrak{A}$. Since a space $(\mathcal{V},\mathbb{T}_{\Omega}(\mathcal{Y}))$. is $\mathcal{N}\mathbb{T}_1$ -space, theorem (3.2) states that every singleton is a $\mathcal{NC}$-set and as a result $\mathfrak{x}\in\{\mathfrak{x}\}\subseteq\mathfrak{A}$.As a result, $\mathfrak{A}$ belongs to the $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$. Consequently, $\mathcal{NBO}(\mathcal{V},\mathcal{Y})\subset\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$ however, $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})\subset\mathcal{NBO}(\mathcal{V},\mathcal{Y})$ in general. Consequently, $\mathcal{NBO}(\mathcal{V},\mathcal{Y})=\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$).

4 Nano B_C – Operators

Definition 4.1

Assume $\mathfrak{A}$ is a subset of the $\mathcal{NTS}\ (\mathcal{V},\mathbb{T}_{\Omega}(\mathcal{Y}))$. A single point $\mathfrak{x}\in\mathcal{V}$ is considered a $\mathcal{NB}_c$-limit point of $\mathfrak{A}$ (the context of short $\mathcal{NB}_c\mathcal{L}$-point) if, for any $\mathcal{NB}_c\mathcal{O}$-set $\mathfrak{G}$ including $\mathfrak{x}$, $\mathfrak{G}\cap(\mathfrak{A}\backslash\{\mathfrak{x}\})\neq\emptyset$. The set of all $\mathcal{NB}_c\mathcal{L}$-points of $\mathfrak{A}$ is known as the nano $\mathcal{B}_c$-derived set of $\mathfrak{A}$, as well it is signified by $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$.

Proposition 4.1

Consider $\mathfrak{A}$ be a subset of $\mathcal{V}$ in $\mathcal{NTS}\ (\mathcal{V},\mathbb{T}_{\Omega}(\mathcal{Y}))$. For each $\mathcal{NC}$- set $\mathfrak{F}$ of $\mathcal{V}$ including $\mathfrak{x}$ Such that $\mathfrak{F}\cap(\mathfrak{A}\backslash\ \{\mathfrak{x}\})\neq\emptyset$, a point $\mathfrak{x}\in\mathcal{V}$ is $\mathcal{NB}_c\mathcal{L}$-points of $\mathfrak{A}$.

Proof**:** presume $\mathfrak{G}$ be any of the $\mathcal{NB}_c\mathcal{O}$-set comprising $\mathfrak{x}$. Afterwards, for any $\mathfrak{x}\in\mathfrak{G}\in\mathcal{NBO}(\mathcal{V},\mathcal{Y})$, there is a $\mathcal{NC}$- set $\mathfrak{F}$ such that $\mathfrak{x}\in\mathfrak{F}\subseteq\mathfrak{G}$. Based on our premise, we have $\mathfrak{F}\cap(\mathfrak{A}\backslash\{\mathfrak{x}\})\neq\emptyset$. Consequently $\mathfrak{G}\cap(\mathfrak{A}\backslash\{\mathfrak{x}\})\neq\emptyset$. As a result, a single point $\mathfrak{x}$ in $\mathcal{V}$ is one of $\mathfrak{A}$'s $\mathcal{NB}_c\mathcal{L}$-points . □

Remarks 4.1

(1) If $\{\mathfrak{x}\}\in\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$ in any of the $\mathcal{NTS}\ (\mathcal{V},\mathbb{T}_{\Omega}(\mathcal{Y}))$, fterwards $\mathfrak{x}\notin\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}\ )$ for any of the subset $\mathfrak{A}\subseteq\mathcal{V}$. Since $\{\mathfrak{x}\}\in\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$ this mean $\{\mathfrak{x}\}$ is $\mathcal{NB}_c\mathcal{O}$-set of $\mathcal{V}$ and $\{\mathfrak{x}\}\cap(\mathfrak{A}\backslash\{\mathfrak{x}\})=\emptyset$ so the definition not satisfy.

(2) If $\mathfrak{A}=\{\mathfrak{i}\}$ singleton set then $\mathfrak{i}\notin\mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$ since $\mathfrak{G}\cap(\{\mathfrak{i}\}\backslash\{\mathfrak{i}\})=\mathfrak{G}\cap\emptyset=\emptyset$.

(3) If $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})=\{\mathcal{V},\emptyset\}$, $\mathfrak{A}\neq\emptyset$ and $\mathfrak{A}$ contain more than one element, then $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A})=\mathcal{V}$ because the only $\mathcal{NB}_c\mathcal{O}$-set is $\mathcal{V}$ for every element in $\mathcal{V}$ and $\mathcal{V}\cap(\mathfrak{A}\backslash\{\mathfrak{x}\})\neq\emptyset$.

Theorem 4.1

Given a $\mathcal{NTS}\ (\mathcal{V},\mathbb{T}_{\Omega}(\mathcal{Y}))$. Let's take $\mathfrak{A}$ and $\mathfrak{B}$ be any subsets of $\mathcal{V}$. After that, we have the following attributes:

(1) $\mathcal{NB}_c\mathfrak{D}(\emptyset)=\emptyset$.

(2) If $\mathfrak{x}\in\mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$, then $\mathfrak{x}\in\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}\backslash\{\mathfrak{x}\})$.

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(3) If $\mathfrak{A} \subseteq \mathfrak{B}$, then $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \subseteq \mathcal{NB}_c\mathfrak{D}(\mathfrak{B})$.
(4) $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \cup \mathcal{NB}_c\mathfrak{D}(\mathfrak{B}) \subseteq \mathcal{NB}_c\mathfrak{D}(\mathfrak{A} \cup \mathfrak{B})$.
(5) $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A} \cap \mathfrak{B}) \subseteq \mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \cap \mathcal{NB}_c\mathfrak{D}(\mathfrak{B})$.
(6) $\mathcal{NB}_c\mathfrak{D}(\mathcal{NB}_c\mathfrak{D}(\mathfrak{A})) \setminus \mathfrak{A} \subseteq \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$.
(7) $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A} \cup \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})) \subseteq \mathfrak{A} \cup \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$.

Proof:

(1) Follows from that $\mathfrak{G} \cap \emptyset = \emptyset$ for each $\mathcal{NB}_c\mathcal{O}$-set .
(2) Straight forward
(3) Let $\mathfrak{A} \subseteq \mathfrak{B}$ and $\mathfrak{x} \in \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$, then for each $\mathcal{NB}_c\mathcal{O}$-set $\mathfrak{G}$ containing $\mathfrak{x}$, $\mathfrak{G} \cap (\mathfrak{A} \setminus \{\mathfrak{x}\}) \neq \emptyset$. Since $\mathfrak{A} \subseteq \mathfrak{B}$, $\mathfrak{G} \cap (\mathfrak{A} \setminus \{\mathfrak{x}\}) \subseteq \mathfrak{G} \cap (\mathfrak{B} \setminus \{\mathfrak{x}\})$ and Consequently $\mathfrak{B} \cap (\mathfrak{A} \setminus \{\mathfrak{x}\}) \neq \emptyset$.Hence $\mathfrak{x} \in \mathcal{NB}_c\mathfrak{D}(\mathfrak{B})$.As a result $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \subseteq \mathcal{NB}_c\mathfrak{D}(\mathfrak{B})$**.**
(4) Since $\mathfrak{A}$ and $\mathfrak{B}$ are subsets of $\mathfrak{A} \cup \mathfrak{B}$, then by part (3) we get the result.
(5) Since $\mathfrak{A} \cap \mathfrak{B}$ is subset of $\mathfrak{A}$and subset of $\mathfrak{B}$, then by part (3) we get the result.
(6) Let $\mathfrak{x} \in \mathcal{NB}_c\mathfrak{D}(\mathcal{NB}_c\mathfrak{D}(\mathfrak{A})) \setminus \mathfrak{A}$ and $\mathfrak{G}$ be any $\mathcal{NB}_c\mathcal{O}$-set containing $\mathfrak{x}$. Consequently $\mathfrak{G} \cap (\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \setminus \{\mathfrak{x}\}) \neq \emptyset$ and $\mathfrak{x} \notin \mathfrak{A}$. Let $\mathfrak{u} \in \mathfrak{G} \cap (\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \setminus \{\mathfrak{x}\})$. Then $\mathfrak{u} \in \mathfrak{G}$ and $\mathfrak{u} \in \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$ But $\mathfrak{u} \neq \mathfrak{x}$. Consequently, $\mathfrak{G} \cap (\mathfrak{A} \setminus \{\mathfrak{u}\}) \neq \emptyset$. Let $\mathfrak{z} \in \mathfrak{G} \cap (\mathfrak{A} \setminus \{\mathfrak{u}\})$. As a result $\mathfrak{z} \in \mathfrak{G}$ and $\mathfrak{z} \in \mathfrak{A}$ but $\mathfrak{z} \neq \mathfrak{u}$. Then $\mathfrak{z} \neq \mathfrak{x}$, since $\mathfrak{z} \in \mathfrak{A}$ but $\mathfrak{x} \notin \mathfrak{A}$. Consequently, $\mathfrak{z} \in \mathfrak{G} \cap (\mathfrak{A} \setminus \{\mathfrak{x}\})$. That is $\mathfrak{G} \cap (\mathfrak{A} \setminus \{\mathfrak{x}\}) \neq \emptyset$. As a result $\mathfrak{x} \in \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$ Consequently $\mathcal{NB}_c\mathfrak{D}(\mathcal{NB}_c\mathfrak{D}(\mathfrak{A})) \setminus \mathfrak{A} \subseteq \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$.
(7) Straight forward. □

In general, the equalities of (4) and (5) of the above theorem does not hold it is shown in the following examples.

Example 4.

Let's take $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$be $\mathcal{NTS}$ defined as in Example (3.1), let $\mathfrak{A} = \{\mathfrak{j}\}$ and $B = \{\mathfrak{l}\}$.Then $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) = \{\mathfrak{i}\}$, $\mathcal{NB}_c\mathfrak{D}(\mathfrak{B}) = \emptyset$ and
$\mathcal{NB}_c\mathfrak{D}(\mathfrak{A} \cup \mathfrak{B}) = \{\mathfrak{i}, \mathfrak{k}\}$ where $\mathfrak{A} \cup \mathfrak{B} = \{\mathfrak{j}, \mathfrak{l}\}$. Consequently $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A} \cup \mathfrak{B}) \nsubseteq$
$\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \cup \mathcal{NB}_c\mathfrak{D}(\mathfrak{B})$

Example 4.2

Let's take $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$ be $\mathcal{NTS}$ described as in the example (3.1), let $\mathfrak{A} = \{\mathfrak{i}, \mathfrak{k}\}$ as well $\mathfrak{B} = \{\mathfrak{j}, \mathfrak{l}\}$. Afterwards, $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) = \{\mathfrak{i}, \mathfrak{j}, \mathfrak{l}\}$, $\mathcal{NB}_c\mathfrak{D}(\mathfrak{B}) = \{\mathfrak{i}, \mathfrak{k}\}$, $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A} \cap \mathfrak{B}) = \emptyset$,where $\mathfrak{A} \cap \mathfrak{B} = \emptyset$ as well $(\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \cap \mathcal{NB}_c\mathfrak{D}(\mathfrak{B})) = \{\mathfrak{i}\}$. Consequently $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \cap \mathcal{NB}_c\mathfrak{D}(\mathfrak{B}) \nsubseteq \mathcal{NB}_c\mathfrak{D}(\mathfrak{A} \cap \mathfrak{B})$.

Corollary 4.1

Let's take $\mathfrak{A}$ be any subset of $\mathcal{V}$ in $\mathcal{NTS}$ $(\mathcal{V}, \mathbb{T}_\Omega(\mathcal{Y}))$, Afterwards $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \subseteq \mathcal{NB}\mathfrak{D}(\mathfrak{A})$.

Proof**:** Remembering that every $\mathcal{NB}_c\mathcal{O}$-set is a $\mathcal{NBO}$-set is enough. □

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Proposition 4.2

Let's take $(\mathcal{V}, \mathbb{T}_{\Omega}(\mathcal{Y}))$ be a $\mathcal{NTS}$ and $\mathfrak{A} \subseteq \mathcal{V}$, afterwards $\mathfrak{A}$ is a $\mathcal{NB}_c\mathcal{C}$-set if and solely if $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \subseteq \mathfrak{A}$.

Proof**:** Considering that $\mathfrak{A}$ is a $\mathcal{NB}_c\mathcal{C}$-set as well that $\mathfrak{x}$ is a $\mathcal{NB}_c\mathcal{L}$- point of $\mathfrak{A}$ that belonging to $\mathcal{V}\backslash\mathfrak{A}$, we could show that $\mathcal{V}\backslash\mathfrak{A}$ is a $\mathcal{NB}_c\mathcal{O}$-set that includes the $\mathcal{NB}_c\mathcal{L}$-point of $\mathfrak{A}$. This gives rise to the contradiction $\mathfrak{A} \cap \mathcal{V}\backslash\mathfrak{A} \neq \emptyset$.

Conversely, Assume $\mathfrak{A}$ contains the set of it is $\mathcal{NB}_c\mathcal{L}$-points. For any of the $\mathfrak{x} \in \mathcal{V}\backslash\mathfrak{A}$, there's a $\mathcal{NB}_c\mathcal{O}$-set $\mathfrak{G}$ including $\mathfrak{x}$ With this in mind $\mathfrak{G} \cap \mathfrak{A} = \emptyset$. This means that the $\mathfrak{x} \in \mathfrak{G} \subseteq \mathcal{V}\backslash\mathfrak{A}$ by proposition (3.15) is a $\mathcal{NB}_c\mathcal{O}$-set, and so $\mathfrak{A}$ is a $\mathcal{NB}_c\mathcal{C}$- set. □

Definition 4.2

Suppose $(\mathcal{V}, \mathbb{T}_{\Omega}(\mathcal{Y}))$ be a $\mathcal{NTS}$ Regarding $\mathcal{Y}$, where $\mathcal{Y}$ is a subset of $\mathcal{V}$. If $\mathfrak{A}$ is a subset of $\mathcal{V}$, then:

- The $\mathcal{NB}_c$-interior of the set $\mathfrak{A}$, signified by $\mathcal{NB}_c\mathfrak{int}(\mathfrak{A})$, is the union of all $\mathcal{NB}_c\mathcal{O}$-sets that are included within $\mathfrak{A}$. In other words, $\mathcal{NB}_c\mathfrak{int}(\mathfrak{A})$ represents the biggest $\mathcal{NB}_c\mathcal{O}$-set encompassing $\mathfrak{A}$.
- The $\mathcal{NB}_c$-exterior point of the set $\mathfrak{A}$, signified by $\mathcal{NB}_c\mathfrak{ext}(\mathfrak{A})$, is the $\mathcal{NB}_c\mathfrak{int}$ of the complement of $\mathfrak{A}$ in other words is $\mathcal{NB}_c\mathfrak{int}(\mathfrak{A}^{\mathcal{C}}))$.
- The $\mathcal{NB}_c$-closure of the set $\mathfrak{A}$, signified by $\mathcal{NB}_c\mathfrak{cl}(\mathfrak{A})$ is The location where all $\mathcal{NB}_c\mathcal{C}$-sets that encompass $\mathfrak{A}$. This implies that $\mathcal{NB}_c\mathfrak{cl}(\mathfrak{A})$, is the smallest $\mathcal{NB}_c\mathcal{C}$-set that consists of $\mathfrak{A}$.

Proposition 4.3

presume $(\mathcal{V}, \mathbb{T}_{\Omega}(\mathcal{Y}))$ be a $\mathcal{NTS}$ and $\mathfrak{A} \subseteq \mathcal{V}$, afterward $\mathcal{NB}_c\mathfrak{cl}(\mathfrak{A}) = \mathfrak{A} \cup \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$.

Proof **:** Being that $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \subseteq \mathcal{NB}_c\mathfrak{cl}(\mathfrak{A})$, as well $\mathfrak{A} \subseteq \mathcal{NB}_c\mathfrak{cl}(\mathfrak{A})$, afterward $\mathfrak{A} \cup \mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \subseteq \mathcal{NB}_c\mathfrak{cl}(\mathfrak{A})$.

To demonstrate that $\mathcal{NB}_c\mathfrak{cl}(\mathfrak{A}), \subseteq \mathfrak{A} \cup \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$ Show us that $\mathfrak{A} \cup \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$ is a $\mathcal{NB}_c\mathcal{C}$-set. Consider $\mathfrak{x} \notin \mathfrak{A} \cup \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$. This suggests $\mathfrak{x} \notin \mathfrak{A}$ and $\mathfrak{x} \notin \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$. Given that $\mathfrak{x} \notin \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$, $\mathfrak{G}_{\mathfrak{x}} \subseteq \mathcal{V}\backslash\mathfrak{A}$ indicates that there is a $\mathcal{NB}_c\mathcal{O}$-set $\mathfrak{G}_{\mathfrak{x}}$of $\mathfrak{x}$ that contains no point of $\mathfrak{A}$. Once more, each point of $\mathfrak{G}_{\mathfrak{x}}$ is a $\mathcal{NB}_c\mathcal{O}$-set. However, no point of $\mathfrak{G}_{\mathfrak{x}}$ can be a $\mathcal{NB}_c\mathcal{L}$-point of $\mathfrak{A}$ because $\mathfrak{G}_{\mathfrak{x}}$ encompasses no points of $\mathfrak{A}$. Consequently, $\mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$ cannot include any point of $\mathfrak{G}_{\mathfrak{x}}$. This indicates that $\mathfrak{G}_{\mathfrak{x}} \subseteq \mathcal{V}\backslash\mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$. Hence, it follows that $\mathfrak{x} \in \mathfrak{G}_{\mathfrak{x}} \subseteq (\mathcal{V}\backslash\mathfrak{A}) \cap (\mathcal{V}\backslash\mathcal{NB}_c\mathfrak{D}(\mathfrak{A}) \subseteq \mathcal{V}\backslash(\mathfrak{A} \cup \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$. Therefore, $\mathfrak{A} \cup \mathfrak{N}\mathcal{B}_c\mathfrak{D}(\mathfrak{A})$ is $\mathcal{NB}_c\mathcal{C}$-set. Hence $\mathcal{NB}_c\mathfrak{cl}(\mathfrak{A}), \subseteq \mathfrak{A} \cup \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$. Thus $\mathcal{NB}_c\mathfrak{cl}(\mathfrak{A}) = \mathfrak{A} \cup \mathcal{NB}_c\mathfrak{D}(\mathfrak{A})$. □

Definition 4.3

Let's take $(\mathcal{V}, \mathbb{T}_{\Omega}(\mathcal{Y}))$ be a $\mathcal{NTS}$ and $\mathfrak{A} \subseteq \mathcal{V}$. A point $\mathfrak{x} \in \mathcal{V}$ is referred to as a $\mathcal{NB}_c$-boundary point of $\mathfrak{A}$ if and only if every $\mathcal{NB}_c\mathcal{O}$-set in $\mathcal{V}$ encompasses $\mathfrak{x}$ consists of at minimum one point

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of $\mathfrak{A}$, as well at minimum one point of $\mathfrak{A}^{\mathcal{C}}$.Where $\mathfrak{A}^{\mathcal{C}}$ is complement of $\mathfrak{A}$. The set of all $\mathcal{NB}_c$-boundary point of $\mathfrak{A}$ is called the $\mathcal{NB}_c$-boundary point of $\mathfrak{A}$ and is signified by $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A})$.

Remarks 4.2

1) If $\{\mathfrak{i}\} \in \mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y})$ in any $\mathcal{NTS}$ $(\mathcal{V},\mathbb{T}_\Omega(\mathcal{Y}))$, $\mathfrak{i} \in \mathcal{V}$, then $\mathfrak{i}$ is not boundary point for any subset $\mathfrak{A}$ in $\mathcal{V}$ since if $\mathfrak{i} \in \mathfrak{A}$, afterward $\{\mathfrak{i}\}\cap\mathfrak{A}^{\mathcal{C}} = \emptyset$ and if $\mathfrak{i} \notin \mathfrak{A}$, afterward $\{\mathfrak{i}\}\cap\mathfrak{A} = \emptyset$, so in this two case $\mathfrak{i} \notin \mathcal{NB}_c\mathfrak{B}(\mathfrak{A})$.

2) If $\mathcal{NB}_c\mathcal{O}(\mathcal{V},\mathcal{Y}) = \{\mathcal{V},\emptyset\}$, $\emptyset \neq \mathfrak{A} \subseteq \mathcal{V}$, afterward $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A}) = \mathcal{V}$ because the only $\mathcal{NB}_c\mathcal{O}$-set is $\mathcal{V}$ for every element in $\mathcal{V}$ and $\mathcal{V}\cap\mathfrak{A} \neq \emptyset$ and $\mathcal{V}\cap\mathfrak{A}^{\mathcal{C}} \neq \emptyset$.

3) Notes that: $\mathcal{V} = \mathcal{NB}_c\mathfrak{int}(\mathfrak{A})\cup\mathcal{NB}_c\mathfrak{ext}(\mathfrak{A})\cup\mathcal{NB}_c\mathfrak{B}(\mathfrak{A})$ as well
$\emptyset = \mathcal{NB}_c\mathfrak{int}(\mathfrak{A})\cap\mathcal{NB}_c\mathfrak{ext}(\mathfrak{A})\cap\mathcal{NB}_c\mathfrak{B}(\mathfrak{A})$, This implies that the sets $\mathcal{NB}_c\mathfrak{int}(\mathfrak{A})$, $\mathcal{NB}_c\mathfrak{ext}(\mathfrak{A})$ as well $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A})$ being a partition for $\mathcal{V}$, also if $\mathfrak{x} \in \mathcal{V}$, afterward $\mathfrak{x} \in \mathcal{NB}_c\mathfrak{int}(\mathfrak{A})$ or $\mathfrak{x} \in \mathcal{NB}_c\mathfrak{ext}(\mathfrak{A})$ or $\mathfrak{x} \in \mathcal{NB}_c\mathfrak{B}(\mathfrak{A})$.

4) The set $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A})$ is $\mathcal{NB}_c\mathcal{C}$-set Being that $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A}) = \mathcal{V}\setminus\mathcal{NB}_c\mathfrak{int}(\mathfrak{A}) \cup\mathcal{NB}_c\mathfrak{ext}(\mathfrak{A})$ and we know that the set $\mathcal{NB}_c\mathfrak{int}(\mathfrak{A})$ as well $\mathcal{NB}_c\mathfrak{ext}(\mathfrak{A})$ are $\mathcal{NB}_c\mathcal{O}$-sets, consequently $\mathcal{NB}_c\mathfrak{int}(\mathfrak{A})\cup\mathcal{NB}_c\mathfrak{ext}(\mathfrak{A})$ is $\mathcal{NB}_c\mathcal{O}$-set, so
$\mathcal{V}\setminus(\mathcal{NB}_c\mathfrak{int}(\mathfrak{A}))\cup\mathcal{NB}_c\mathfrak{ext}(\mathfrak{A}))$ is $\mathcal{NB}_c\mathcal{C}$-set, hence $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A})$is $\mathcal{NB}_c\mathcal{C}$-set

Theorem 4.1

Suppose $\mathfrak{A}$ be a subset of $\mathcal{V}$ in $\mathcal{NTS}$ $(\mathcal{V},\mathbb{T}_\Omega(\mathcal{Y}))$. Afterward,

(1) $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A}) = \mathcal{NB}_c\mathfrak{B}(\mathfrak{A}^{\mathcal{C}})$.
(2) $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A}) = \mathcal{NB}_c\mathfrak{cl}(\mathfrak{A})\setminus\mathcal{NB}_c\mathfrak{int}(\mathfrak{A})$.
(3) $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A}) = \mathcal{NB}_c\mathfrak{cl}(\mathfrak{A})\cap\mathcal{NB}_c\mathfrak{cl}(\mathfrak{A}^{\mathcal{C}})$.
(4) $\mathfrak{A}$ is $\mathcal{NB}_c\mathcal{O}$-set if and only if $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A}) \subseteq \mathfrak{A}^{\mathcal{C}}$.
(5) $\mathfrak{A}$ is $\mathcal{NB}_c\mathcal{C}$-set if and only if $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A}) \subseteq \mathfrak{A}$.
(6) $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A}) = \emptyset$ if and only if $\mathfrak{A}$ is $\mathcal{NB}_c$ -clopen set .

Proof:

(1) Let $\mathfrak{x} \in \mathcal{NB}_c\mathfrak{B}(\mathfrak{A})$,then by definition (4.3) for each $\mathcal{NB}_c\mathcal{O}$-set $\mathfrak{G}$ in $\mathcal{V}$ encompasses $\mathfrak{x}$, $\mathfrak{G}\cap\mathfrak{A} \neq \emptyset$ and $\mathfrak{G}\cap\mathfrak{A}^{\mathcal{C}} \neq \emptyset$,then $\mathfrak{G}\cap(\mathfrak{A}^{\mathcal{C}})^{\mathcal{C}} \neq \emptyset$ and $\mathfrak{G}\cap\mathfrak{A}^{\mathcal{C}} \neq \emptyset$. Consequently $\mathfrak{x} \in \mathcal{NB}_c\mathfrak{B}(\mathfrak{A}^{\mathcal{C}})$, as a result $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A}) \subseteq \mathcal{NB}_c\mathfrak{B}(\mathfrak{A}^{\mathcal{C}})$. In the same way we can prove $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A}^{\mathcal{C}}) \subseteq \mathcal{NB}_c\mathfrak{B}$, so $\mathcal{NB}_c\mathfrak{B}(\mathfrak{A}) = \mathcal{NB}_c\mathfrak{B}(\mathfrak{A}^{\mathcal{C}})$

Proof : (2,3,4,5and 6) Straight forward . □

5 CONCLUSION

Finally, we have introduced and investigated $\mathcal{NB}_c\mathcal{O}$-sets, a novel class of $\mathcal{NBO}$-sets in $\mathcal{NTS}$. The properties and relationships of this new class with existing types of $\mathcal{NO}$-sets were examined in detail. Our results show that this novel idea can lead to new advances in $\mathcal{NTS}$ and can offer a deeper understanding of the structure of $\mathcal{NTS}$. Future work may explore further applications or generalization within other Nano topological or algebraic structure.

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Conflict of interests.

There are non-conflicts of interest.

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