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A New Neutrosophic Sets in Neutrosophic Topology

Asia I. Saeed[1*], Hardi A. SHareef [2]

[1] College of Science, University of Sulaimani, asiaibraim080@gmail.com, Sulaimania, Iraq.
[2] College of Science, University of Sulaimani, hardy.shareef@univsul.edu.iq, Sulaimania, Iraq.
***Corresponding author email:** hardy.shareef@univsul.edu.iq; mobile: 07701543561

مجموعة نيوتروسوفيكية جديدة في الطوبولوجيا النيتروسوفية

اسيا ابراهيم سعيد[1*]، هردي علي شريف[2]

1 كلية العلوم ، جامعة السليمانية، asiaibraim080@gmail.com، السليمانية ، العراق
2 كلية العلوم، جامعة السليمانية، hardy.shareef@univsul.edu.iq، السليمانية ، العراق

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ABSTRACT

The neutrosophic sets approach was proposed by Smarandache. Uncertain data is handled using neutrosophic sets. Within this article , we introduce a novel forms of somewhat open set within neutrosophic topological space also we investigate several their basic characteristics and examples within neutrosophic topological space. Further neutrosophic somewhat interior, neutrosophic somewhat closure, neutrosophic somewhat derived set, neutrosophic somewhat frontier operators are discussed. Also, the notion of neutrosophic somewhat functions and somewhat irresolute functions were introduced in neutrosophic topological spaces.

Keywords: Neutrosophic S_w-open set, Neutrosophic S_w-interior, NeutrosophicS_w-closure and Neutrosophic S_w-frontier.

1 INTRODUCTION

Fuzzy sets and fuzzy logic were initially suggested by L. Zadeh in 1965. That serves as a crucial idea for managing the unknown in daily existence, since every component have an associated role [9]. An extension of the theory of fuzzy sets, fuzzy sets with intuition were introduced by Attanassov in 1986 [11]. Intuitionistic sets of fuzzy elements are defined by each element's member function and non-membership function. In contrast, we must deal with incompatibility and uncertainty in everyday existence. Within this situation, Smarandache used neutrosophic theory of sets to deal with current issues in real-life situations. Smarandache's neutrosophic theory of sets was centred on the sciences of society, engineering, medicine care, and other areas [10, 12]. Members, uncertainty, and non-membership functions are characteristics of neutrosophic groups [13, 8]. In 2012, Salama and Alblowi defined neutrosophic topological spaces using neutrosophic sets [3]. Additional studies were conducted to look at the distinct characteristics of set neutrosophic within other domains [2 – 6, 15]. Velico [16] introduced a novel class of a topological space set known as δ-open set. In 1987 Popa [19] introduced the concept of somewhat preopen (or somewhat nearly open) set. Crossely and Hilderband proposed the idea of irresolute function in1972 [20]. Baker [17] studied the notion of somewhat open functions in 1996. This notation was introduced by Centry, Karel and Hoyle in 1971. On the other hand Sarak in 2006 studied the notation of somewhat continuous functions [18]. This concept was also introduced by Frolik in 1961. The purpose of somewhat open set in the area of neutrosophic topological space is discussed throughout the present research. We generated ideas and proposed the novel class of set called neutrosophic somewhat open set exercised with theorems and appropriate examples.

2 PRELIMINARIES

Definition 2.1 [2] Suppose that Ӽ is an exact set that is not empty. A variety of neutrosophic set Á is an item with the shape $\text{Á} = \{\langle \text{ӽ}, \text{ϫ}_{\text{Á}}(\text{ӽ}), \lambda_{\text{Á}}(\text{ӽ}), \text{ɤ}_{\text{Á}}(\text{ӽ})\rangle : \text{ӽ} \in \text{Ӽ}\}$, when the qualifications of membership , uncertainty , and non-membership are presented by $(\text{ϫ}_{\text{Á}}(\text{ӽ}), \lambda_{\text{Á}}(\text{ӽ}), \text{ɤ}_{\text{Á}}(\text{ӽ}))$, respectively. $\mathcal{N}(\text{Ӽ})$ will represent a category that contains every neutrosophic sets in Ӽ. The total values $\text{ϫ}_{\text{Á}}(\text{ӽ})$, $\text{ɤ}_{\text{Á}}(\text{ӽ})$, and $\lambda_{\text{Á}}(\text{ӽ})$ is unrestricted. So $^{-}0 \leq \text{ϫ}_{\text{Á}}(\text{ӽ}) + \lambda_{\text{Á}}(\text{ӽ}) + \text{ɤ}_{\text{Á}}(\text{ӽ}) \leq 3^{+}$, for each $\text{ӽ} \in \text{Ӽ}$.

Remark 2.1 [3] An organised triplet $\langle \text{ϫ}_{\text{Á}}, \lambda_{\text{Á}}, \text{ɤ}_{\text{Á}}\rangle$ in $]^{-}0, 1^{+}[$ on Ӽ could be recognised as a neutrosophic set $\text{Á} = \{\langle \text{ӽ}, \text{ϫ}_{\text{Á}}(\text{ӽ}), \lambda_{\text{Á}}(\text{ӽ}), \text{ɤ}_{\text{Á}}(\text{ӽ})\rangle : \text{ӽ} \in \text{Ӽ}\}$.

Definition 2.2 [3] Let $\text{Á} = \langle \text{ӽ}, \text{ϫ}_{\text{Á}}, \lambda_{\text{Á}}, \text{ɤ}_{\text{Á}}\rangle$ be a $\mathcal{NS}$ of Ӽ. Then the completeness of neutrosophic set Á, represented as Á^{c}, and it is laid out by three types of complements.

C_1. $\text{Á}^{c} = \{\langle \text{ӽ}, \text{ɤ}_{\text{Á}}(\text{ӽ}), 1 - \lambda_{\text{Á}}(\text{ӽ}), \text{ϫ}_{\text{Á}}(\text{ӽ})\rangle : \text{ӽ} \in \text{Ӽ}\}$
C_2. $\text{Á}^{c} = \{\langle \text{ӽ}, \text{ɤ}_{\text{Á}}(\text{ӽ}), \lambda_{\text{Á}}(\text{ӽ}), \text{ϫ}_{\text{Á}}(\text{ӽ})\rangle : \text{ӽ} \in \text{Ӽ}\}$
C_3. $\text{Á}^{c} = \{\langle \text{ӽ}, 1 - \text{ϫ}_{\text{Á}}(\text{ӽ}), 1 - \lambda_{\text{Á}}(\text{ӽ}), 1 - \text{ɤ}_{\text{Á}}\rangle : \text{ӽ} \in \text{Ӽ}\}$.

Definition 2.3 [3] The following is a representation of two neutrosophic sets $1_{\mathcal{N}}$ and $0_{\mathcal{N}}$ in Ӽ:
$(1_1) 1_{\mathcal{N}} = \{\langle \text{ӽ}, 1,0,0\rangle : \text{ӽ} \in \text{Ӽ}\}$
$(1_2) 1_{\mathcal{N}} = \{\langle \text{ӽ}, 1,0,1\rangle : \text{ӽ} \in \text{Ӽ}\}$
$(1_3) 1_{\mathcal{N}} = \{\langle \text{ӽ}, 1,1,0\rangle : \text{ӽ} \in \text{Ӽ}\}$
$(1_4) 1_{\mathcal{N}} = \{\langle \text{ӽ}, 1,1,1\rangle : \text{ӽ} \in \text{Ӽ}\}$

$(0_1)\ 0_{\mathcal{N}} = \{\langle \text{ӽ}, 0,0,1\rangle : \text{ӽ} \in \text{Ӽ}\}$
$(0_2)\ 0_{\mathcal{N}} = \{\langle \text{ӽ}, 0,1,1\rangle : \text{ӽ} \in \text{Ӽ}\}$
$(0_3)\ 0_{\mathcal{N}} = \{\langle \text{ӽ}, 0,1,0\rangle : \text{ӽ} \in \text{Ӽ}\}$

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$(0_4)\ 0_{\mathcal{N}} = \{\langle \text{ӽ}, 0,0,0\rangle : \text{ӽ} \in \text{Ӽ}\}$.

Definition 2.4 [3] $\acute{A} \cap \acute{C}$ is characterized as follows if $\acute{A}$ and $\acute{C}$ are any pair of neutrosophic sets on Ӽ:

i. $\acute{A} \cap \acute{C} = \langle \text{ӽ}, \text{ꭓ}_{\acute{A}}(\text{ӽ}) \wedge \text{ꭓ}_{\acute{C}}(\text{ӽ}), \lambda_{\acute{A}}(\text{ӽ}) \vee \lambda_{\acute{C}}(\text{ӽ})\ \&\ \text{ɤ}_{\acute{A}}(\text{ӽ}) \vee \text{ɤ}_{\acute{C}}(\text{ӽ})\rangle$
ii. $\acute{A} \cap \acute{C} = \langle \text{ӽ}, \text{ꭓ}_{\acute{A}}(\text{ӽ}) \wedge \text{ꭓ}_{\acute{C}}(\text{ӽ}), \lambda_{\acute{A}}(\text{ӽ}) \wedge \lambda_{\acute{C}}(\text{ӽ})\ \&\ \text{ɤ}_{\acute{A}}(\text{ӽ}) \vee \text{ɤ}_{\acute{C}}(\text{ӽ})\rangle$.

Definition 2.5 [3] $\acute{A} \cup \acute{C}$ is characterized as follows if $\acute{A}$ and $\acute{C}$ are any pair of neutrosophic sets on Ӽ:

i. $\acute{A} \cup \acute{C} = \langle \text{ӽ}, \text{ꭓ}_{\acute{A}}(\text{ӽ}) \vee \text{ꭓ}_{\acute{C}}(\text{ӽ}), \lambda_{\acute{A}}(\text{ӽ}) \wedge \lambda_{\acute{C}}(\text{ӽ})\ \&\ \text{ɤ}_{\acute{A}}(\text{ӽ}) \wedge \text{ɤ}_{\acute{C}}(\text{ӽ})\rangle$
ii. $\acute{A} \cup \acute{C} = \langle \text{ӽ}, \text{ꭓ}_{\acute{A}}(\text{ӽ}) \vee \text{ꭓ}_{\acute{C}}(\text{ӽ}), \lambda_{\acute{A}}(\text{ӽ}) \vee \lambda_{\acute{C}}(\text{ӽ})\ \&\ \text{ɤ}_{\acute{A}}(\text{ӽ}) \wedge \text{ɤ}_{\acute{C}}(\text{ӽ})\rangle$.

Definition 2.6 [3] $\acute{A} \subseteq \acute{C}$ is characterized as follows if $\acute{A}$ and $\acute{C}$ are any pair of neutrosophic sets on Ӽ:

i. $\acute{A} \subseteq \acute{C}$ if and only if $\text{ꭓ}_{\acute{A}}(\text{ӽ}) \leq \text{ꭓ}_{\acute{C}}(\text{ӽ}), \lambda_{\acute{A}}(\text{ӽ}) \geq \lambda_{\acute{C}}(\text{ӽ})\ \&\ \text{ɤ}_{\acute{A}}(\text{ӽ}) \geq \text{ɤ}_{\acute{C}}(\text{ӽ})\ \forall\ \text{ӽ} \in \text{Ӽ}$
ii. $\acute{A} \subseteq \acute{C}$ if and only if $\text{ꭓ}_{\acute{A}}(\text{ӽ}) \leq \text{ꭓ}_{\acute{C}}(\text{ӽ}), \lambda_{\acute{A}}(\text{ӽ}) \leq \lambda_{\acute{C}}(\text{ӽ})\ \&\ \text{ɤ}_{\acute{A}}(\text{ӽ}) \geq \text{ɤ}_{\acute{C}}(\text{ӽ})\ \forall\ \text{ӽ} \in \text{Ӽ}$.

Definition 2.8 [4] Let Đ & Ƙ be any neutrosophic sets in Ӽ, then the following are exists:
i. $(\text{Đ})^{\text{ç}} \cup (\text{Ƙ})^{\text{ç}} = (\text{Đ} \cap \text{Ƙ})^{\text{ç}}$

ii. $(\text{Đ})^{\text{ç}} \cap (\text{Ƙ})^{\text{ç}} = (\text{Đ} \cup \text{Ƙ})^{\text{ç}}$.

Definition 2.7 [14] Let $\{\acute{A}_i : i \in I$ and $\acute{A}_i$ are $\mathcal{NS}$ in $\text{Ӽ}\}$ be any arbitrary collection, therefore:

1. $\cup\, \acute{A}_i$ could be characterized in the following way:
i. $\cup\, \acute{A}_i = \left(\vee_{i\in I}\, \text{ꭓ}_{\acute{A}_i}(\text{ӽ}),\ \wedge_{i\in I}\, \lambda_{\acute{A}_i}(\text{ӽ}),\ \wedge_{i\in I}\, \text{ɤ}_{\acute{A}_i}(\text{ӽ})\right)$
ii. $\cup\, \acute{A}_i = \left(\vee_{i\in I}\, \text{ꭓ}_{\acute{A}_i}(\text{ӽ}),\ \vee_{i\in I}\, \lambda_{\acute{A}_i}(\text{ӽ}),\ \wedge_{i\in I}\, \text{ɤ}_{\acute{A}_i}(\text{ӽ})\right)$.
2. $\cap\, \acute{A}_i$ could be characterized in the following way:
i. $\cap\, \acute{A}_i = \left(\wedge_{i\in I}\, \text{ꭓ}_{\acute{A}_i}(\text{ӽ}),\ \vee_{i\in I}\, \lambda_{\acute{A}_i}(\text{ӽ}),\ \vee_{i\in I}\, \text{ɤ}_{\acute{A}_i}(\text{ӽ})\right)$
ii. $\cap\, \acute{A}_i = \left(\wedge_{i\in I}\, \text{ꭓ}_{\acute{A}_i}(\text{ӽ}),\ \wedge_{i\in I}\, \lambda_{\acute{A}_i}(\text{ӽ}),\ \vee_{i\in I}\, \text{ɤ}_{\acute{A}_i}(\text{ӽ})\right)$.

Definition 2.9 [3] Suppose that $\mathcal{T}_{\mathcal{N}}$ is the collection of neutrosophic subset of Ӽ. A $\mathcal{T}_{\mathcal{N}}$ is said to be a neutrosophic topology ($\mathcal{NT}$) when:

1. $0_{\mathcal{N}}$ and $1_{\mathcal{N}}$ belong to $\mathcal{T}_{\mathcal{N}}$
2. For every two $\mathcal{NS}$'s $\acute{A}_1, \acute{A}_2 \in \mathcal{T}_{\mathcal{N}}$, then $\acute{A}_1 \cap \acute{A}_2$ belong to $\mathcal{T}_{\mathcal{N}}$
3. For every family of $\mathcal{NS}$'s $\{\acute{A}_i : i \in I\} \subseteq \mathcal{T}_{\mathcal{N}}$, then $\cup\, \acute{A}_i$ belong to $\mathcal{T}_{\mathcal{N}}$.

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Thus the neutrosophic topological space ($\mathcal{NTS}$) over $Ӽ$ is represented by ($Ӽ,\mathcal{T}_{\mathcal{N}}$). We refer to the components of $\mathcal{T}_{\mathcal{N}}$ as neutrosophic open ($\mathcal{NO}$) sets. And a neutrosophic set Á is closed ($\mathcal{NC}$) if the complement $Á^c$ is a neutrosophic open.

Definition 2.10 [1] The neutrosophic interior and closure of a $\mathcal{NS}$ Á in a $\mathcal{NTS}$ ($Ӽ,\mathcal{T}_{\mathcal{N}}$) are determined as:

$\mathcal{N}int(Á) = \bigcup_{K \subseteq Á} K$, where K is a $\mathcal{NOS}$ in $Ӽ$
$\mathcal{N}cl(Á) = \bigcap_{Á \subseteq K} K$, where K is a $\mathcal{NCS}$ in $Ӽ$.

Definition 2.11 [6] Suppose that Á is a $\mathcal{NS}$ on $Ӽ$, and ($Ӽ,\mathcal{T}_{\mathcal{N}}$) is a $\mathcal{NTS}$ on $Ӽ$. Then a $\mathcal{NS}$ Á is neutrosophic pre-open set ($\mathcal{NPOS}$), neutrosophic semi-open set ($\mathcal{NSOS}$), neutrosophic regular-open set ($\mathcal{NROS}$), neutrosophic δ-open set ($\mathcal{N}\delta\mathcal{OS}$), neutrosophic α-open set ($\mathcal{N}\alpha\mathcal{OS}$), neutrosophic β-open set ($\mathcal{N}\beta\mathcal{OS}$) if and only if $Á \subseteq \mathcal{N}int(\mathcal{N}cl(Á))$, $Á \subseteq \mathcal{N}cl(\mathcal{N}int(Á)$, $Á = \mathcal{N}int(\mathcal{N}cl(Á))$, $Á = \mathcal{N}\delta int(Á) = \bigcup_{K \subseteq Á} K$,where K is a $\mathcal{NROS}$ in $Ӽ$. [5] $Á \subseteq \mathcal{N}int(\mathcal{N}cl(\mathcal{N}int(Á)))$, $Á \subseteq \mathcal{N}cl(\mathcal{N}int(\mathcal{N}cl(Á))))$, respectively.

And the complement of a $\mathcal{NPOS}$, $\mathcal{NSOS}$, $\mathcal{N}\alpha\mathcal{OS}$, $\mathcal{NROS}$, $\mathcal{N}\delta\mathcal{OS}$ & $\mathcal{N}\beta\mathcal{OS}$ is considered as a neutrosophic pre-closed set, semi-closed set, α-closed set, regular-closed set, δ -closed set & β-closed set ($\mathcal{NPCS}$, $\mathcal{NSCS}$, $\mathcal{N}\alpha\mathcal{CS}$, $\mathcal{NRCS}$, $\mathcal{N}\delta\mathcal{CS}$ & $\mathcal{N}\beta\mathcal{CS}$, for short) in $Ӽ$, respectively.

The Family members of each $\mathcal{NPOS}$, $\mathcal{NPCS}$, $\mathcal{NSOS}$, $\mathcal{NSCS}$, $\mathcal{N}\alpha\mathcal{OS}$, $\mathcal{N}\alpha\mathcal{CS}$, $\mathcal{NROS}$, $\mathcal{NRCS}$, $\mathcal{N}\delta\mathcal{OS}$, $\mathcal{N}\delta\mathcal{CS}$, $\mathcal{N}\beta\mathcal{OS}$ & $\mathcal{N}\beta\mathcal{CS}$ of $Ӽ$, is indicated as $\mathcal{NPOS}(Ӽ)$, $\mathcal{NPCS}(Ӽ)$, $\mathcal{NSOS}(Ӽ)$, $\mathcal{NSCS}(Ӽ)$, $\mathcal{N}\alpha\mathcal{OS}(Ӽ)$, $\mathcal{N}\alpha\mathcal{CS}(Ӽ)$, $\mathcal{NROS}(Ӽ)$, $\mathcal{NRCS}(Ӽ)$, $\mathcal{N}\delta\mathcal{OS}(Ӽ)$, $\mathcal{N}\delta\mathcal{CS}(Ӽ)$, $\mathcal{N}\beta\mathcal{OS}(Ӽ)$ & $\mathcal{N}\beta\mathcal{CS}(Ӽ)$, respectively.

Definition 2.12 [7] Suppose that $Ӽ$ is a set that is not empty. The neutrosophic set $P = \{\langle ӽ,\ \varkappa_P(ӽ),\ \lambda_P(ӽ),\ ɤ_P(ӽ)\rangle : ӽ \in Ӽ\}$ is referred to as a neutrosophic point ($\mathcal{N}P$, for short) in $Ӽ$, if and only if for each y belong to $Ӽ$, and $ȴ, ȶ, ȵ$ are truly standards or nonstandard subset of $]^-0, 1^+[$, $\varkappa_P(y) = 0$, $\lambda_P(y) = 1$, $ɤ_P(y) = 1$ for $ӽ \neq y$, and $\varkappa_P(y) = ȴ$, $\lambda_P(y) = ȶ$, $ɤ_P(y) = ȵ$ for $ӽ = y$. Where $0 < ȴ \leq 1, 0 \leq ȶ < 1, 0 \leq ȵ < 1$, the degree of membership value is $ȴ$, the degree of uncertainty value is $ȶ$ and the degree of non-membership value $ȵ$. A neutrosophic point will denoted as $ӽ_{ȴ,ȶ,ȵ}$ or $P = \langle ӽ, ȴ, ȶ, ȵ\rangle$. And $ӽ \in Ӽ$ is referred to as the support of $ӽ_{ȴ,ȶ,ȵ}$,

Definition 2.13 [7] Suppose that $Ӽ$ is any set that is not empty. A neutrosophic set $Á = \{\langle ӽ, \varkappa_{Á}(ӽ), \lambda_{Á}(ӽ), ɤ_{Á}(ӽ)\rangle : ӽ \in Ӽ\}$ ($P \in Á$) will be considered to include a neutrosophic point $P = \langle ӽ, ȴ, ȶ, ȵ\rangle$ specified on $Ӽ$, if $ȴ \leq \varkappa_{Á}(ӽ)$, $ȶ \geq \lambda_{Á}(ӽ)$ and $ȵ \geq ɤ_{Á}(ӽ)$.

Definition 2.14 [4] Let ($Ӽ,\mathcal{T}_{\mathcal{N}}$) and ($\mathcal{Y},\sigma_{\mathcal{N}}$) be two $\mathcal{NTS}$'s. If the inverse image of every neutrosophic open set in ($\mathcal{Y},\sigma_{\mathcal{N}}$) is a neutrosophic open set in ($Ӽ,\mathcal{T}_{\mathcal{N}}$), then the function

$f:(Ӽ,\mathcal{T}_{\mathcal{N}})\to(\mathcal{Y},\sigma_{\mathcal{N}})$ is said to be a neutrosophic continuous ($\mathcal{N}$-continuous, for short) function.

Definition 8.1.2 [21] Suppose that $P=\langle ӽ, ᶅ, ƫ, ᶇ\rangle$ is a neutrosophic point of a $(Ӽ,\mathcal{T}_{\mathcal{N}})$. A neutrosophic set Á of Ӽ is said to be a neutrosophic neighborhood of $P=\langle ӽ, ᶅ, ƫ, ᶇ\rangle$ if there exist a neutrosophic open set Đ such that $\langle ӽ, ᶅ, ƫ, ᶇ\rangle\in Đ\subseteq Á$.

3 NEUTROSOPHIC SOMEWHAT OPEN SET

The notion of a neutrosophic somewhat open set we introduces in this section and as well as it is characteristics are described.

Definition 3.1 In $\mathcal{N}TS$ $(Ӽ,\mathcal{T}_{\mathcal{N}})$ a $\mathcal{N}S$ Á with $0_{\mathcal{N}}$ is said to be a neutrosophic somewhat open set ($\mathcal{N}S_wOS$, for short) if $\mathcal{N}int(Á)\neq 0_{\mathcal{N}}$. And the family of all $\mathcal{N}S_wOS$'s in Ӽ is represented as $\mathcal{N}S_wOS(Ӽ)$ or $\mathcal{N}S_wOS(Ӽ,\mathcal{T}_{\mathcal{N}})$. A neutrosophic S_w-closed set is the complement of a neutrosophic S_w-open set, and $\mathcal{N}S_wCS(Ӽ,\mathcal{T}_{\mathcal{N}})$ or $\mathcal{N}S_wCS(Ӽ)$ is a collection that includes all neutrosophic S_w-closed sets in Ӽ.

Example 3.1 Suppose that $Ӽ=\{l,m,n\}$, and describe NS's $Á_1$ & $Á_2$ in Ӽ as follows:

$$Á_1=\langle(0.3,0.5,07),(0.4,0.5,0.8),(0.5,0.5,0.7)\rangle$$
$$Á_2=\langle(0.2,0.5,0.9),(0.2,0.5,0.9),(0.4,0.5,0.7)\rangle$$

Were $\mathcal{T}_{\mathcal{N}}=\{0_{\mathcal{N}},Á_1,Á_2,1_{\mathcal{N}}\}$, then $(Ӽ,\mathcal{T}_{\mathcal{N}})$ is a $\mathcal{N}TS$, now $0_{\mathcal{N}},Á_1,Á_2$ and $1_{\mathcal{N}}$ are $\mathcal{N}S_wOS'$s.

Proposition 3.1 Let $(Ӽ,\mathcal{T}_{\mathcal{N}})$ be a neutrosophic topological space, then the following statements are true.

1. Any $\mathcal{N}OS$ is a $\mathcal{N}S_wOS$, and any $\mathcal{N}CS$ is a $\mathcal{N}S_wCS$.
2. Any $\mathcal{N}ROS$ is a $\mathcal{N}S_wOS$, and any $\mathcal{N}RCS$ is a $\mathcal{N}S_wCS$.
3. Any $\mathcal{N}SOS$ is a $\mathcal{N}S_wOS$, and any $\mathcal{N}SCS$ is a $\mathcal{N}S_wCS$.
4. Any $\mathcal{N}\alpha OS$ is a $\mathcal{N}S_wOS$, and any $\mathcal{N}\alpha CS$ is a $\mathcal{N}S_wCS$.
5. Any $\mathcal{N}\delta OS$ is a $\mathcal{N}S_wOS$, and any $\mathcal{N}\delta CS$ is a $\mathcal{N}S_wCS$.

Proof.

1. Let Á be a $\mathcal{N}OS$, then $Á=\mathcal{N}int(Á)$. Therefore, Á is a $\mathcal{N}S_wOS$.
2. Let Á be a $\mathcal{N}ROS$, then $Á=\mathcal{N}int(\mathcal{N}cl(Á))$. As every $\mathcal{N}ROS$ is $\mathcal{N}OS$ so by (1) in Proposition (3.1) Á is a $\mathcal{N}S_wOS$.

3. Let $\acute{A}$ be a $\mathcal{N}SOS$, thus $\acute{A} \subseteq \mathcal{N}\mathrm{cl}(\mathcal{N}\mathrm{int}(\acute{A}))$.If $\mathcal{N}\mathrm{int}(\acute{A}) = 0_{\mathcal{N}}$, then $\mathcal{N}\mathrm{cl}(\mathcal{N}\mathrm{int}(\acute{A})) = 0_{\mathcal{N}}$ so we get contradiction as $\acute{A}$ is a $\mathcal{N}SOS$, therefore must $\mathcal{N}\mathrm{int}(\acute{A}) \neq 0_{\mathcal{N}}$, hence $\acute{A}$ is a $\mathcal{N}S_wOS$.
4. Let $\acute{A}$ be a $\mathcal{N}\alpha OS$, then $\acute{A} \subseteq \mathcal{N}\mathrm{int}(\mathcal{N}\mathrm{cl}(\mathcal{N}\mathrm{int}(\acute{A})))$. As every $\mathcal{N}\alpha OS$ is $\mathcal{N}SOS$ so by (3) in Proposition (3.1) $\acute{A}$ is a $\mathcal{N}S_wOS$.
5. Let $\acute{A}$ be a $\mathcal{N}\delta OS$, then $\acute{A} = \mathcal{N}\delta\mathrm{int}(\acute{A}) = \cup_{K \subseteq \acute{A}} K$,where K is a $\mathcal{N}ROS$ in Ӿ. So by (2) in Proposition (3.1) and axiom (3) in defintion of $\mathcal{N}TS$, we get $\acute{A}$ is a $\mathcal{N}S_wOS$.□

The example that follows demonstrates that the reverse is untrue in Proposition (3.1).

Example 3.2 i. Suppose that Ӿ $= \{l\}$, and describe $\mathcal{N}S$'s $\acute{A}_1, \acute{A}_2$ & $\acute{A}_3$ in Ӿ as follows:

$$\acute{A}_1 = \langle(0.5,\ 0.5,\ 0.5)\rangle, \acute{A}_2 = \langle(0.5,\ 0.2,\ 0.3)\rangle, \acute{A}_3 = \langle(0.5,\ 0.1,\ 0.3)\rangle$$

Where $\mathcal{T}_{\mathcal{N}} = \{0_{\mathcal{N}}, \acute{A}_1, \acute{A}_3, 1_{\mathcal{N}}\}$, then (Ӿ $, \mathcal{T}_{\mathcal{N}})$ is a $\mathcal{N}TS$, and as $\mathcal{N}int(\acute{A}_2) = \acute{A}_1$, then $\acute{A}_2$ is a $\mathcal{N}S_wOS$, and it is not a $\mathcal{N}OS$ since $\acute{A}_2 \notin \mathcal{T}_{\mathcal{N}}$, also not a $NROS$ (resp. $N\delta OS$).

ii. Suppose that Ӿ $= \{l\}$, and describe $\mathcal{N}S$'s $\acute{A}_1$ & $\acute{A}_2$ in Ӿ as follows:

$$\acute{A}_1 = \langle(0.5,0.5,0.5)\rangle, \acute{A}_2 = \langle(0.6,0.4,0.4)\rangle$$

Where $\mathcal{T}_{\mathcal{N}} = \{0_{\mathcal{N}}, \acute{A}_1, 1_{\mathcal{N}}\}$, then (Ӿ $, \mathcal{T}_{\mathcal{N}})$ is a $\mathcal{N}TS$, and $\acute{A}_2$ is a $\mathcal{N}S_wOS$ but not $\mathcal{N}SOS$, $\mathcal{N}\alpha OS$.

Lemma 3.1 Every $\mathcal{N}\beta OS$ (resp. $\mathcal{N}POS$) is not a $\mathcal{N}S_wOS$, and every $\mathcal{N}S_wOS$ is not a $\mathcal{N}\beta OS$ (resp. $\mathcal{N}POS$).

Example 3.3 i. Suppose that Ӿ $= \{l\}$, and describe $\mathcal{N}S$'s $\acute{A}_1$ & $\acute{A}_2$ in Ӿ as follows:

$$\acute{A}_1 = \langle(0.5,0.3,0.4)\rangle, \acute{A}_2 = \langle(0.5,0.2,0.5)\rangle$$

Where $\mathcal{T}_{\mathcal{N}} = \{0_{\mathcal{N}}, \acute{A}_2, 1_{\mathcal{N}}\}$, then (Ӿ $, \mathcal{T}_{\mathcal{N}})$ is a $\mathcal{N}TS$, and $\acute{A}_1$ is a $\mathcal{N}\beta OS$ (resp. $\mathcal{N}POS$) is not a $\mathcal{N}S_wOS$.

ii. Suppose that Ӿ $= \{l\}$, and describe $\mathcal{N}S$'s $\acute{A}_1$ & $\acute{A}_2$ in Ӿ as follows:

$$Á_1 = \langle(0.4,0.6,0.6)\rangle, Á_2 = \langle(0.5,0.5,0.5)\rangle$$

Where $\mathcal{T}_{\mathcal{N}} = \{0_{\mathcal{N}}, Á_1, 1_{\mathcal{N}}\}$, then $(Ӽ, \mathcal{T}_{\mathcal{N}})$ is a $\mathcal{N}TS$, and $Á_2$ is a $\mathcal{N}S_wOS$ but not $\mathcal{N}POS$.
iii. Suppose that $Ӽ = \{l\}$, and describe $\mathcal{N}S$'s $Á_1, Á_2, Á_3$ & $Á_4$ in $Ӽ$ as follows:

$$Á_1 = \langle(0.4,0.6,0.6)\rangle, Á_3 = \langle(0.4,0.5,0.5)\rangle$$
$$Á_2 = \langle(0.6,0.4,0.4)\rangle, Á_4 = \langle(0.5,0.5,0.4)\rangle$$

Where $\mathcal{T}_{\mathcal{N}} = \{0_{\mathcal{N}}, Á_1, Á_3, 1_{\mathcal{N}}\}$, then $(Ӽ, \mathcal{T}_{\mathcal{N}})$ is a $\mathcal{N}TS$, and $Á_2$ is a $\mathcal{N}S_wOS$, but not a $\mathcal{N}\beta OS$.

Proposition 3.2 The union any pair of $\mathcal{N}S_wOS$ is also a $\mathcal{N}S_wOS$.
Proof. Assume that the two $\mathcal{N}S_wOS$'s $Á$ and $Ć$, and then $\mathcal{N}int(Á) \neq 0_{\mathcal{N}}$ & $\mathcal{N}int(Ć) \neq 0_{\mathcal{N}}$, therefor $\mathcal{N}int(Á \cup Ć) \neq 0_{\mathcal{N}}$. Thus $Á \cup Ć$ is a $\mathcal{N}S_wOS$. □

But the intersection not need to be $\mathcal{N}S_wOS$ (resp. $\mathcal{N}S_wCS$). And we will show it by the following example.

Example 3.4 Suppose that $Ӽ = \{l, m\}$, and we describe $\mathcal{N}S$'s $Á_1$, $Á_2$, $Á_3$, $Á_4$, $Á_5$ and $Á_6$ of $Ӽ$ by:

$$\begin{aligned}
Á_1 &= \langle(0.5,\ 0.8,\ 0.8), (0.2,\ 0.2,\ 0.2)\rangle \\
Á_2 &= \langle(0.5,\ 0.2,\ 0.2), (0.5,\ 0.2,\ 0.2)\rangle \\
Á_3 &= \langle(0.2,\ 0.2,\ 0.2), (0.5,\ 0.8,\ 0.8)\rangle \\
Á_4 &= \langle(0.6,\ 0.2,\ 0.2), (0.6,\ 0.2,\ 0.2)\rangle \\
Á_5 &= \langle(0.2,\ 0.2,\ 0.2), (0.6,\ 0.6,\ 0.6)\rangle \\
Á_6 &= \langle(0.6,\ 0.6,\ 0.6), (0.2,\ 0.2,\ 0.2)\rangle
\end{aligned}$$

Where $\mathcal{T}_{\mathcal{N}} = \{Á_1, Á_2, Á_3, Á_4, 0_{\mathcal{N}}, 1_{\mathcal{N}}\}$, then $(Ӽ, \mathcal{T}_{\mathcal{N}})$ is a $\mathcal{N}TS$, but $Á_6 \cap Á_5$ is not a $\mathcal{N}S_wOS$ (resp. $\mathcal{N}S_wCS$).

Remark 3.1 Suppose that $Á$ and $Ć$ are any two $\mathcal{N}S$'s in $\mathcal{N}TS$ $(Ӽ, \mathcal{T}_{\mathcal{N}})$, if $Á$ is a $\mathcal{N}POS$ (resp. $\mathcal{N}\beta OS$) and $Ć$ is a $\mathcal{N}S_wOS$, then $(Á \cup Ć)$ is not a $\mathcal{N}S_wOS$. In Example 3.3 (i) $Á_1$ is a $\mathcal{N}POS$ (resp. $\mathcal{N}\beta OS$) and $0_{\mathcal{N}}$ is a $\mathcal{N}S_wOS$, but $(Á_1 \cup 0_{\mathcal{N}})$ is not a $\mathcal{N}S_wOS$.

Corollary 3.1 Suppose that $Á$ and $Ć$ are any two $\mathcal{N}S$'s in $\mathcal{N}TS$ $(Ӽ, \mathcal{T}_{\mathcal{N}})$, if $Á$ is a $\mathcal{N}OS$ (resp. $\mathcal{N}SOS$, $\mathcal{N}\alpha OS$, $\mathcal{N}ROS$ and $\mathcal{N}\delta OS$) and $Ć$ is a $\mathcal{N}S_wOS$ then $(Á \cup Ć)$ is $\mathcal{N}S_wOS$.

Proof. Let $Á$ be a $\mathcal{N}OS$ (resp. $\mathcal{N}SOS$, $\mathcal{N}\alpha OS$, $\mathcal{N}ROS$ and $\mathcal{N}\delta OS$) and $Ć$ be a $\mathcal{N}S_wOS$ then by using Proposition (3.1) $Á$ is a $\mathcal{N}S_wOS$, and by Proposition (3.2) $(Á \cup Ć)$ is $\mathcal{N}S_wOS$. □

Lemma 3.2 i. Suppose that $\acute{A}$ and $\acute{C}$ are any two $\mathcal{NS}$'s in $\mathcal{NTS}$ $(Ӽ, \mathcal{T}_{\mathcal{N}})$, if $\acute{A}$ is a $\mathcal{N}OS$ (resp. $\mathcal{N}SOS$, $\mathcal{N}\alpha OS$, $\mathcal{N}ROS$, $\mathcal{N}POS$, $\mathcal{N}\delta OS$ and $\mathcal{N}\beta OS$), and $\acute{C}$ is $\mathcal{N}S_wOS$ then $(\acute{A} \cap \acute{C})$ is not a $\mathcal{N}S_wOS$. For examples:

(a) Suppose that $Ӽ = \{l\}$, and describe $\mathcal{NS}$'s $\acute{A}_1, \acute{A}_2$ & $\acute{A}_3$ in $Ӽ$ as follows:

$$\acute{A}_1 = \langle (0.5,\ 0.5,\ 0.5) \rangle, \acute{A}_2 = \langle (0.6,\ 0.2,\ 0.2) \rangle, \acute{A}_3 = \langle (0.4,\ 0.4,\ 0.4) \rangle$$

Where $\mathcal{T}_{\mathcal{N}} = \{0_{\mathcal{N}}, \acute{A}_1, 1_{\mathcal{N}}\}$, then $(Ӽ, \mathcal{T}_{\mathcal{N}})$ is a is $\mathcal{NTS}$, and $\acute{A}_3$ is a $\mathcal{N}\beta OS$ (resp. $\mathcal{N}POS$), and $\acute{A}_2$ is a $\mathcal{N}S_wOS$, but $\acute{A}_1 \cap \acute{A}_2$ is not a $\mathcal{N}S_wOS$.

(b) In Example (3.4) $\acute{A}_3$ is a $\mathcal{N}OS$ (resp. $\mathcal{N}SOS$, $\mathcal{N}\alpha OS$, $\mathcal{N}ROS$ and $\mathcal{N}\beta OS$), and $\acute{A}_6$ is $\mathcal{N}SwOS$, but $\acute{A}_3 \cap \acute{A}_6$ is not a $\mathcal{N}S_wOS$.

ii. Let $\acute{A}$ and $\acute{C}$ be $\mathcal{NS}$'s in $\mathcal{NTS}$ $(Ӽ, \mathcal{T}_{\mathcal{N}})$, if $\acute{A}$ is a $\mathcal{N}OS$ (resp. $\mathcal{N}SOS$, $\mathcal{N}\alpha OS$, $\mathcal{N}ROS$ and $\mathcal{N}SOS$), and $\acute{C}$ is a $\mathcal{N}S_wOS$ then $\acute{A} \cap \acute{C})$ is not a $\mathcal{N}OS$ (resp. $\mathcal{N}SOS$, $\mathcal{N}\alpha OS$, $\mathcal{N}ROS$ and $\mathcal{N}\delta OS$). In Example (3.2) $\acute{A}_1$ is a $\mathcal{N}S_wOS$ and $1_{\mathcal{N}}$ is a $\mathcal{N}OS$ (resp. $\mathcal{N}SOS$, $\mathcal{N}\alpha OS$, $\mathcal{N}ROS$ and $\mathcal{N}\delta OS$), but $(\acute{A}_1 \cap 1_{\mathcal{N}})$ is not a $\mathcal{N}OS$ (resp. $\mathcal{N}SOS$, $\mathcal{N}\alpha OS$, $\mathcal{N}ROS$ and $\mathcal{N}\delta OS$).

iii. Let $\acute{A}$ and $\acute{C}$ be $\mathcal{NS}$'s in $\mathcal{NTS}$ ($Ӽ, \mathcal{T}_{\mathcal{N}}$), if $\acute{A}$ is a $\mathcal{N}POS$, and $\acute{C}$ is $\mathcal{N}S_wOS$ then $(\acute{A} \cap \mathcal{B})$ is not a $\mathcal{N}POS$. In Example 3.3 (ii) $1_{\mathcal{N}}$ is a $\mathcal{N}POS$, and $\acute{A}_2$ is $\mathcal{N}S_wOS$, but $(1_{\mathcal{N}} \cap \acute{A}_2)$ is not a $\mathcal{N}POS$.

4. Let $\acute{A}$ and $\acute{C}$ be $\mathcal{NS}$'s in $\mathcal{NTS}$ $(Ӽ, \mathcal{T}_{\mathcal{N}})$, if $\acute{A}$ is a $\mathcal{N}\beta OS$, and $\acute{C}$ is $\mathcal{N}SwOS$ then $(\acute{A} \cap \acute{C})$ is not a $\mathcal{N}\beta OS$. In Example 3.3 (iii) $1_{\mathcal{N}}$ is a $\mathcal{N}\beta OS$, and $\acute{A}_2$ is $\mathcal{N}S_wOS$, but $(1_{\mathcal{N}} \cap \acute{A}_2)$ is not a $\mathcal{N}\beta OS$.

Proposition 3.3 The combination or union of arbitrary collection of $\mathcal{N}S_wOS$ is a $\mathcal{N}S_wOS$.
Proof. Obvious. □

4 NEUTROSOPHIC S_w-INTERIOR IN $\mathcal{NTS}$S

The neutrosophic S_w-interior operator and its characteristics in neutrosophic topological space are delineated in the following part.

Definition 4.1 The neutrosophic S_w-interior of a neutrosophic set $\acute{A}$ in $\mathcal{NTS}$ $(Ӽ, \mathcal{T}_{\mathcal{N}})$ is defined as follow: (briefly. $\mathcal{N}S_wint(\acute{A})$)

$$\mathcal{N}S_wint(\acute{A}) = \bigcup\nolimits_{K \subseteq \acute{A}} K\ ,, \text{where } K \text{ is a } \mathcal{N}S_wOS \text{ in } Ӽ.$$

Example 4.1 Suppose that $Ӽ = \{l, m, n\}$ and define $\mathcal{NS}$'s $\acute{A}_1, \acute{A}_2, \acute{A}_3, \acute{A}_4$ & $\acute{A}_5$ in $Ӽ$ as follows:

$$
\begin{aligned}
\acute{A}_1 &= \langle(0.1,\ 0.2,\ 0.7),(0.4,\ 0.3,\ 0.9),(0.8,\ 0.5,\ 0.5)\rangle\\
\acute{A}_2 &= \langle(0.5,\ 0.7,\ 0.9),(0.1,\ 0.3,\ 0.4),(0.3,\ 0.6,\ 0.8)\rangle\\
\acute{A}_3 &= \langle(0.9,\ 0.1,\ 0.3),(0.6,\ 0.2,\ 0.2),(0.9,\ 0.5,\ 0.3)\rangle\\
\acute{A}_4 &= \langle(0.2,\ 0.5,\ 0.9),(0.1,\ 0.5,\ 0.3),(0.4,\ 0.5,\ 0.6)\rangle\\
\acute{A}_5 &= \langle(0.6,\ 0.2,\ 0.7),(0.8,\ 0.3,\ 0.3),(0.8,\ 0.5,\ 0.5)\rangle\\
\acute{A}_1 \cap \acute{A}_2 &= \langle(0.1,\ 0.7,\ 0.9),(0.1,\ 0.3,\ 0.9),(0.3,\ 0.6,\ 0.8)\rangle\\
\acute{A}_1 \cup \acute{A}_2 &= \langle(0.5,\ 0.2,\ 0.7),(0.4,\ 0.3,\ 0.4),(0.8,\ 0.5,\ 0.5)\rangle
\end{aligned}
$$

Where $\mathcal{T}_{\mathcal{N}} = \{0_{\mathcal{N}}, \acute{A}_1, \acute{A}_2, \acute{A}_1 \cap \acute{A}_2, \acute{A}_1 \cup \acute{A}_2, 1_{\mathcal{N}}\}$, then $(\text{Ӽ}, \mathcal{T}_{\mathcal{N}})$ is a $\mathcal{N}TS$, and $\mathcal{N}S_w int(\acute{A}_3) = \acute{A}_1 \cup \acute{A}_2$.

Proposition 4.1 The neutrosophic S_w-interior operator accomplishes:

1. $\mathcal{N}S_w int(\acute{A}) \subseteq \acute{A}$.
2. $\acute{A} \subseteq \acute{C} \Rightarrow \mathcal{N}S_w int(\acute{A}) \subseteq \mathcal{N}S_w int(\acute{C})$.
3. $\mathcal{N}S_w int(\acute{A})$ is the greatest $\mathcal{N}S_w OS$ contained in Á.
4. $\mathcal{N}S_w int(\acute{A}) = \acute{A}$ if and only if Á is a $\mathcal{N}S_w OS$.
5. $\mathcal{N}S_w int(\mathcal{N}S_w int(\acute{A})) = \mathcal{N}S_w int(\acute{A})$.
6. $(\mathcal{N}S_w int(\acute{A}))^c = \mathcal{N}S_w cl(\acute{A}^c)$.
7. $\mathcal{N}S_w int(0_{\mathcal{N}}) = 0_{\mathcal{N}}, \mathcal{N}S_w int(1_{\mathcal{N}}) = 1_{\mathcal{N}}$

Proof.

1. $\mathcal{N}S_w int(\acute{A}) = \bigcup_{K \subseteq \acute{A}} K$, where K is a $\mathcal{N}S_w OS$ in Ӽ. Thus, $\mathcal{N}S_w int(\acute{A}) \subseteq \acute{A}$.
2. $\mathcal{N}S_w int(\acute{C}) = \cup\,\{K : K$ is a $\mathcal{N}S_w OS$ in Ӽ & $K \subseteq \acute{C}\} \supseteq \cup\,\{K : K$ is a $\mathcal{N}S_w OS$ in Ӽ , $K \subseteq \acute{C}\} \supseteq \mathcal{N}S_w int(\acute{A})$. Thus, $\mathcal{N}S_w int(\acute{A}) \subseteq \mathcal{N}S_w int(\acute{C})$.
3. Let K be any $\mathcal{N}S_w OS$ which $K \subseteq \acute{A}$, then $K \subseteq \mathcal{N}S_w int(\acute{A})$. Hence, $\mathcal{N}S_w int$ (Á) is the greatest $\mathcal{N}S_w OS$ contained in Á.
4. Suppose that Á is any $\mathcal{N}S_w OS$ of Ӽ. Then the greatest $\mathcal{N}S_w OS$ containing Á is itself. Therefore, $\mathcal{N}S_w int(\acute{A}) = \acute{A}$.
5. By (4), the greatest $\mathcal{N}S_w OS$ containing $\mathcal{N}S_w int(\acute{A})$ is itself. Hence, $\mathcal{N}S_w int(\mathcal{N}S_w int(\acute{A})) = \mathcal{N}S_w int(\acute{A})$.
6. $\mathcal{N}S_w int(\acute{A})$ is the greatest $\mathcal{N}S_w OS$ contained in Á. The complement is the smallest $\mathcal{N}S_w CS$ containing $\acute{A}^c$. Therefore, $(\mathcal{N}S_w int(\acute{A}))^c = \mathcal{N}S_w cl(\acute{A}^c)$.

7. $\mathcal{N}S_w int(0_{\mathcal{N}}) = 0_{\mathcal{N}}$ and $\mathcal{N}S_w int(1_{\mathcal{N}}) = 1_{\mathcal{N}}$ are obvious by (4) and definition. □

Theorem 4.1 Suppose that $(Ҳ, \mathcal{T}_{\mathcal{N}})$ is a $\mathcal{NTS}$. Next, we have the following for any neutrosophic subset Đ and Ꝁ of a $\mathcal{NTS}$.

1. $\mathcal{N}S_w int(\text{Đ} \cap \text{Ꝁ}) \subseteq \mathcal{N}S_w int(\text{Đ}) \cap \mathcal{N}S_w int(\text{Ꝁ})$
2. $\mathcal{N}S_w int(\text{Đ} \cup \text{Ꝁ}) \supseteq \mathcal{N}S_w int(\text{Đ}) \cup \mathcal{N}S_w int(\text{Ꝁ})$.

Proof.

1. Assume that $\mathcal{L} \subseteq (\text{Đ} \cap \text{Ꝁ})$, then $\mathcal{L} \subseteq \text{Đ}$ and $\mathcal{L} \subseteq \text{Ꝁ}$, by using Proposition 4.1 (2), then $\mathcal{N}S_w int(\mathcal{L}) \subseteq \mathcal{N}S_w int(\text{Đ})$ and $\mathcal{N}S_w int(\mathcal{L}) \subseteq \mathcal{N}S_w int(\text{Ꝁ})$. Thus, $\mathcal{N}S_w int(\mathcal{L}) \subseteq \mathcal{N}S_w int(\text{Đ}) \cap \mathcal{N}S_w int(\text{Ꝁ})$. Therefore, $\mathcal{N}S_w int(\text{Đ} \cap \text{Ꝁ}) \subseteq \mathcal{N}S_w int(\text{Đ}) \cap \mathcal{N}S_w int(\text{Ꝁ})$.
2. As $\text{Đ} \subseteq \text{Đ} \cup \text{Ꝁ}$ and $\text{Ꝁ} \subseteq \text{Đ} \cup \text{Ꝁ}$, by using Proposition 4.1 (2), $\mathcal{N}S_w \text{int}(\text{Đ}) \subseteq \mathcal{N}S_w int(\text{Đ} \cup \text{Ꝁ})$ and $\mathcal{N}S_w int(\text{Ꝁ}) \subseteq \mathcal{N}S_w int(\text{Đ} \cup \text{Ꝁ})$. Therefore, $\mathcal{N}S_w int(\text{Đ}) \cup \mathcal{N}S_w int(\text{Ꝁ}) \subseteq \mathcal{N}S_w int(\text{Ꝁ} \cup \text{Đ})$. □

The example that follows demonstrates the fact that equality does not have to be preserved in Theorem (4.1).

Example 4.2 Suppose that $Ҳ = \{l, m\}$ and define in Ҳ, the $\mathcal{N}S$'s $\text{Á}_1, \text{Á}_2, \text{Á}_3, \text{Á}_4, \text{Á}_5$ & Á_6 as follows:

$$\begin{aligned}
\text{Á}_1 &= \langle (0.5,\ 0.8,\ 0.8), (0.2,\ 0.2,\ 0.2) \rangle \\
\text{Á}_2 &= \langle (0.5,\ 0.2,\ 0.2), (0.5,\ 0.2,\ 0.2) \rangle \\
\text{Á}_3 &= \langle (0.2,\ 0.2,\ 0.2), (0.5,\ 0.8,\ 0.8) \rangle \\
\text{Á}_4 &= \langle (0.6,\ 0.2,\ 0.2), (0.6,\ 0.2,\ 0.2) \rangle \\
\text{Á}_5 &= \langle (0.2,\ 0.2,\ 0.2), (0.6,\ 0.6,\ 0.6) \rangle \\
\text{Á}_6 &= \langle (0.6,\ 0.6,\ 0.6), (0.2,\ 0.2,\ 0.2) \rangle
\end{aligned}$$

Where $\mathcal{T}_{\mathcal{N}} = \{\text{Á}_1, \text{Á}_2, \text{Á}_3, \text{Á}_4, 0_{\mathcal{N}}, 1_{\mathcal{N}}\}$, then $(Ҳ, \mathcal{T}_{\mathcal{N}})$ is a $\mathcal{NTS}$, and $\mathcal{N}S_w int(\text{Á}_6) \cup \mathcal{N}S_w int(\text{Á}_5) = \text{Á}_2$, $\mathcal{N}S_w int(\text{Á}_6 \cup \text{Á}_5) = \text{Á}_4$ thus, $\mathcal{N}S_w int(\text{Á} \cup \text{Ć}) \neq \mathcal{N}S_w int(\text{Á}) \cup \mathcal{N}S_w int(\text{Ć})$. And $\mathcal{N}S_w int(\text{Á}_6) \cap \mathcal{N}S_w int(\text{Á}_5) = \langle (0.2,0.8,0.8), (0.2,0.8,0.8) \rangle$, $\mathcal{N}S_w \text{int}(\text{Á}_6 \cap \text{Á}_5) = 0_{\mathcal{N}}$ thus $\mathcal{N}S_w int(\text{Á} \cap \text{Ć}) \neq \mathcal{N}S_w int(\text{Á}) \cap \mathcal{N}S_w int(\text{Ć})$.

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5 NEUTROSOPHIC S_w-DRIVED SET IN $\mathcal{NTS}$S

The neutrosophic S_w-drived operator and its characteristics in neutrosophic topological space are delineated in the following section.

Definition 5.1 A $\mathcal{N}P$ $\langle ӽ, ȴ, ȶ, ȵ \rangle$ is a neutrosophic S_w-limit point of a neutrosophic set Á. If each one of $\mathcal{N}S_wO$ set $\mathcal{L}$ which includes $P = \langle ӽ, ȴ, ȶ, ȵ \rangle$, then $\mathcal{L} \cap \left(Á - \{\langle ӽ, ȴ, ȶ, ȵ \rangle\}\right) \neq 0_{\mathcal{N}}$. And the neutrosophic S_w-drived set of Á, represented as $\mathcal{N}S_wd(Á)$ is the collection of every neutrosophic S_w-limit points of Á.

Example 5.1 Suppose that $Ӽ = \{l, m\}$ and define neutrosophic sets $Á_1, Á_2, Á_3$ & $Á_4$ in $Ӽ$ as follows:

$$Á_1 = \langle (0.5,\ 0.8,\ 0.8), (0.2,\ 0.2,\ 0.2) \rangle$$
$$Á_2 = \langle (0.5,\ 0.2,\ 0.2), (0.5,\ 0.2,\ 0.2) \rangle$$
$$Á_3 = \langle (0.2,\ 0.2,\ 0.2), (0.5,\ 0.8,\ 0.8) \rangle$$
$$Á_4 = \langle (0.6,\ 0.2,\ 0.2), (0.6,\ 0.2,\ 0.2) \rangle$$

Where $\mathcal{T}_{\mathcal{N}} = \{Á_1, Á_2, Á_3, Á_4, 0_{\mathcal{N}}, 1_{\mathcal{N}}\}$, then $(Ӽ, \mathcal{T}_{\mathcal{N}})$ is a $\mathcal{N}TS$, and $\mathcal{N}S_wd\left(Á_1\right) = 1_{\mathcal{N}}$

Proposition 5.1 Let $(Ӽ, \mathcal{T}_{\mathcal{N}})$ be a $\mathcal{N}TS$, then For pair of $\mathcal{N}S$'s Đ and K of $(Ӽ, \mathcal{T}_{\mathcal{N}})$ the following claims are true:

1. $\mathcal{N}S_wd(0_{\mathcal{N}}) = 0_{\mathcal{N}}, \mathcal{N}S_wd(1_{\mathcal{N}}) = 1_{\mathcal{N}}$.
2. $\mathcal{N}S_wd(Đ) \subseteq \mathcal{N}d(Đ)$, where $\mathcal{N}d(Đ)$ is neutrosophic drived set of Á.
3. If Á ⊆ K, then $\mathcal{N}S_wd(Đ) \subseteq \mathcal{N}S_wd(K)$.
4. $\mathcal{N}S_wd(Đ \cup K) = \mathcal{N}S_wd(Đ) \cup \mathcal{N}S_wd(K)$.
5. $\mathcal{N}S_wd(Đ \cap K) \subseteq \mathcal{N}S_wd(Đ) \cap \mathcal{N}S_wd(K)$.

Proof.

1. Obvious.
2. It suffices to observe that every $\mathcal{N}OS$ is a $\mathcal{N}S_wOS$.
3. Let $Đ \subseteq K$ and $P = \langle ӽ, ȴ, ȶ, ȵ \rangle \in \mathcal{N}S_wd(Đ)$, then for each $\mathcal{N}S_wOS$ $\mathcal{L}$ of $P = \langle ӽ, ȴ, ȶ, ȵ \rangle, \mathcal{L} \cap (Đ - \{\langle ӽ, ȴ, ȶ, ȵ \rangle\}) \neq 0_{\mathcal{N}}$. As $Đ \subseteq K \therefore \mathcal{L} \cap (K - \{\langle ӽ, ȴ, ȶ, ȵ \rangle\}) \neq 0_{\mathcal{N}}$, so $\langle ӽ, ȴ, ȶ, ȵ \rangle \in \mathcal{N}S_wd(K)$.

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4. Let $P = \langle ӽ, ȴ, ȶ, ȵ \rangle \in \mathcal{N}S_w d(Đ \cup Ƙ)$, then for each $\mathcal{N}S_wOS$ $\mathcal{L}$ of $P = \langle ӽ, ȴ, ȶ, ȵ \rangle$, $\mathcal{L} \cap ((Đ \cup Ƙ) - \{\langle ӽ, ȴ, ȶ, ȵ \rangle\}) \neq 0_{\mathcal{N}}$, so $\mathcal{N}S_wOS$ $\mathcal{L} \cap (Đ - \{\langle ӽ, ȴ, ȶ, ȵ \rangle\}) \neq 0_{\mathcal{N}} \cup \mathcal{L} \cap (Ƙ - \{\langle ӽ, ȴ, ȶ, ȵ \rangle\}) \neq 0_{\mathcal{N}}$, there for $P = \langle ӽ, ȴ, ȶ, ȵ \rangle \in \mathcal{N}S_w d(Đ)$ or $P = \langle ӽ, ȴ, ȶ, ȵ \rangle \in \mathcal{N}S_w d(Ƙ)$.Thus $P = \langle ӽ, ȴ, ȶ, ȵ \rangle \in \mathcal{N}S_w d(Đ) \cup \mathcal{N}S_w d(Ƙ)$. And as $Đ \subseteq (Đ \cup Ƙ)$, $Ƙ \subseteq (Đ \cup Ƙ)$, then by using (2) in Proposition (5.1) $\mathcal{N}S_w d(Đ) \subseteq \mathcal{N}S_w d(Đ \cup Ƙ)$ and $\mathcal{N}S_w d(Ƙ) \subseteq \mathcal{N}S_w d(Đ \cup Ƙ)$. Thus $\mathcal{N}S_w d(Đ) \cup \mathcal{N}S_w d(Ƙ) \subseteq \mathcal{N}S_w d(Đ \cup Ƙ)$.
5. As $Đ \cap Ƙ \subseteq Đ$ and $Đ \cap Ƙ \subseteq Ƙ$, by using (2) in Proposition (5.1) $\mathcal{N}S_w d(Đ \cap Ƙ) \subseteq \mathcal{N}S_w d(Đ)$ and $\mathcal{N}S_w d(Đ \cap Ƙ) \subseteq \mathcal{N}S_w d(Ƙ) \therefore \mathcal{N}S_w d(Đ \cap Ƙ) \subseteq \mathcal{N}S_w d(Đ) \cap \mathcal{N}S_w d(Ƙ)$. □

The example that follows demonstrates that the equality of (5) does not have to be preserved in Proposition (5.1).

Example 5.2 Suppose that $Ӽ = \{l, m\}$, and define neutrosophic sets $Á_1$ & $Á_2$ in $Ӽ$ as follows:

$$Á_1 = \langle (1.0, 0.0, 0.0), (0.0, 1.0, 1.0) \rangle, Á_2 = \langle (0.0, 1.0, 1.0), (1.0, 0.0, 0.0) \rangle$$

Where $\mathcal{T}_{\mathcal{N}} = \{0_{\mathcal{N}}, Á_1, Á_2, 1_{\mathcal{N}}\}$, then $(Ӽ, \mathcal{T}_{\mathcal{N}})$ is a $\mathcal{NTS}$, now $\mathcal{N}S_w d(Á_1 \cap Á_2) = 0_{\mathcal{N}}$ and $\mathcal{N}S_w d(Á) = \mathcal{N}S_w d(Á) = 1_{\mathcal{N}}$, hence $\mathcal{N}S_w d(Đ \cap Ƙ) \neq \mathcal{N}S_w d(Đ) \cap \mathcal{N}S_w d(Ƙ)$.

6 NEUTROSOPHIC S_w-CLOSURE IN $\mathcal{NTS}$S

The neutrosophic S_w-closure operator and its characteristics in neutrosophic topological space are delineated in this part.

Definition 6.1 The neutrosophic S_w-closure of a neutrosophic set $Á$ in $\mathcal{NTS}$ $(Ӽ, \mathcal{T}_{\mathcal{N}})$ is defined as: (briefly. $\mathcal{N}S_w cl(Á)$)

$$\mathcal{N}S_w cl(Á) = \bigcap_{Á \subseteq Ƙ} Ƙ \text{ , where } Ƙ \text{ is a } \mathcal{N}S_wC \text{ in } Ӽ.$$

Example 6.1 Suppose that $Ӽ = \{l, m\}$, and define neutrosophic sets $Á_1, Á_2, Á_3$ & $Á_4$ in $Ӽ$ as follows:

$$\acute{A}_1 = \langle (0.5,\ 0.8,\ 0.8), (0.2,\ 0.2,\ 0.2) \rangle$$
$$\acute{A}_2 = \langle (0.5,\ 0.2,\ 0.2), (0.5,\ 0.2,\ 0.2) \rangle$$
$$\acute{A}_3 = \langle (0.2,\ 0.2,\ 0.2), (0.5,\ 0.8,\ 0.8) \rangle$$
$$\acute{A}_4 = \langle (0.6,\ 0.2,\ 0.2), (0.6,\ 0.2,\ 0.2) \rangle$$

Where $\mathcal{T}_{\mathcal{N}} = \{\acute{A}_1, \acute{A}_2, \acute{A}_3, \acute{A}_4, 0_{\mathcal{N}}, 1_{\mathcal{N}}\}$, then (Ӽ , $\mathcal{T}_{\mathcal{N}}$) is a $\mathcal{N}TS$, and $\mathcal{N}S_wcl\ (\acute{A}) = \langle(0.8,0.2,0.5), (0.2,0.2,0.2)\rangle, \mathcal{N}S_wcl(\acute{A}_2) = 1_{\mathcal{N}}$.

Proposition 6.1 If Á is a neutrosophic set in $\mathcal{N}TS$ (Ӽ , $\mathcal{T}_{\mathcal{N}}$), then $\mathcal{N}S_wcl(\acute{A}) = \mathcal{N}S_wd(\acute{A}) \cup \acute{A}$.

Proof. Since $\mathcal{N}S_wd(\acute{A}) \subseteq \mathcal{N}S_wcl(\acute{A})$, then $\acute{A} \cup \mathcal{N}S_wd(\acute{A}) \subseteq \mathcal{N}S_wcl(\acute{A})$. On the other hand, let P = ⟨ӽ , ᶅ, ƫ, ᶇ⟩ ∈ $\mathcal{N}Swcl\ (\acute{A})$. If P = ⟨ӽ , ᶅ, ƫ, ᶇ⟩ ∈ Á, thus proof is done. If P = ⟨ӽ , ᶅ, ƫ, ᶇ⟩ ∉ Á, each $\mathcal{N}S_wOS\ \mathcal{L}$ containing P = ⟨ӽ , ᶅ, ƫ, ᶇ⟩ intersects Á at a point distinct from P = ⟨ӽ , ᶅ, ƫ, ᶇ⟩, so P = ⟨ӽ , ᶅ, ƫ, ᶇ⟩ ∈ $\mathcal{N}S_wd(\acute{A})$. Thus $\mathcal{N}S_wcl(\acute{A}) \subseteq \acute{A} \cup \mathcal{N}S_wd(\acute{A})$. □

Proposition 6.2 The neutrosophic S_w-closure oparatore satisfies:

1. $\acute{A} \subseteq \mathcal{N}S_wcl(\acute{A})$.
2. $Đ \subseteq K \Rightarrow \mathcal{N}S_wcl(Đ) \subseteq \mathcal{N}S_wcl(K)$.
3. The smallest $\mathcal{N}S_wCS$ that contains Á is $\mathcal{N}S_wcl(\acute{A})$.
4. $\mathcal{N}S_wcl(\acute{A}) = \acute{A}$ if and only if Á is a $\mathcal{N}S_wCS$.
5. $\mathcal{N}S_wcl(\mathcal{N}Swcl(\acute{A})) = \mathcal{N}S_wcl(\acute{A})$.
6. $(\mathcal{N}S_wcl(\acute{A}))^c = \mathcal{N}S_wint(\acute{A}^c)$.
7. $\mathcal{N}S_wcl(0_{\mathcal{N}}) = 0_{\mathcal{N}},\ \mathcal{N}S_wcl(1_{\mathcal{N}}) = 1_{\mathcal{N}}$.

Proof.

1. $\mathcal{N}S_wcl(\acute{A}) = \bigcap_{\acute{A} \subseteq K} K$, where K is a $\mathcal{N}S_wCS$ in Ӽ. Thus, $\acute{A} \subseteq \mathcal{N}S_wcl(\acute{A})$.
2. Let $Đ \subseteq K \Rightarrow \mathcal{N}S_wd(Đ) \subseteq \mathcal{N}S_wd(K)$. Thus $Đ \cup \mathcal{N}S_wd(Đ) \subseteq K \cup \mathcal{N}S_wd(K) \Rightarrow \mathcal{N}S_wcl(Đ) \subseteq \mathcal{N}S_wcl(K)$, by Proposition (6.1).
3. If K is any $\mathcal{N}S_wCS$ containing Á, then $\mathcal{N}S_wcl(\acute{A}) \subseteq K$. Hence $\mathcal{N}S_wcl(\acute{A})$ is the smallest $\mathcal{N}S_wCO$ containg Á.
4. Suppose Á is any $\mathcal{N}S_wCS$ of Ӽ. Then the smallest $\mathcal{N}S_wCS$ contained in Á is itself. Therefore, $\mathcal{N}S_wcl(\acute{A}) = \acute{A}$.

5. By (4), the smallest $\mathcal{N}S_wCS$ contained in $\mathcal{N}S_wcl(\acute{A})$ is itself. Hence, $\mathcal{N}S_wcl(\mathcal{N}S_wcl(\acute{A})) = \mathcal{N}S_wcl(\acute{A})$.
6. $\mathcal{N}S_wcl(\acute{A})$ is the smallest $\mathcal{N}S_wCS$ containing Á. The complement is the greatest $\mathcal{N}S_wOS$ contained in $\acute{A}^c$. Therefore, $(\mathcal{N}S_wcl(\acute{A}))^c = \mathcal{N}S_wint(\acute{A}^c)$.
7. $\mathcal{N}S_wcl(0_\mathcal{N}) = 0_\mathcal{N}$ and $\mathcal{N}S_wcl(1_\mathcal{N}) = 1_\mathcal{N}$ are obvious by (4) and definition. □

Theorem 6.1 Suppose that $(Ӽ, \mathcal{T}_\mathcal{N})$ is a $\mathcal{N}TS$. Next, we have the following for each of the neutrosophic subsets Đ and K of a $\mathcal{N}TS$.

1. $\mathcal{N}S_wcl(Đ \cap K) \subseteq \mathcal{N}S_wcl(Đ) \cap \mathcal{N}S_wcl(K)$
2. $\mathcal{N}S_wcl(Đ \cup K) \subseteq \mathcal{N}S_wcl(Đ) \cup \mathcal{N}S_wcl(K)$.

Proof.

1. As $Đ \cap K \subseteq K$ and $Đ \cap K \subseteq Đ$, then by using Proposition 6.1 (2) $\mathcal{N}S_wcl(Đ \cap K) \subseteq \mathcal{N}S_wcl(Đ), \mathcal{N}S_wcl(Đ \cap K) \subseteq \mathcal{N}S_wcl(K)$.Therefor $\mathcal{N}S_wcl(Đ \cap K) \subseteq \mathcal{N}S_wcl(Đ) \cap \mathcal{N}S_wcl(K)$.
2. As $Đ \subseteq Đ \cup K$ and $K \subseteq Đ \cup K$, by using Proposition 6.1 (2), $\mathcal{N}S_wcl(Đ) \subseteq \mathcal{N}S_wcl(Đ \cup K)$ and $\mathcal{N}S_wcl(K) \subseteq \mathcal{N}S_wcl(Đ \cup K)$. Therefor $\mathcal{N}S_wcl(Đ) \cup \mathcal{N}S_wcl(K) \subseteq \mathcal{N}S_wcl(Đ \cup K)$. □

The example that follows demonstrates that equality does not have to be preserved in Theorem (6.1).

Example 6.2 In Example (6.1) $\mathcal{N}S_wcl(\acute{A}_1) \cup \mathcal{N}S_wcl(\acute{A}_3) = \langle(0.8,0.2,0.2), (0.8,0.2,0.2)\rangle$, $\mathcal{N}S_wcl(\acute{A}_1 \cup \acute{A}_3) = 1_\mathcal{N}$ thus $\mathcal{N}S_wcl(Đ \cup K) \neq \mathcal{N}S_wcl(Đ) \cup \mathcal{N}S_wcl(K)$.And $\mathcal{N}S_wcl(\acute{A}_1) \cap \mathcal{N}S_wcl(\acute{A}_3) = \langle(0.2,0.2,0.5), (0.2,0.2,0.5)\rangle$, $\mathcal{N}S_wcl(\acute{A}_1 \cap \acute{A}_3) = \langle(0.2,0.8,0.6), (0.2,0.8,0.6)\rangle$, thus $\mathcal{N}S_wcl(Đ \cap K) \neq \mathcal{N}S_wcl(Đ) \cap \mathcal{N}S_wcl(K)$.

7 NEUTROSOPHIC S_w-FRONTIER IN $\mathcal{N}TSS$

The neutrosophic S_w-frontier operator and its characteristics in neutrosophic topological space are presented in this part.

Definition 7.1 The neutrosophic S_w-frontier of a neutrosophic set Ą in $\mathcal{N}TS$ $(Ӽ, \mathcal{T}_\mathcal{N})$ is $\mathcal{N}S_wfr(Ą) = \mathcal{N}S_wcl(Ą) \cap \mathcal{N}S_wcl(Ą^c)$. ($\mathcal{N}S_wfr(Ą)$, for short).

Example 7.1 In Example (6.1) $\mathcal{N}S_wfr(\acute{A}_1) = \langle(0.8,0.2,0.5), (0.8,0.2,0.5)\rangle = \mathcal{N}S_wCS({\acute{A}_1}^c)$ and $\mathcal{N}S_wfr(\acute{A}_2) = \langle(0.2,0.2,0.5), (0.2,0.2,0.5)\rangle$.

Proposition 7.1 The neutrosophic S_w-frontier operator satisfies the following in $\mathcal{N}TS\ (\text{Ҳ}, \mathcal{T}_{\mathcal{N}})$.

1. $\mathcal{N}S_{w}int(\text{Á}) \cup \mathcal{N}S_{w}fr(\text{Á}) \subseteq \mathcal{N}S_{w}cl(\text{Á})$.
2. $\mathcal{N}S_{w}fr(\text{Á}) = \mathcal{N}S_{w}fr(\text{Á}^{c})$.
3. $\mathcal{N}S_{w}fr(\text{Á})$ is a $\mathcal{N}S_{w}CS$.
4. $\mathcal{N}S_{w}fr(\mathcal{N}S_{w}fr(\text{Á})) \subseteq \mathcal{N}S_{w}fr(\text{Á})$.
5. $\mathcal{N}S_{w}fr(\mathcal{N}S_{w}int(\text{Á})) \subseteq \mathcal{N}S_{w}fr(\text{Á})$.
6. $\mathcal{N}S_{w}fr(0_{\mathcal{N}}) = 0_{\mathcal{N}}, \mathcal{N}S_{w}fr(1_{\mathcal{N}}) = 0_{\mathcal{N}}$.

Proof.

1. $\mathcal{N}S_{w}int(\text{Á}) \cup \mathcal{N}S_{w}fr(\text{Á}) \subseteq \text{Á} \cup \left(\mathcal{N}S_{w}cl(\text{Á}) \cap \mathcal{N}S_{w}\text{cl}(\text{Á}^{c})\right) \subseteq \mathcal{N}S_{w}cl(\text{Á}) \cup \mathcal{N}S_{w}cl(\text{Á}) = \mathcal{N}S_{w}cl(\text{Á})$.
2. $\mathcal{N}S_{w}fr(\text{Á}) = \mathcal{N}S_{w}cl(\text{Á}) \cap \mathcal{N}S_{w}cl(\text{Á}^{c}) = \mathcal{N}S_{w}cl(\text{Á}^{c^{c}}) \cap \mathcal{N}S_{w}cl(\text{Á}^{c}) = \mathcal{N}S_{w}fr(\text{Á}^{c})$.
3. $\mathcal{N}S_{w}cl(\mathcal{N}S_{w}fr(\text{Á})) = \mathcal{N}S_{w}cl\left(\mathcal{N}S_{w}cl(\text{Á})) \cap \mathcal{N}S_{w}cl(\text{Á}^{c})\right) \subseteq \mathcal{N}S_{w}cl(\mathcal{N}S_{w}cl(\text{Á})) \cap \mathcal{N}S_{w}cl\left(\mathcal{N}S_{w}cl(\text{Á}^{c})\right) = \mathcal{N}S_{w}cl(\text{Á}) \cap \mathcal{N}S_{w}cl(\text{Á}^{c}) = \mathcal{N}S_{w}fr(\text{Á})$. Thus by (4) $\mathcal{N}S_{w}fr(\text{Á})$ is $\mathcal{N}S_{w}CS$.
4. $\mathcal{N}S_{w}fr(\mathcal{N}S_{w}fr(\text{Á})) = \mathcal{N}S_{w}cl(\mathcal{N}S_{w}fr(\text{Á}) \cap \mathcal{N}S_{w}cl(\mathcal{N}S_{w}fr(\text{Á}))^{c} \subseteq \mathcal{N}S_{w}cl(\mathcal{N}S_{w}fr(\text{Á})) = \mathcal{N}S_{w}fr(\text{Á})$.
5. $\mathcal{N}S_{w}fr(\mathcal{N}S_{w}int(\text{Á})) = \mathcal{N}S_{w}cl(\mathcal{N}S_{w}int(\text{Á})) \cap \mathcal{N}S_{w}cl(\mathcal{N}S_{w}int(\text{Á}))^{c} = \mathcal{N}S_{w}cl(\mathcal{N}S_{w}int(\text{Á})) \cap \mathcal{N}S_{w}cl(\text{Á}^{c}) \subseteq \mathcal{N}S_{w}cl(\text{Á}) \cap \mathcal{N}S_{w}cl(\text{Á}^{c}) = \mathcal{N}S_{w}fr(\text{Á})$.
6. $\mathcal{N}S_{w}fr(0_{\mathcal{N}}) = \mathcal{N}S_{w}cl(0_{\mathcal{N}}) \cap \mathcal{N}S_{w}cl({0_{\mathcal{N}}}^{c}) = 0_{\mathcal{N}},\ \mathcal{N}S_{w}fr(1_{\mathcal{N}}) = \mathcal{N}S_{w}cl(1_{\mathcal{N}}) \cap \mathcal{N}S_{w}cl({1_{\mathcal{N}}}^{c}) = 0_{\mathcal{N}}$. □

The example that follows demonstrates that the equality of (1, 4 and 5) need not be hold in Proposition (7.1).

Example 7.2 In Example (6.1) $\mathcal{N}S_{w}int(\text{Á}_1) = \cup\ \mathcal{N}S_{w}fr(\text{Á}_1) = \text{Á}_1$, but $\mathcal{N}S_{w}int(\text{Á}_1) = \langle(0.8,0.2,0.5), (0.2,0.2,0.2)\rangle = {\text{Á}_1}^{c}$. Thus $\mathcal{N}S_{w}cl(\text{Á}) \neq \mathcal{N}S_{w}int(\text{Á}) \cup \mathcal{N}S_{w}fr(\text{Á})$. And in Example (4.2) $\mathcal{N}S_{w}fr(\mathcal{N}S_{w}int(\text{Á}_1) = \text{Á}_1$, but $\mathcal{N}S_{w}fr(\text{Á}_6) = {\text{Á}_1}^{c}$. Thus $\mathcal{N}S_{w}fr(\mathcal{N}S_{w}int(\text{Á})) \neq \mathcal{N}S_{w}fr(\text{Á})$.

Lemma 7.1 If $(Ӽ, \mathcal{T}_{\mathcal{N}})$ is a $\mathcal{N}TS$, and for any neutrosophic subset Á of a $\mathcal{N}TS$. Then $\mathcal{N}S_w int(Á) \cap \mathcal{N}S_w fr(Á) \neq 0_{\mathcal{N}}$. In Example (6.1) $\mathcal{N}S_w fr(Á_4) = \langle(0.2,0.2,0.5),(0.2,0.2,0.5)\rangle$, and $\mathcal{N}S_w int(Á_4) = Á_4$.

8 NEUTROSOPHIC CONTINUOUS FUNCTIONS

8.1 Neutrosophic S_w-Continuous Functions

The notion of a neutrosophic S_w-continuous function in neutrosophic topological spaces is presented in this part. And, we examine several of the main results depending on neutrosophic S_w-open sets.

Definition 8.1.1 If $f: (Ӽ, \mathcal{T}_{\mathcal{N}}) \to (Ƴ, \sigma_{\mathcal{N}})$ is a function. Then $f: (Ӽ, \mathcal{T}_{\mathcal{N}}) \to (Ƴ, \sigma_{\mathcal{N}})$ is said to be a neutrosophic S_w-continuous ($\mathcal{N}S_w$-continuous, for short) function if $f^{-1}(\mathcal{V})$ is a $\mathcal{N}S_wOS$ in Ӽ for each $\mathcal{N}OS$ $\mathcal{V}$ in Ƴ.

Example 8.1.1 Let $Ӽ = Ƴ = \{l, m\}$ and define $\mathcal{N}S$'s as follows:

$$Á_1 = \langle(0.2,0.5,0.6),(0.3,0.5,0.4)\rangle$$
$$Á_2 = \langle(0.3,0.5,0.4),(0.2,0.5,0.6)\rangle$$

Now $\mathcal{T}_{\mathcal{N}} = \{0_{\mathcal{N}}, Á_1, 1_{\mathcal{N}}\}$ is neutrosophic topology on Ӽ and $\sigma_{\mathcal{N}} = \{0_{\mathcal{N}}, Á_2, 1_{\mathcal{N}}\}$ is neutrosophic topology on Ƴ, then $(Ӽ, \mathcal{T}_{\mathcal{N}})$ and $(Ƴ, \sigma_{\mathcal{N}})$ are $\mathcal{N}TS$'s. Also we define $f: (Ӽ, \mathcal{T}_{\mathcal{N}}) \to (Ƴ, \sigma_{\mathcal{N}})$ as: $f(l) = m, f(m) = l$. As every $\mathcal{N}OS$ in Ƴ has an inverse image that is a $\mathcal{N}S_wOS$ in Ӽ. Thus f is a neutrosophic S_w-continuous function.

Theorem 8.1.1 All continuous functions that are neutrosophic are also neutrosophic S_w-continuous functions.

Proof. Assume that $f: (Ӽ, \mathcal{T}_{\mathcal{N}}) \to (Ƴ, \sigma_{\mathcal{N}})$ is a neutrosophic continuous function. Let $\mathcal{V}$ be a neutrosophic open set in $(Ƴ, \sigma_{\mathcal{N}})$. Thus we get $f^{-1}(\mathcal{V})$ is a $\mathcal{N}OS$ in $(Ӽ, \mathcal{T}_{\mathcal{N}})$. Since each $\mathcal{N}OS$ is a $\mathcal{N}S_wOS$, so $f^{-1}(\mathcal{V})$ is $\mathcal{N}S_wOS$ in $(Ӽ, \mathcal{T}_{\mathcal{N}})$. Therefore, f is $\mathcal{N}S_w$-continuous function. □

The example that follows demonstrates that Theorem (8.1.1)'s opposite is untrue.

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Example 8.1.2 Let $Ҳ = ƴ = \{l, m\}$ and define $\mathcal{NS}$'s as follows:

$$\acute{A}_1 = \langle (0.2,0.5,0.6), (0.2,0.5,0.4) \rangle$$
$$\acute{A}_2 = \langle (0.2,0.4,0.4), (0.2,0.5,0.4) \rangle$$
$$\acute{A}_3 = \langle (0.2,0.5,0.4), (0.2,0.4,0.4) \rangle$$
$$\acute{A}_4 = \langle (0.2,0.5,0.4), (0.2,0.5,0.6) \rangle$$

Now $\mathcal{T}_{\mathcal{N}} = \{0_{\mathcal{N}}, \acute{A}_1, 1_{\mathcal{N}}\}$ is neutrosophic topology on Ҳ and $\sigma_{\mathcal{N}} = \{0_{\mathcal{N}}, \acute{A}_3, \acute{A}_4, 1_{\mathcal{N}}\}$ is neutrosophic topology on ƴ, then $(Ҳ, \mathcal{T}_{\mathcal{N}})$ and $(ƴ, \sigma_{\mathcal{N}})$ are $\mathcal{NTS}$'s. Also we define $f\colon (Ҳ, \mathcal{T}_{\mathcal{N}}) \to (ƴ, \sigma_{\mathcal{N}})$ as: $f(l) = m$, $f(m) = l$. And f is not a neutrosophic continuous function and it is a $\mathcal{N}S_w$-continuouse function.

Theorem 8.1.2 Let $f\colon (Ҳ, \mathcal{T}_{\mathcal{N}}) \to (ƴ, \sigma_{\mathcal{N}})$ be neutrosophic S_w-continuous function and $\mathrm{g}\colon (ƴ, \sigma_{\mathcal{N}}) \to (\mathcal{Z}, \gamma_{\mathcal{N}})$ be a neutrosophic continuous function. Then $\mathrm{g}of\colon (Ҳ, \mathcal{T}_{\mathcal{N}})) \to (\mathcal{Z}, \gamma_{\mathcal{N}})$ is a neutrosophic S_w-continuous function.

Proof. Assume that K is a $\mathcal{NOS}$ in $\mathcal{Z}$. As $\mathrm{g}: (ƴ, \sigma_{\mathcal{N}}) \to (\mathcal{Z}, \gamma_{\mathcal{N}})$ is a neutrosophic continuous , $\mathrm{g}^{-1}(\mathrm{K})$ is a neutrosophic open set in ƴ. So $f^{-1}(\mathrm{g}^{-1}(\mathrm{K}))$ is a $\mathcal{N}S_wOS$ in Ҳ, as f is a $\mathcal{N}S_w$-continuous function. However $f^{-1}(\mathrm{g}^{-1}(\mathrm{K})) = (\mathrm{g}of)^{-1}(\mathrm{K})$. Then $(\mathrm{g}of)^{-1}(\mathrm{K})$ is $\mathcal{N}S_wOS$ in Ҳ. Therefore, $\mathrm{g}of$ is a $\mathcal{N}S_w$-continuous function. □

Corollary 8.1.1 Let $(Ҳ, \mathcal{T}_{\mathcal{N}})$ and $(ƴ, \sigma_{\mathcal{N}})$ represent two neutrosophic topological spaces. Then prove that a function $f\colon (Ҳ, \mathcal{T}_{\mathcal{N}}) \to (ƴ, \sigma_{\mathcal{N}})$ is neutrosophic S_w-continuous if and only if $f^{-1}(\mathrm{K})$ is $\mathcal{N}S_wCS$ in Ҳ for each $\mathcal{NCS}$ K in ƴ.

Proof. Assume that K is a $\mathcal{NCS}$ in ƴ. Then K^c is a $\mathcal{NOS}$ in ƴ. As f is $\mathcal{N}S_w$-continuous. Thus $f^{-1}(\mathrm{K}^c)$ is a $\mathcal{N}S_wOS$ in Ҳ. As $f^{-1}(\mathrm{K}^c) = [f^{-1}(\mathrm{K})]^c$ so $f^{-1}(\mathrm{K})$ is $\mathcal{N}S_wCS$ in Ҳ. Conversely, Assume that K is a $\mathcal{NOS}$ in ƴ. Then K^c is a $\mathcal{NCS}$ in ƴ. Via assumptions, $f^{-1}(\mathrm{K}^c)$ is $\mathcal{N}S_wCS$ in Ҳ. As $f^{-1}(\mathrm{K}^c) = [f^{-1}(\mathrm{K})]^c$ so $f^{-1}(\mathrm{K})$ is $\mathcal{N}S_wOS$ in Ҳ. Therefore, f is $\mathcal{N}S_w$-continuous. □

Proposition 8.1.1 Let $f\colon (Ҳ, \mathcal{T}_{\mathcal{N}}) \to (ƴ, \sigma_{\mathcal{N}})$ be a function. Then f is a $\mathcal{N}S_w$-continuous Function if and only if $f(\mathcal{N}S_wcl(\mathcal{V})) \subseteq \mathcal{N}cl(f(\mathcal{V}))$ for each $\mathcal{NS}$ $\mathcal{V}$ in Ҳ.

Proof. Assume that $\mathcal{V}$ is a $\mathcal{NS}$ in Ҳ and f is a $\mathcal{N}S_w$-continuous function. Then evidently $f(\mathcal{V}) \subseteq \mathcal{N}cl(f(\mathcal{V}))$. Now, $\mathcal{V} \subseteq f^{-1}(f(\mathcal{V})) \subseteq f^{-1}(\mathcal{N}cl(f(\mathcal{V}))$ and $\mathcal{N}S_wcl(\mathcal{V}) \subseteq \mathcal{N}S_wcl[f^{-1}(\mathcal{N}cl(f(\mathcal{V})))]$. As f is a $\mathcal{N}S_w$-continuous function and $f^{-1}(\mathcal{N}cl(f(\mathcal{V})))$ is a $\mathcal{N}S_wCS$. Thus $\mathcal{N}S_wcl(f^{-1}(\mathcal{N}cl(f(\mathcal{V})))) = f^{-1}(\mathcal{N}cl(f(\mathcal{V})))$. Hence, $f(\mathcal{N}S_wcl(\mathcal{V})) \subseteq \mathcal{N}cl(f(\mathcal{V}))$. Conversely, let $f\big(\mathcal{N}S_wcl(\mathcal{V})\big) \subseteq \mathcal{N}cl(f(\mathcal{V}))$, for all $\mathcal{NS}$ $\mathcal{V}$ in Ҳ, and $\mathcal{L}$ be a $\mathcal{NCS}$ in ƴ. Then $\mathcal{N}cl(f(f^{-1}(\mathcal{L})) = \mathcal{N}cl(\mathcal{L}) = \mathcal{L}$. Via assumptions, $f\big(\mathcal{N}S_wcl(f^{-1}(\mathcal{L}))\big) \subseteq$

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$\mathcal{N}cl(f(f^{-1}(\mathcal{L})) \subseteq \mathcal{L}$ and hence $\mathcal{N}S_wcl(f^{-1}(\mathcal{L})) \subseteq f^{-1}(\mathcal{L})$. Since $f^{-1}(\mathcal{L}) \subseteq \mathcal{N}cl(f^{-1}(\mathcal{L}))$ so $f^{-1}(\mathcal{L}) = \mathcal{N}S_wcl(f^{-1}(\mathcal{L}))$. This indicates that $f^{-1}(\mathcal{L})$ is a $\mathcal{N}S_wCS$ in Ҳ. Thus by Corollary (8.1.1), f is a $\mathcal{N}S_w$-continuous function. □

Proposition 8.1.2 Suppose that $f:(\text{Ҳ},\mathcal{T}_\mathcal{N}) \to (\text{Ƴ},\sigma_\mathcal{N})$ is a function. Then f is a $\mathcal{N}S_w$-continuous function if and only if $\mathcal{N}S_wcl(f^{-1}(\mathcal{L})) \subseteq f^{-1}(\mathcal{N}cl(\mathcal{L}))$ for every $\mathcal{N}S$ $\mathcal{L}$ in Ƴ.

Proof. Assume that $\mathcal{L}$ is any $\mathcal{N}S$ in Ƴ and f is a $\mathcal{N}S_w$-continuous function. Clearly $f^{-1}(\mathcal{L}) \subseteq f^{-1}(\mathcal{N}cl(\mathcal{L}))$. Then, $\mathcal{N}S_wcl(f^{-1}(\mathcal{L})) \subseteq \mathcal{N}S_wcl(f^{-1}(\mathcal{N}cl(\mathcal{L})))$. Since $\mathcal{N}cl(\mathcal{L})$ is a $\mathcal{N}CS$ in Ƴ. So by Corollary (8.1.1), $f^{-1}(\mathcal{N}cl(\mathcal{L}))$ is a $\mathcal{N}S_wCS$ in Ҳ. Thus, $\mathcal{N}S_wcl(f^{-1}(\mathcal{L})) \subseteq \mathcal{N}S_wcl(f^{-1}(\mathcal{N}cl(\mathcal{L}))) = f^{-1}(\mathcal{N}cl(\mathcal{L}))$. Conversely, let $\mathcal{N}S_wcl(f^{-1}(\mathcal{L})) \subseteq f^{-1}(\mathcal{N}cl(\mathcal{L}))$ for every $\mathcal{N}S$ $\mathcal{L}$ in Ƴ, and $\mathcal{L}$ be a $\mathcal{N}CS$ in Ƴ. And by assumption, $\mathcal{N}S_wcl(f^{-1}(\mathcal{L})) \subseteq f^{-1}(\mathcal{N}cl(\mathcal{L})) = f^{-1}(\mathcal{L})$. This show that $f^{-1}(\mathcal{L})$ is a $\mathcal{N}S_wCS$ in Ҳ. Thus by Corollary (8.1.1), f is a $\mathcal{N}S_w$-continuous function. □

Proposition 8.1.3 Let $f:(\text{Ҳ},\mathcal{T}_\mathcal{N}) \to (\text{Ƴ},\sigma_\mathcal{N})$ be a bijective function. Then f is $\mathcal{N}S_w$-continuous if and only if $\mathcal{N}int\left(f(\mathcal{V})\right) \subseteq f(\mathcal{N}S_wint(\mathcal{V}))$ for each $\mathcal{N}S$ $\mathcal{V}$ in Ҳ.

Proof. Assume that $\mathcal{V}$ is any $\mathcal{N}S$ in Ҳ and f is a $\mathcal{N}S_w$-continuous bijective. Let $f(\mathcal{V}) = \mathcal{L}$. Clearly $f^{-1}(\mathcal{N}int(\mathcal{L})) \subseteq f^{-1}(\mathcal{L})$. Since f is an injective mapping, $f^{-1}(\mathcal{L}) = \mathcal{V}$, so that $f^{-1}(\mathcal{N}int(\mathcal{L})) \subseteq \mathcal{V}$. Therefore, $\mathcal{N}S_wint(f^{-1}(\mathcal{N}int(\mathcal{L}))) \subseteq \mathcal{N}S_wint\ (\mathcal{V})$. Since f is a $\mathcal{N}S_w$-continuous, so $f^{-1}(\mathcal{N}int(\mathcal{L}))$ is $\mathcal{N}S_wOS$ in Ҳ and $f^{-1}(\mathcal{N}int(\mathcal{L})) \subseteq \mathcal{N}S_wint(\mathcal{V})$, $f(f^{-1}(\mathcal{N}int(\mathcal{L}))) \subseteq f(\mathcal{N}S_wint(\mathcal{V}))$. Thus, $\mathcal{N}int\left(f(\mathcal{V})\right) \subseteq f(\mathcal{N}S_wint(\mathcal{V}))$. Conversely, $\mathcal{N}int\left(f(\mathcal{V})\right) \subseteq f(\mathcal{N}S_wint(\mathcal{V}))$ for every neutrosophic set $\mathcal{V}$ in Ҳ. Let $\mathcal{L}$ be a $\mathcal{N}OS$ set in Ƴ. As f is surjective and by assumption, $\mathcal{N}int(\mathcal{L}) = \mathcal{N}int\left(f\left(f^{-1}(\mathcal{L})\right)\right) \subseteq f\left(\mathcal{N}S_wint(f^{-1}(\mathcal{L}))\right)$. It follows that $f^{-1}(\mathcal{L}) \subseteq \mathcal{N}S_wint(f^{-1}(\mathcal{L})$. Hence, $f^{-1}(\mathcal{L})$ is a $\mathcal{N}S_wOS$ in Ҳ. Therefore, f is a $\mathcal{N}S_w$-continuous function. □

Proposition 8.1.4 Suppose that $f:(\text{Ҳ},\mathcal{T}_\mathcal{N}) \to (\text{Ƴ},\sigma_\mathcal{N})$ is a function. Then f is a $\mathcal{N}S_w$-continuous function if and only if $f^{-1}(\mathcal{N}int(\mathcal{L})) \subseteq \mathcal{N}S_wint(f^{-1}(\mathcal{L}))$ for every $\mathcal{N}S$ $\mathcal{L}$ in Ƴ.

Proof. Assume that $\mathcal{L}$ is any $\mathcal{N}S$ in Ƴ and f is a $\mathcal{N}S_w$-continuous mapping. Clearly $f^{-1}(\mathcal{N}int(\mathcal{L})) \subseteq f^{-1}(\mathcal{L})$ then $\mathcal{N}S_wint(f^{-1}(\mathcal{N}int(\mathcal{L}))) \subseteq \mathcal{N}S_wint(f^{-1}(\mathcal{L}))$. As $\mathcal{N}int(\mathcal{L})$ is $\mathcal{N}OS$ in Ƴ and f is $\mathcal{N}S_w$-continuous, $f^{-1}(\mathcal{N}int(\mathcal{L}))$ is $\mathcal{N}S_wOS$ in Ҳ. Hence $\mathcal{N}S_wint(f^{-1}(\mathcal{N}int(\mathcal{L}))) = f^{-1}(\mathcal{N}int(\mathcal{L})) \subseteq \mathcal{N}S_wint(f^{-1}(\mathcal{L}))$. Conversely, $f^{-1}(\mathcal{N}int(\mathcal{L})) \subseteq \mathcal{N}S_wint(f^{-1}(\mathcal{L}))$ for all $\mathcal{N}S$ $\mathcal{L}$ in Ƴ. And Đ be any $\mathcal{N}OS$ in Ƴ. Then $f^{-1}(\text{Đ}) = f^{-1}(\mathcal{N}int(\text{Đ})) \subseteq \mathcal{N}S_wint(f^{-1}(\text{Đ}))$ and thus, $f^{-1}(\text{Đ}) = \mathcal{N}S_wint(f^{-1}(\text{Đ}))$. This implies that $f^{-1}(\text{Đ})$ is $\mathcal{N}S_wOS$ in Ҳ. Hence f is a $\mathcal{N}S_w$-continuous function. □

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Proposition 8.1.5 Suppose that $f:(Ӽ,\mathcal{T}_{\mathcal{N}})\rightarrow(Ƴ,\sigma_{\mathcal{N}})$ is a bijective function. Then f is a $\mathcal{N}S_w$-continuous function if and only if $f(\mathcal{N}S_wfr(\mathcal{V}))\subseteq\mathcal{N}S_wfr(f(\mathcal{V}))$ for each $\mathcal{N}S$ $\mathcal{V}$ in Ӽ.

Proof. Assume that f is a $\mathcal{N}S_w$-continuous bijective function, and let $\mathcal{V}$ be a $\mathcal{N}S$ in Ӽ. By Definition, $\mathcal{N}S_wfr(\mathcal{V})=\mathcal{N}S_wcl(\mathcal{V})\cap\mathcal{N}S_wcl(\mathcal{V}^c)$. And by Proposition (8.1.1), $f(\mathcal{N}S_wcl(\mathcal{V}))\subseteq\mathcal{N}cl(f(\mathcal{V}))$ then, $f\big(\mathcal{N}S_wfr(\mathcal{V})\big)=f\big(\mathcal{N}S_wcl(\mathcal{V})\big)\cap f\big(\mathcal{N}S_wcl(\mathcal{V}^c)\big)\subseteq\mathcal{N}cl(f(\mathcal{V}))\cap\mathcal{N}cl(f(\mathcal{V}))^c=\mathcal{N}fr(f(\mathcal{V}))$. Conversely, $f(\mathcal{N}S_wfr(\mathcal{V}))\subseteq\mathcal{N}fr(f(\mathcal{V}))$ for each $\mathcal{N}S$ $\mathcal{V}$ in Ӽ. Thus $f(\mathcal{N}S_wcl(\mathcal{V}))=f(\mathcal{N}S_wint(\mathcal{V}))\cup f(\mathcal{N}S_wfr(\mathcal{V}))\subseteq f(\mathcal{V})\cup\mathcal{N}fr(f(\mathcal{V}))\subseteq\mathcal{N}cl(f(\mathcal{V}))$. Therefore, by Proposition (8.1.1), f is a $\mathcal{N}S_w$-continuous function. □

Proposition 8.1.6 Suppose that $f:(Ӽ,\mathcal{T}_{\mathcal{N}})\rightarrow(Ƴ,\sigma_{\mathcal{N}})$ is a neutrosophic bijective function. Then f is a $\mathcal{N}S_w$-continuous function if and only if $\mathcal{N}S_wfr(f^{-1}(\mathcal{L}))\subseteq f^{-1}(\mathcal{N}fr(\mathcal{L}))$ for each $\mathcal{N}S$ $\mathcal{L}$ in Ƴ.

Proof. Assume that f is a $\mathcal{N}S_w$-continuous bijective function, and let $\mathcal{L}$ be a $\mathcal{N}S$ in Ƴ. By Proposition (8.1.2), $\mathcal{N}S_wcl\big(f^{-1}(\mathcal{L})\big)\subseteq f^{-1}\big(\mathcal{N}cl(\mathcal{L})\big)$. So $f^{-1}\big(\mathcal{N}fr(\mathcal{L})\big)=f^{-1}\big(\mathcal{N}cl(\mathcal{L})\cap\mathcal{N}cl(\mathcal{L}^c)\big)=\big(\mathcal{N}cl(\mathcal{L})\big)\cap f^{-1}\big(\mathcal{N}cl(\mathcal{L}^c)\big)\supseteq\mathcal{N}S_wcl(f^{-1}(\mathcal{L}))\cap\mathcal{N}S_wcl\big(f^{-1}(\mathcal{L}^c)\big)=\mathcal{N}S_wcl(f^{-1}(\mathcal{L}))\cap\mathcal{N}S_wcl[(f^{-1}(\mathcal{L}))^c]=\mathcal{N}S_wfr(f^{-1}(\mathcal{L}))$. Thus , $\mathcal{N}S_wfr(f^{-1}(\mathcal{L}))\subseteq f^{-1}(\mathcal{N}fr(\mathcal{L}))$. Conversely since $\mathcal{N}S_wfr(f^{-1}(\mathcal{L}))\subseteq f^{-1}(\mathcal{N}fr(\mathcal{L}))$ for all $\mathcal{N}S$ $\mathcal{L}$ in Ƴ. This follow that $\mathcal{N}S_wcl(f^{-1}(\mathcal{L}))\subseteq f^{-1}(\mathcal{N}cl(\mathcal{L}))$. Therefore, by Proposition (8.1.2) f is a $\mathcal{N}S_w$-continuous function. □

Theorem 8.1.3 Let $f:(Ӽ,\mathcal{T}_{\mathcal{N}})\rightarrow(Ƴ,\sigma_{\mathcal{N}})$ be a neutrosophic function, then following claims are equivalence.

1. f is a $\mathcal{N}S_w$-continuous function.
2. There exist a $\mathcal{N}S_wOS$ $\mathcal{G}$ so that $P=\langle ӽ, ᶅ, ƫ, ᶇ\rangle\in\mathcal{G}\subseteq f^{-1}(Á)$. For any neutrosophic neighborhood Á of $f(\langle ӽ, ᶅ, ƫ, ᶇ\rangle)$, and any neutrosophic point $\langle ӽ, ᶅ, ƫ, ᶇ\rangle\in Ӽ$,
3. There exist a neutrosophic S_w-open Set $\mathcal{G}$ in Ӽ such that $\langle ӽ, ᶅ, ƫ, ᶇ\rangle\in\mathcal{G}$ and $f(\mathcal{G})\subseteq Á$. For any neutrosophic point $\langle ӽ, ᶅ, ƫ, ᶇ\rangle\in Ӽ$, and any neutrosophic neighborhood Á of $f(\langle ӽ, ᶅ, ƫ, ᶇ\rangle)$.

Proof. (1) → (2) : Assume that Á is a neutrosophic neighborhood of $f(\langle ӽ, ᶅ, ƫ, ᶇ\rangle)$, and $\langle ӽ, ᶅ, ƫ, ᶇ\rangle$ is a neutrosophic point in Ӽ. Then there exist a $\mathcal{N}OS$ $\mathcal{G}$ in Ӽ, such as $f(\langle ӽ, ᶅ, ƫ, ᶇ\rangle)\in\mathcal{G}\subseteq Á$. As f is a $\mathcal{N}S_w$-continuous, therefore it implies that is $f^{-1}(\mathcal{G})$ is a $\mathcal{N}S_wOS$ in Ӽ and $\langle ӽ, ᶅ, ƫ, ᶇ\rangle\in f^{-1}(f(\langle ӽ, ᶅ, ƫ, ᶇ\rangle))\subseteq f^{-1}(\mathcal{G})\subseteq f^{-1}(Á)$. This implies (2) is true.

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(2) → (3) : Assume that Á is a neutrosophic neighborhood of f(⟨ӽ , ɫ, ŧ, ɳ⟩), and ⟨ӽ , ɫ, ŧ, ɳ⟩ is a neutrosophic point in Ӽ. The criteria for (2) indicates that there exist a $\mathcal{N}S_wOS$ $\mathcal{G}$ in Ӽ such as ⟨ӽ , ɫ, ŧ, ɳ⟩ $\in \mathcal{G} \subseteq f^{-1}$(Á). Thus ⟨ӽ , ɫ, ŧ, ɳ⟩ $\in \mathcal{G}$ and $f(\mathcal{G}) \subseteq f^{-1}(f$(Á)). Hence (3) is true.

(3) → (1): Let $\mathcal{L}$ be a $\mathcal{N}OS$ in $\mathcal{Y}$ and let ⟨ӽ , ɫ, ŧ, ɳ⟩ $\in f^{-1}(\mathcal{L})$. As $\mathcal{L}$ is a $\mathcal{N}OS$, f(⟨ӽ , ɫ, ŧ, ɳ⟩) $\in \mathcal{L}$, and so $\mathcal{L}$ is neutrosophic neighborhood of f(⟨ӽ , ɫ, ŧ, ɳ⟩). It follows from (3) that there exist a $\mathcal{N}S_wOS$ $\mathcal{G}$ in Ӽ such that ⟨ӽ , ɫ, ŧ, ɳ⟩ $\in \mathcal{G}$ and $f(\mathcal{G}) \subseteq \mathcal{L}$ so that ⟨ӽ , ɫ, ŧ, ɳ⟩ $\in \mathcal{G} \subseteq f^{-1}(f(\mathcal{G})) \subseteq f^{-1}(\mathcal{L})$. Hence by definition $f^{-1}(\mathcal{L})$ is a $\mathcal{N}S_wOS$ in Ӽ, and f is a $\mathcal{N}S_w$-continuous function. □

8.2 Neutrosophic S_w-Irresolute Functions

The notion of neutrosophic S_w-irresolute function in neutrosophic topological spaces is presented in this part. We also talk about connection with the $\mathcal{N}S_w$-continuous function.

Definition 8.2.1 Let $(\mathcal{X}, \mathcal{T}_\mathcal{N})$ and $(\mathcal{Y}, \sigma_\mathcal{N})$ be two $\mathcal{N}TS$'s. Then the neutrosophic function f: (Ӽ , $\mathcal{T}_\mathcal{N}$) → $(\mathcal{Y}, \sigma_\mathcal{N})$ is said to be a neutrosophic S_w-irresolute ($\mathcal{N}S_w$-irresolute , for short) function if the Inverse image of each neutrosophic S_w-open Set in $(\mathcal{Y}, \sigma_\mathcal{N})$ is a neutrosophic S_w-open set in (Ӽ , $\mathcal{T}_\mathcal{N}$).

Example 8.2.1 In Example (8.1.1), since The reverse of each $\mathcal{N}S_wOS$ in $\mathcal{Y}$ is $\mathcal{N}S_wOS$ in Ӽ. Thus f is a $\mathcal{N}S_w$-irresolute function.

Theorem 8.2.1 Neutrosophic S_w-irresolute functions are all $\mathcal{N}$-continuous functions.

Proof. Assume that f: (Ӽ , $\mathcal{T}_\mathcal{N}$) → $(\mathcal{Y}, \sigma_\mathcal{N})$ is a $\mathcal{N}$-continuous function, and $\mathcal{V}$ is a $\mathcal{N}OS$ in $(\mathcal{Y}, \sigma_\mathcal{N})$. Thus $f^{-1}(\mathcal{V})$ is $\mathcal{N}OS$ in (Ӽ , $\mathcal{T}_\mathcal{N}$). As each $\mathcal{N}OS$ is $\mathcal{N}S_wOS$, so $\mathcal{V}$ and $f^{-1}(\mathcal{V})$ are both $\mathcal{N}S_wOS$'s. Hence, f is neutrosophic S_w-irresolute function. □

The example that follows demonstrates that Theorem (8.2.1)'s opposite is untrue.

Example 8.2.2 In Example (8.1.2), f is not a $\mathcal{N}$-continuous function, and it is a $\mathcal{N}S_w$-irresolute function.

Corollary 8.2.1 Let f: (Ӽ , $\mathcal{T}_\mathcal{N}$) → $(\mathcal{Y}, \sigma_\mathcal{N})$ be a $\mathcal{N}S_w$-irresolute function , then f is a neutrosophic S_w-continuous function.

Proof. Let f: (Ӽ , $\mathcal{T}_\mathcal{N}$) → $(\mathcal{Y}, \sigma_\mathcal{N})$ be a neutrosophic S_w-irresolute function. And $\mathcal{V}$ be a neutrosophic open set in $(\mathcal{Y}, \sigma_\mathcal{N})$. As each $\mathcal{N}OS$ is a $\mathcal{N}S_wOS$, $\mathcal{V}$ is a $\mathcal{N}S_wOS$. And $f^{-1}(\mathcal{V})$ is a $\mathcal{N}S_wOS$ in (Ӽ , $\mathcal{T}_\mathcal{N}$), as f is a $\mathcal{N}S_w$-irresolute function. Therefore, f is a $\mathcal{N}S_w$-continuous function. □

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Theorem 8.2.2 Let $f:(\text{Ҳ},\mathcal{T}_{\mathcal{N}})\rightarrow(\text{Ƴ},\sigma_{\mathcal{N}})$ be a neutrosophic S_w-irresolute function and $g:(\text{Ƴ},\sigma_{\mathcal{N}})\rightarrow(\mathcal{Z},\gamma_{\mathcal{N}})$ be a neutrosophic S_w-irresolute function. Then $gof:(\text{Ҳ},\mathcal{T}_{\mathcal{N}})\rightarrow(\mathcal{Z},\gamma_{\mathcal{N}})$ is a neutrosophic S_w-irresolute function.

Proof. Assume that Đ is a $\mathcal{N}S_wOS$ in $\mathcal{Z}$. As $g:(\text{Ƴ},\sigma_{\mathcal{N}})\rightarrow(\mathcal{Z},\gamma_{\mathcal{N}})$ is a $\mathcal{N}S_w$-irresolute, $g^{-1}(\text{Đ})$ is a $\mathcal{N}S_wOS$ in Ƴ. As also f is a $\mathcal{N}S_w$-irresolute function, $f^{-1}(g^{-1}(\text{Đ}))$ is $\mathcal{N}S_wOS$ in Ҳ. But $f^{-1}(g^{-1}(\text{Đ}))=(gof)^{-1}(\text{Đ})$. Thus $(gof)^{-1}(\text{Đ})$ is $\mathcal{N}S_wOS$ in Ҳ. Therefore, gof is a $\mathcal{N}S_w$-irresolute function. □

Theorem 8.2.3 Let $f:(\text{Ҳ},\mathcal{T}_{\mathcal{N}})\rightarrow(\text{Ƴ},\sigma_{\mathcal{N}})$ be a neutrosophic S_w-irresolute function and $g:(\text{Ƴ},\sigma_{\mathcal{N}})\rightarrow(\mathcal{Z},\gamma_{\mathcal{N}})$ be a neutrosophic S_w-continuous function. Then $gof:(\text{Ҳ},\mathcal{T}_{\mathcal{N}})\rightarrow(\mathcal{Z},\gamma_{\mathcal{N}})$ is a neutrosophic S_w-irresolute function.

Proof. Assume that Đ is a $\mathcal{N}OS$ in $\mathcal{Z}$. As $g:(\text{Ƴ},\sigma_{\mathcal{N}})\rightarrow(\mathcal{Z},\gamma_{\mathcal{N}})$ is a $\mathcal{N}S_w$-continuous, $g^{-1}(\text{Đ})$ is a $\mathcal{N}S_wOS$ in Ƴ. As f is a $\mathcal{N}S_w$-irresolute function, $f^{-1}(g^{-1}(\text{Đ}))$ is $\mathcal{N}S_wOS$ in Ҳ. But $f^{-1}(g^{-1}(\text{Đ}))=(gof)^{-1}(\text{Đ})$. Thus $(gof)^{-1}(\text{Đ})$ is a $\mathcal{N}S_wOS$ in Ҳ. Therefore, gof is a $\mathcal{N}S_w$-irresolute function. □

CONCLUSION

In this work, we have redefined and investigated $\mathcal{N}S_wOS$, a novel class of $\mathcal{N}OS$ in neutrosophic topological spaces. After that, we discussed a several of its characteristics. Therefore, we anticipate that the results presented within the above data will facilitate improvement and advancing future research on neutrosophic topological spaces.

Conflict of interests.

There are non-conflicts of interest.

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