Article

JOURNAL OF UNIVERSITY OF BABYLON
For Pure and Applied Sciences (JUBPAS)

Vol. 34 ; No. 2 | 2024

ISSN: 2312-8135 | Print ISSN: 1992-0652

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# Investigation of Isovector Balance and Effective Deformation for $^{106}$Mo and $^{104}$Zr Isotones within IBM-2 Framework

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# دراسة التوازن الايزوفكتوري والتشوه الفعال لنظيري $^{106}$Mo و $^{104}$Zr ضمن (IBM-2)

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**Accepted:** 6/5/2026 **Published:** 30/6/2026

## ABSTRACT

Background: The Interacting boson model (IBM-2) Has been applied to study the effect of nuclear structure properties on the spectral features of $^{106}$Mo and $^{104}$Zr isotones which have neutron number N=64.

Materials and Methods: The Interacting Boson Model (IBM-2) was applied to analyze the deviation $\Delta R$ from ideal behavior, for the U(5) to SU(3) and the weight of each determination in each nucleus. The energy ratio $R(4/2)$, $(E0_2^+/E0_1^+)$ shape coexistence, the effective nuclear deformation parameter $\chi_{eff}$ and the isovector balance (IV) have been determined to confirm the approach to rotational distortion. Electric quadrupole transition probabilities B(E2) and magnetic dipole transition probabilities B(M1) have been checked.

Results: The effective nuclear deformation parameter $\chi_{eff} < 0$, in $^{106}$Mo and $^{104}$Zr was negative, stay away from zero. The isovector balance (IV) describes the relationship between the proton $\chi_P$ and neutron $\chi_V$ parameters within the IBM-2 framework, in $^{106}$Mo and $^{104}$Zr, it was found to reinforce the idea of transitioning from vibrational to rotational behavior, this applies perfectly to the results of electrical transition ratios ($\Re$, $\Re$' and $\Re$"). The energy level sequences and ratios indicated that the $^{106}$Mo and $^{104}$Zr are transitional nuclei between vibrational U(5) and rotational SU(3) symmetries. Collective states associated with proton-neutron symmetry and symmetry mixing play an important role in studying collective nuclear properties by changing the Majorana parameter $(\xi_2)$ showed a significant impact was shown on determining the mixing locations of some levels , $2_3^+$, $4_3^+$, $6_3^+$ and $1_1^+$ while other levels remained unaffected and their values remained constant at specific values, such as the levels $2_2^+$ $3_1^+$ and $5_1^+$ in $^{106}$Mo and $^{104}$Zr. No experimental values of the $1_1^+$ level are available.

Conclusion: The study shows that the $^{106}$Mo and $^{104}$Zr isotones exhibit transitional behavior between the vibrational and rotational symmetries, with a clear tendency toward rotational deformation. The negative values of the effective nuclear deformation parameter $\chi_{eff}$ and the isovector balance emphasize the importance of proton-neutron symmetry in describing the collective properties of these isotones.

Key words: IBM-2, MSS $^{106}$Mo and $^{104}$Zr isotones, $\Delta R$ $\chi_{eff}$ , $w$ and IV

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## INTRODUCTION

A fundamental component of nuclear physics, the Interacting Boson Model (IBM) provides a straight forward but effective depiction of intricate nuclear interactions. It focuses on the collective behavior of paired nucleons (protons or neutrons) as opposed to their individual dynamics, treating them as bosons. Nucleons are divided into two primary boson kinds in this model ş-bosons (spin 0) and ɗ-bosons (spin 2). IBM is a vital tool for researching medium and heavy nuclei since these acts as building blocks to simulate nuclear energy levels, collective excitations, and shape transformations [1]. Atomic nuclei are known to exhibit changes in their levels of energy and electromagnetic rates of transition when the number of protons (or neutrons) is altered, which causes the shape to change from one type of collective behavior to another [2]-[4] In recent decades, there has been a notable. emphasis on the study of dynamical symmetries due to their remarkable ability to reveal the properties of complex systems [5],[6]. The interacting boson model (IBM) is a theoretical framework that uses interacting bosons to explain collective nuclear states. The IBM models nuclei as a system of interacting bosons that depict the behavior of nucleons as a whole. A Hamiltonian that incorporates numerous symmetries is used by the bosons to interact [5]. Given that the Hamiltonian can be defined in terms of a group of generators that satisfy particular Lie algebra, the IBM has the property of dynamical symmetry. Neutron-rich nuclei in the $40 \leq 50$ region have garnered theoretical and experimental interest throughout the past years. $^{106}Mo$ and $^{104}Zr$ nuclei show an evolution from vibration to near rotation [7],[8]. Such changes in the degree of collectivity are even stronger in the zirconium isotopic. Many authors are interested in the region of neutron excess nuclei at mass close to A-100 due to the detection of the phase transition (from spherical to well deformed nuclei and from well deformed to gamma soft) [9].

## MATERIALS AND METHODS

An additional development of the interacting boson model-1, is the interacting boson model-2. Since the interacting boson model-2 can theoretically be constructed from the shell model, this technique provides a microscopic foundation for the interacting boson model IBM-1, representation of collective nuclear states. This advancement is predicated on the notion of generalized fermion seniority [10],[11], has been presented by Arima et al [12]-[17]. A direct physical explanation of the bosons as coupled pairs of particles with angular momentum $J^{\pi} = 0$ and $J^{\pi} = 2$ has been provided by the model. Three components make up the Hamiltonian operator $\hat{U}$ in interacting boson model-2: one for each of the proton $\hat{U}_{P}$ and neutron $\hat{U}_{v}$ bosons, and a third portion that describes the interaction between the two $V_{Pv}$ [18]-[20].

$$\hat{U} = \hat{U}_{P} + \hat{U}_{v} + V_{Pv} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

A simple schematic Hamiltonian that takes into account microscopic factors is provided by [18]-[20]

$$\hat{U} = \varepsilon\left(n_{dP} + n_{dv}\right) + \kappa Q_{P}.Q_{v} + V_{PP} + V_{vv} + M_{Pv} \dots\dots\dots\dots\dots\dots\dots\dots (2)$$

where

$$Q_{f} = (d_{f}^{\dagger}s_{f} + s_{f}^{\dagger}d_{f})_{f}^{2} + \chi_{f}(d_{f}^{\dagger}d_{f})_{f}^{2} \quad f = P, v \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

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$s^\dagger$ and $d^\dagger$ are the creation operators and $s$ are $d$ the annihilation operators, The Hamiltonian operator $\hat{U}$ in the interacting boson model-2 consists of several terms, where $\varepsilon$ represents the single boson energy parameter, $n_{d\pi}$ and $n_{d\nu}$ denote the proton and neutron $d$-boson number operators, respectively, $\kappa$ is the quadrupole-quadrupole interaction strength parameter, $Q_\pi$ and $Q_\nu$ are the proton and neutron quadrupole operators, $V_{\pi\pi}$ and $V_{\nu\nu}$ describe the interactions between proton-proton and neutron-neutron bosons, respectively, while $M_{\pi\nu}$ represents the Majorana interaction. term between proton and neutron bosons.

$$V_{\rho\rho} = \sum_{L=0,2,4} \frac{1}{2}(2L+1)^{\frac{1}{2}} C_L^{\rho}\left[(d_\rho^\dagger d_\rho^\dagger)^{(L)}.(d_\rho d_\rho)^{(L)}\right]^{(0)} \dots\dots\dots\dots\dots\dots\dots\dots (4)$$

Proton and neutron energies are represented by $\varepsilon_\pi, \varepsilon_\nu$ respectively, and are taken to be equal $\varepsilon_\pi = \varepsilon_\nu = \varepsilon$.The operators $(s^\dagger, d^\dagger)$ represent creation operators, while $(s, d)$ denote annihilation operators. The Majorana operator $M_{\pi\nu}$, represented by the final term in Eq.(2), is usually used to remove states of mixed proton-neutron symmetry. This term can be written as [19]-[21]

$$M_{\pi\nu} = \xi_2(s_\nu^\dagger d_\pi^\dagger - d_\nu^\dagger s_\pi^\dagger)^{(2)}.(s_\nu d_\pi - d_\nu s_\pi)^{(2)} + \sum_{k=1,3} \xi_k (d_\nu^\dagger d_\pi^\dagger)^{(k)} - (d_\nu d_\pi)^{(k)} \dots\dots\dots (5)$$

If there is experimental evidence for a so-called "mixed symmetry state," the Majorana parameter changes to ascertain where these states are located in the spectrum. The Hamiltonian Eq. (2) is diagonalized to obtain the energy levels then permitting the parameters $\varepsilon, \kappa, \chi_\pi, \chi_\nu$ and $C_L$ to change until Eq. (2) yields the best fit to the experimental spectrum [21]. $U(5)$ limit is when $\varepsilon \gg \kappa$, $SU(3)$ limit is when $\varepsilon \ll \kappa$ and $\chi_\pi = \chi_\nu = -\sqrt{7}/2$, and $O(6)$ limit is when $\varepsilon \ll \kappa$ and $\chi_\nu = -\chi_\pi$. The majority of nuclei fall between two of these three limiting situations rather than strictly falling into one of them. For a number of isotones, the IBM allows for a seamless transition between the limiting instances. The form of the generic single boson transition operator of angular momentum $\ell$ is identical to IBM, with the exception that each term must take into account $\pi, \nu$ the degree of freedom [22]-[26].

$$T^{(l)} = \alpha_{2\rho}\delta_{\ell 2}[d^\dagger s + s^\dagger d]_\rho^{(2)} + \beta_{\ell\rho}[d^\dagger d]_\rho^{(\ell)} + \gamma_{0\rho}\delta_{\ell 0}[s^\dagger s]_\rho^{(0)} \dots \rho = \pi \; or \; \nu \dots\dots\dots\dots\dots\dots (6)$$

For $E2$ operator [24],[25]

$$T^{E2} = e_\pi Q_\pi + e_\nu Q_\nu \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

where $Q_\rho$ is the exact same as in Eq. (3) and $\alpha_2, \beta_l, \gamma_0$ are the coefficients of the various terms in the operators. $e_\pi, e_\nu$ are boson effective charges that, depending on the boson number $N$, can have any value that matches the experimental results. The nuclear structure affects the effective charge values, however for the $0_1^+$ and $2_1^+$ states, the ratio $e_\nu/e_\pi$ depends simply on the maximal $F-$spin assumption. It is possible to get the two effective charges $e_\pi$ and $e_\nu$ [27][29] by utilizing boson number $N_\pi$, $N_\nu$ and experimental $B(E2; 2_1^+ \to 0_1^+)$ values for a series of isotones to produce a graph between $N_\nu/N_\pi$ and where described as letting $\ell = 1$ in Eq.(6) yields the M1 operator, which is expressed as [21]- [25]

$$T^{(M1)} = [\tfrac{3}{4\pi}]^{\frac{1}{2}}(g_\pi L_\pi^{(1)} + g_\nu L_\nu^{(1)}) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (8)$$

Where $g_\pi$ , $g_\nu$ are the boson $g-$factors in units of magneton Bohr $\mu_N$ and $L_\rho^{(1)} = \sqrt{10}(d^\dagger d)_\rho^{(1)}$, this operator can also be expressed as:

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$$Ŧ^{(M1)} = [\frac{3}{4Ᵽ}]^{\frac{1}{2}}[\frac{1}{2}(g_Ᵽ + g_ν)(L_Ᵽ^{(1)} + L_ν^{(1)}) + \frac{1}{2}(g_Ᵽ - g_ν)(L_Ᵽ^{(1)} - L_ν^{(1)})] \ldots\ldots (9)$$

The previous Eq. can be stated as follows for the Ṃ1 transition, with the first term on the right is diagonal:

$$Ŧ^{(M1)} = 0.77\left[\left(d^{\dagger}d\right)_{Ᵽ}^{(1)} - \left(d^{\dagger}d\right)_{ν}^{(1)}\right](g_Ᵽ - g_ν) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

There are few experimental absolute B(M1) transitions available to support IBM estimates. Within the dynamical symmetries U(5), SU(3) and O(6), the branching ratios of the decreased transition probability for electric quadrupole transitions follow the following relations [12].

$$\Re = \frac{B(E2;4_1^+ \to 2_1^+)}{B(E2;2_1^+ \to 0_1^+)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$\Re' = \frac{B(E2;2_2^+ \to 2_1^+)}{B(E2;2_1^+ \to 0_1^+)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$\Re'' = \frac{B(E2;0_2^+ \to 2_1^+)}{B(E2;2_1^+ \to 0_1^+)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

They can be expressed in terms of N, the total number of bosons [13]:

$$\Re = \Re' = \Re'' = \frac{2(N-1)}{N} < 2: \quad U(5) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

$$\Re \approx \frac{10}{7};\ \Re' = \Re'' = 0: \quad SU(3) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

$$\Re = \Re' < \frac{10}{7};\ \Re'' = 0: \quad O(6) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The deviation from ideal behavior, $\Delta R$ for the SU(3) and O(6) using the following Eq..

$$\Delta R = R^{Exp.}{}_{4/2} - R^{Ideal.}{}_{4/2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

then, the weight of each limit for these nuclei was evaluated using the Eq.

$$w(U5) = (R_{SU3} - R_{\frac{4}{2}})/(R_{SU3} - R_{U5}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

$$w(SU3) = (R_{4/2} - R_{U5})/(R_{SU3} - R_{U5}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

The effective average nuclear deformation was calculated using the Eq.

$$\chi_{eff} = \frac{N_Ᵽ\chi_Ᵽ + N_ν\chi_ν}{N_Ᵽ + N_ν} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

The Isovector balance (IV) was analyzed to study the relationship between the shape of deformation parameters of protons $\chi_Ᵽ$ and neutrons $\chi_ν$ using the Eq. (21,22)[10],[16]. According to the contribution of the boson number $IV_1$ or without $IV_2$.

$$IV_1 = \frac{N_Ᵽ\chi_Ᵽ - N_ν\chi_ν}{N_Ᵽ\chi_Ᵽ + N_ν\chi_ν} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

$$IV_2 = |\chi_Ᵽ - \chi_ν|/|\chi_Ᵽ| + |\chi_ν| \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

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## RESULTS AND DISCUSSION:

The isotones $^{106}$Mo and $^{104}$Zr have proton numbers 42 and 40 which are equivalent to (4 and 5) hole proton boson numbers with neutron number 64 which is equivalent to (7) neutron particle bosons. The interacting boson model-2 Hamiltonian have been employed to study the nuclear structure of the $^{106}$Mo and $^{104}$Zr isotones through parameters which are listed in table (1). The deviation from ideal behavior, $\Delta R$, for the U(5) and SU(3) limits and the weight of each determination in each nucleus was also determined using Eq. (17-19) as shown in figure(1). The effective deformation parameter $\chi_{eff.}$ was calculated based on deformation parameters of protons $\chi_\pi$ and neutrons $\chi_\nu$ , which serves as a fundamental indicator for determining the nature of nuclear deformation using Eq. (20),also the isovector balance according to the contribution of the boson number $IV_1$ or without $IV_2$ was analyzed to study the relationship between the shape parameters of protons $\chi_\pi$ and neutrons $\chi_\nu$ using Eq. (21,22) as shown in figure (2). Shape coexistence,($E0_2^+/E0_1^+$) and ($E_{4/2}$ , $E_{6/2}$ and $E_{8/2}$) limits was calculated as drawn in figure (3). The energy levels computed with IBM-2 and the available experimental data are compared in figure (4).

Table 1: The parameters have been applied to $^{106}$Mo and $^{104}$Zr isotones (in MeV) unless $\chi$ in the interacting boson model-2 Hamiltonian.

| isotones | The parameter | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $N_\nu$ | $N_\pi$ | $\varepsilon$ | $\kappa$ | $\chi_\nu$ | $\chi_\pi$ | $\xi_2$ | $\xi_{1,3}$ | $C_\nu^L$ | $C_\pi^L$ |
| $^{106}$Mo | 7 | 4 | 0.54 | -0.15 | -0.2 | -0.5 | 0.04 | -0.012 | -0.1, -0.3, 0.03 | 0.01,0.02,0.01 |
| $^{104}$Zr | 7 | 5 | 0.44 | -0.16 | -0.2 | -0.6 | 0.05 | -0.012 | -0.1, -0.3, 0.03 | 0.01,0.02, 0.03 |

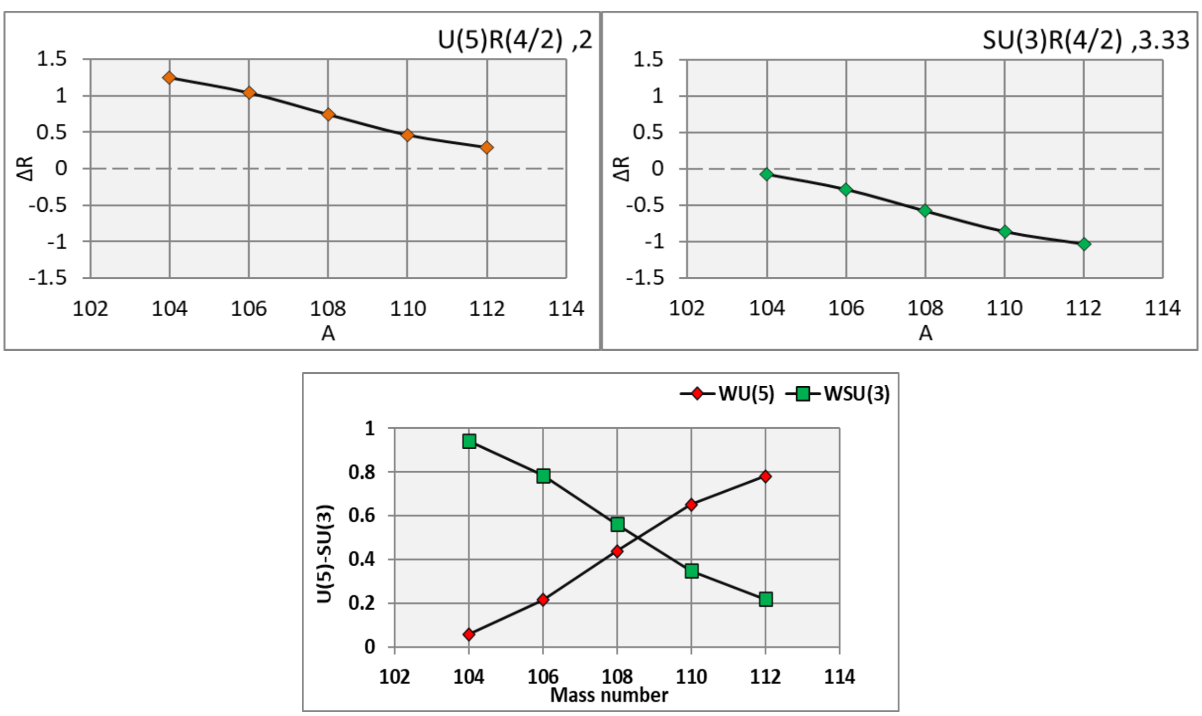


Figure 1: The magnitude of the deviation from ideal behavior $\Delta R$, between determinations U(5) and SU(3), and the weighting of each determination

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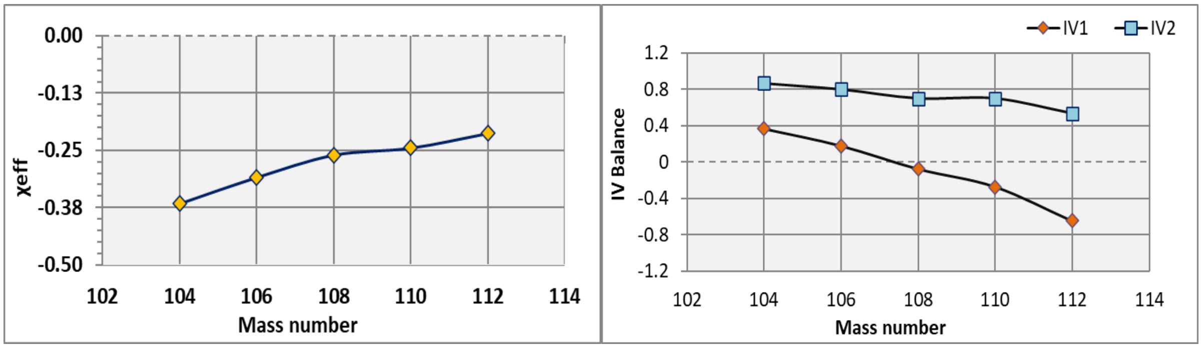


Figure 2: The effective deformation parameter $\chi_{eff.}$ , Isovector balance according to the contribution of the boson number $IV_1$ or without $IV_2$ for $^{106}$Mo and $^{104}$Zr isotones as a function of mass numbers.

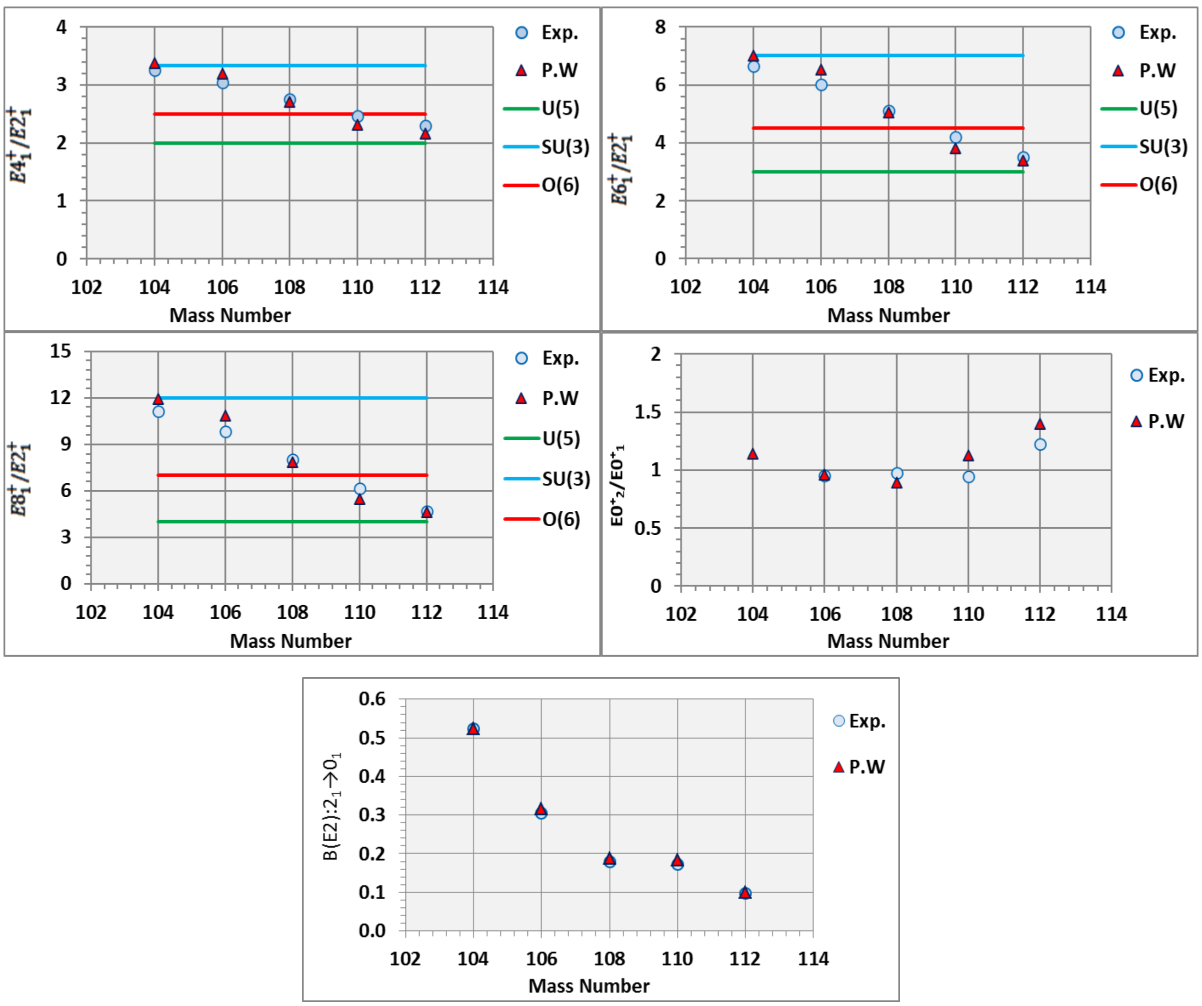


Figure 3: Theoretical, and experimental [29],[30] energy ratios ($E_{4/2}$ , $E_{6/2}$ and $E_{8/2}$) and shape coexistence, ($E0_2^+/E0_1^+$) and B(E2) to the $2_1^+$level for $^{106}$Mo and $^{104}$Zr isotopes as a function of mass numbers respectively.

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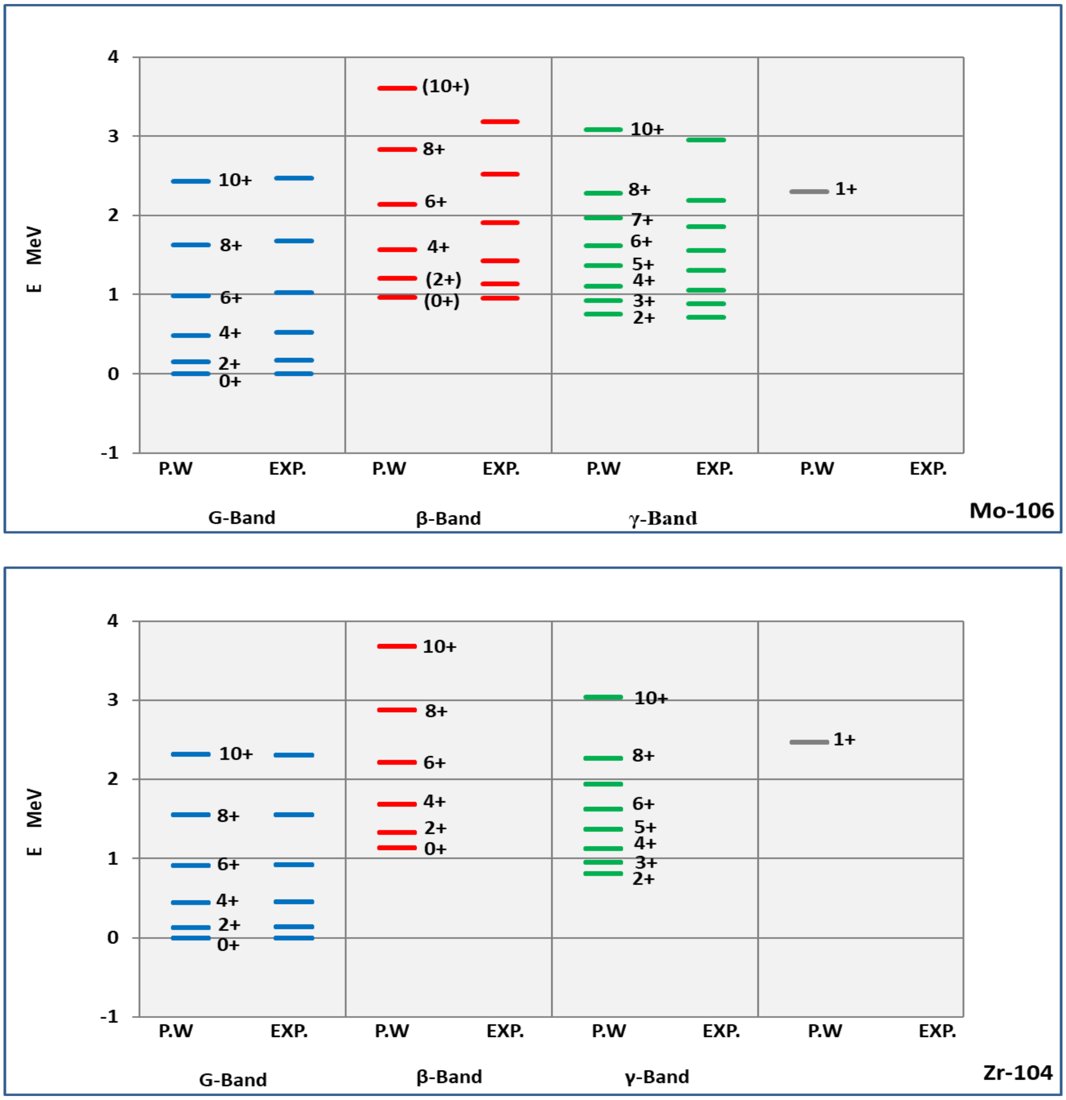


Figure 4: Comparison of the estimated and experimental states [29],[30] for $^{106}$Mo and $^{104}$Zr isotones.

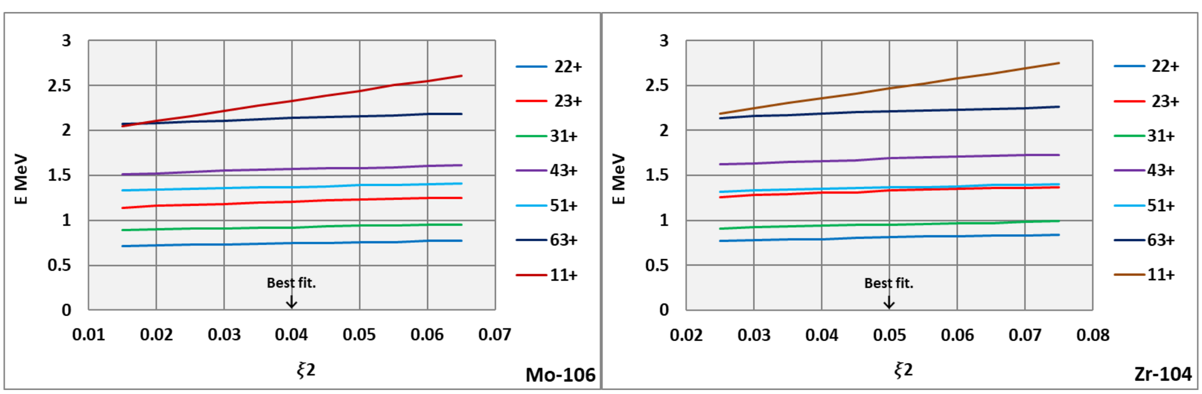


Figure 5: Mixed symmetry states (MSS) for $^{106}$Mo and $^{104}$Zr isotones.

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The parameters (effective charge) used to calculate the reduced electrical transitions B(E2), Eq. (7) are listed in Table 2, with the calculated transition ratios from the Eq. (11-13). The calculated B(E2) values were compared with available experimental data [29],[30] and presented in table (3). In this table, E.V (Experimental Value) denotes the measured experimental values, while M.V (Model Value) represents the values calculated using the adopted theoretical model.

Table2: The used effective boson charges in IBM-2 to calculate transition for $^{106}$Mo and $^{104}$Zr isotones [27] and the ratios of transitions $\mathfrak{R}$, $\mathfrak{R}'$ and $\mathfrak{R}''$.

| Isotones | E2 effective boson charges | | $\mathfrak{R}$ | | $\mathfrak{R}'$ | | $\mathfrak{R}''$ | |
|---|---|---|---|---|---|---|---|---|
| | $e_\pi$ | $e_\nu$ | *E.V* | *M.V* | *E.V* | *M.V* | *E.V* | *M.V* |
| $^{106}$Mo | 0.18 | 0.05 | 1.37 | 1.42 | ----- | 0.103 | ----- | 0.004 |
| $^{104}$Zr | 0.23 | 0.06 | ----- | 1.41 | ----- | 0.05 | ---- | 0.001 |

Table3: Comparison of the calculated electric quadruple transition probability B(E2) with the experimental data [29],[30] for $^{106}$Mo and $^{104}$Zr isotones.

| Isotones | B(E2) values in ($e^2b^2$) | | | |
|---|---|---|---|---|
| | $^{106}_{42}Mo$ | | $^{104}_{40}Zr$ | |
| $J_f^+ \to J_i^+$ | *M.V* | *E.V* | *M.V* | *E.V* |
| $2_1^+ \to 0_1^+$ | 0.316 | 0.304 | 0.524 | 0.523 |
| $4_1^+ \to 2_1^+$ | 0.450 | 0.417 | 0.742 | ----- |
| $6_1^+ \to 4_1^+$ | 0.490 | 0.387 | 0.800 | ----- |
| $8_1^+ \to 6_1^+$ | 0.498 | ----- | 0.807 | ------ |
| $2_1^+ \to 2_2^+$ | 0.032 | ----- | 0.028 | ----- |
| $2_2^+ \to 2_3^+$ | 0.0007 | ----- | 0.007 | ----- |
| $2_1^+ \to 0_2^+$ | 0.006 | ----- | 0.002 | ----- |
| $4_1^+ \to 2_2^+$ | 0.0009 | ----- | 0.001 | ----- |
| $4_2^+ \to 2_1^+$ | 0.001 | ----- | 0.002 | ----- |
| $4_2^+ \to 2_2^+$ | 0.182 | ----- | 0.284 | ----- |
| $2_2^+ \to 0_2^+$ | 0.028 | ----- | 0.024 | ----- |
| $6_2^+ \to 4_2^+$ | 0.343 | ----- | 0.548 | ----- |
| $8_2^+ \to 6_2^+$ | 0.398 | ----- | 0.636 | ----- |
| $3_1^+ \to 2_2^+$ | 0.448 | ----- | 0.793 | ----- |
| $5_1^+ \to 4_2^+$ | 0.206 | ----- | 0.381 | ----- |
| $7_1^+ \to 6_2^+$ | 0.099 | ----- | 0.187 | ----- |
| $4_3^+ \to 2_3^+$ | 0.114 | ----- | 0.449 | ----- |
| $5_1^+ \to 3_1^+$ | 0.270 | ----- | 0.441 | ----- |
| $7_1^+ \to 5_1^+$ | 0.361 | ----- | 0.589 | ----- |
| $2_1^+ \to 1_1^+$ | 0.020 | ----- | 0.035 | ----- |
| $2_2^+ \to 1_1^+$ | 0.0009 | ----- | 0.001 | ----- |
| $3_1^+ \to 1_1^+$ | 0.001 | ----- | 0.001 | ----- |
| $4_2^+ \to 4_1^+$ | 0.031 | | 0.030 | ----- |
| $2_3^+ \to 0_1^+$ | $9.51\times 10^{-5}$ | ----- | $6.87\times 10^{-5}$ | ----- |
| $4_3^+ \to 2_1^+$ | $9.61\times 10^{-10}$ | ----- | $1.16\times 10^{-6}$ | ----- |

The effective g-factors for proton $g_\pi$ and neutron $g_\nu$ for $^{106}$Mo and $^{104}$Zr isotones are $g_\pi = 0.3\mu n$ and $g_\nu = 0.45\mu n$, Eq. (10) has been used to determine the B(M1) transition probabilities, as shown in Table (4).

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Table 4: Theatrical reduced magnetic transition probabilities B(M1) for $^{106}$Mo and $^{104}$Zr. There are no experimental values for comparison

| isotones | B(M1) ($\mu_n^2$) | | | |
|---|---|---|---|---|
| | $^{106}_{42}Mo$ | | $^{104}_{40}$Zr | |
| $J_f^+ \rightarrow J_i^+$ | *M.V* | *E.V* | *M.V* | *E.V* |
| $2_1^+ \rightarrow 1_1^+$ | 0.004 | ----- | 0.005 | ----- |
| $2_2^+ \rightarrow 1_1^+$ | 0.0001 | ----- | $5.64\times 10^{-6}$ | ----- |
| $2_3^+ \rightarrow 1_1^+$ | 0.0001 | ----- | 0.0001 | ----- |
| $2_1^+ \rightarrow 2_2^+$ | $1.94\times 10^{-6}$ | ----- | $4.79\times 10^{-7}$ | ----- |
| $2_2^+ \rightarrow 2_3^+$ | $2.6\times 10^{-5}$ | ----- | $4.01\times 10^{-6}$ | ----- |
| $2_1^+ \rightarrow 2_3^+$ | $8.07\times 10^{-6}$ | ----- | $4.79\times 10^{-7}$ | ----- |
| $3_1^+ \rightarrow 2_1^+$ | $6.58\times 10^{-6}$ | ----- | $2.05\times 10^{-6}$ | ----- |
| $3_1^+ \rightarrow 2_2^+$ | 0.0002 | ----- | 0.0003 | ----- |
| $4_1^+ \rightarrow 3_1^+$ | $2.9\times 10^{-7}$ | ----- | $6.07\times 10^{-8}$ | ----- |
| $4_2^+ \rightarrow 4_1^+$ | $6.57\times 10^{-6}$ | ----- | $9.41\times 10^{-7}$ | ----- |
| $5_1^+ \rightarrow 4_1^+$ | $2.1\times 10^{-5}$ | ----- | $7.04\times 10^{-6}$ | ----- |

## CONCLUSIONS

In this study, the second interacting bosons model has been the most flexible nuclear model because of its ability in the characterization of nuclei with complex composition in heavy mass number nuclei. The Molybdenum$^{106}$Mo and Zirconium $^{104}$Zr nuclei with A = 106,104 exhibit transitional respectively. The behavior of the nuclei $^{106}$Mo was determined to be that of a transitional nucleus between U(5) to SU(3) ,while $^{104}$Zr nucleus exhibits a sharp morphological transition from spherical U(5) to distorted shape SU(3), based on the ratio between the two states $(E0_2^+/E0_1^+)$ shape coexistence, or the existence of two different nuclear shapes-one spherical or nearly spherical and the other deformed. The energy bands indicate the presence of a transitional behavior from the vibrational limit to the rotational limit for nuclei $^{106}$Mo and$^{104}$Zr. The energy ratios and dynamic symmetry have been studied in the IBM-2. $B(E2;\, 2_1^+ \rightarrow 0_1^+)$, which represents the probability of an electric transition from the first excited state $2_1^+$ to the ground state $0_1^+$, is one of the most important structural properties in nuclear physics, the value of B(E2) in spherical nuclei is usually small. In deformed nuclei, the value of B(E2) is very large, reflecting a non-spherical charge distribution. Therefore, a high value indicates a nucleus with an irregular shape, often elongated or oblate it means that a large number of nucleons are involved in the transition, on the other hand small values, indicate behavior closer to single-particle behavior as illustrated in Figure 3 a gradual transition toward collective behavior is observed in $^{104}$Zr isotopes, which exhibit relatively larger values B(E2). The deviation $\Delta R$ from ideal behavior, for the U(5) to SU(3) limits and the weight of each determination in each nucleus was also determined confirming the approach to rotational distortion. The purpose of calculating the weights is to represent the behavior of each nucleus as a mixture of vibrational (U(5)) and rotational (SU(3)); the ratio R(4/2) was used, which clearly reflects the type of collective structure of the nucleus and the degree of deviation from the sequential ratio for each determination. The effective nuclear deformation parameter $\chi_{eff}$ was determined, where the value $\chi_{eff} > 0$ indicates the shape is oblate (flat), while $\chi_{eff} < 0$, it is prolate (elongated ). Values close to zero indicate γ-soft or spherical/elastic, in $^{106}$Mo and$^{104}$Zr the values are negative, and stay away from zero. The isovector balance (IV) describes the relationship between the proton $\chi_P$ and neutron $\chi_v$ shape parameters within the IBM-2 framework, $\chi_P$and $\chi_v$ represent the shape of the proton and neutron components, i.e., the shape of the nucleus (spherical, unstable gamma, deformed). The relationship between them gives what is known as the Isovector-Isoscalar balance. In IBM-2, the concept of Isovector balance allows us to define two parts of the operator, the isoscalar part (reflecting the combined collective motion of protons and neutrons),and the isovector (reflecting the difference or asymmetry between the proton and neutron) components.The nucleus is said to achieve perfect isovector balance if $\chi_\nu \approx \chi_\pi$, if $\chi_\nu \neq \chi_\pi$, said to achieve unbalanced contributions, transitional or near-transitional change in form such as in $^{106}$Mo and$^{104}$Zr, They are found to reinforce the idea of transitioning from vibrational to rotational behavior, this applies perfectly to the results of electrical transition ratios ($\Re$, $\Re'$ and $\Re''$). The energy level sequences and ratios indicated that the $^{106}$Mo and $^{104}$Zr are transitional nuclei between vibrational U(5) and rotational SU(3) symmetries. Collective states associated with proton-neutron symmetry and symmetry mixing play an important role in studying collective nuclear properties by changing the majorana parameter ($\xi_2$) showed a significant impact was shown on determining the mixing locations of some levels , $2_3^+$, $4_3^+, 6_3^+$ and $1_1^+$ while other levels remained unaffected and their values remained constant at specific values, such as the levels $2_2^+$ $3_1^+$ and $5_1^+$ in $^{106}$Mo and $^{104}$Zr. No experimental values of the $1_1^+$ level are available.

## Acknowledgments:

I would like to sincerely thank Dr. Heiyam Najy Hady for her valuable support and guidance throughout this work.

## Conflict of interests.

There are non-conflicts of interest.

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## الخلاصة

**المقدمة:** تم تطبيق أنموذج البوزون المتفاعلة الثاني (IBM−2) لدراسة تأثير خصائص البنية النووية على السمات الطيفية لنظائر $^{106}$Mo و$^{104}$Z التي تحتوي على عدد نيوتروني N=64

**طرق العمل:** تم تطبيق أنموذج البوزونات المتفاعلة (IBM−2) لتحليل الانحراف $\Delta R$ عن السلوك المثالي ضمن الانتقال من التناظر U(5) إلى SU(3) وتحديد وزن كل تناظر في كل نواة. كما تم دراسة نسبة $R(4/2)$، وظاهرة التعايش الشكلي $(E0_2^+/E0_1^+)$، وتم حساب معامل التشوه النووي الفعال $\chi_{eff}$ والتوازن الايزوفكتوري (IV) لتأكيد اقتراب النوى من التشوّه الدوراني. بالإضافة إلى ذلك، تم فحص احتمالية الانتقال الكهربائية رباعي القطب B(E2) واحتمالية الانتقال ثنائي القطب المغناطيسي B(M1).

**الاستنتاجات:** تظهر الدراسة أن نظيري $^{106}$Mo و$^{104}$Zr يظهران سلوكا انتقاليا بين التناظرين الاهتزازي والدوراني، مع ميل واضح نحو التشوه الدوراني. تؤكد القيم السالبة لمعامل التشوه النووي الفعال $\chi_{eff}$ والتوازن الايزوفكتوري (IV) أهمية تناظر البروتون–نيوترون في وصف الخصائص الجماعية لهذين النظيرين.

**الكلمات المفتاحية:** أنموذج البوزونات المتفاعلة الثاني IBM−2, MSS, نظائر $^{106}$Mo و $^{104}$Zr, $\Delta R$, و IV