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Existence, Stability, and Numerical Analysis of a Cubic Nonlinear Differential Equation via Picard Iteration

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وجود واستقرار وتحليل عددي لمعادلة تفاضلية غير خطية تكعيبية باستخدام تكرار بيكارد

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ABSTRACT

This paper presents the investigation of the presence, stability, and convergence of the numerical solution for a third-order differential equation with cubic nonlinearity using the integral iterative method and numerical computation. The nonlinear initial-value problem is converted into an integral equation, and the iterative procedure in the spirit of Picard iteration is introduced to generate the solution. Analysis of stability is accomplished by obtaining the equilibrium points of the nonlinear differential system and then using the derivative test to determine their nature. Numerical simulations are done in MATLAB utilizing the built-in ODE45 Runge–Kutta algorithm as the reference solution. The comparison of the results of the integral iterative solution and the numerical reference solution indicates that the equilibrium point $u = 0$ is asymptotically stable, while the other equilibrium points are unstable. The findings illustrate that the iteration scheme can produce the exact solution only in the stable domain; however, the solution diverges numerically due to the influence of the cubic nonlinearity in the unstable domain. The error analysis by calculating the error with maximum norm and L2 norm proves the instability of the numerical method for long time intervals.

Keywords: nonlinear differential equation; Picard iteration; equilibrium analysis; stability analysis; Runge–Kutta method; numerical divergence; error analysis.

INTRODUCTION

Nonlinear differential equations are one of the essential tools for modeling complex dynamic systems in the fields of science and engineering. Unlike linear systems, nonlinear dynamics deals with systems whose equations of motion are nonlinear with respect to the state variables. This results in complex dynamic behaviors that are not possible with linear systems. In fact, even the addition of a nonlinear term like a quadratic or cubic expression in the equations of motion can result in highly non-trivial behaviors. In this regard, nonlinear systems are an essential part of modern mathematical analysis and are closely related to differential equations and modeling, with numerous applications in physics, biology, engineering, and other fields of science [1]. In this regard, it is essential to analyze the existence and stability of solutions to ensure that the mathematical model of the system is properly defined and has non-trivial dynamic behaviors.

One of the most important ideas in the qualitative analysis of nonlinear systems is that of equilibrium points. An equilibrium point is defined as a point where the time derivative is zero, that is:

Dx/dt = f(x) = 0. Dx/dt = f1(x, y), and Dy/dt = f2(x, y), we must solve the following equations to obtain our equilibrium point(s): F1(x, y) = 0 and F2(x, y) = 0. Once you have found your equilibrium point, denoted as (x*, y*), you linearize about it to determine stability using your Jacobian matrix. The eigenvalues, denoted by the symbol λ, determine the nature of your equilibrium point. The eigenvalues can be found by solving the following equation, which is derived from the characteristic equation of your Jacobian matrix: Det $(J - \lambda I) = 0$. The following guidelines can help you determine the nature of your equilibrium point: If $\mathrm{Re}(\lambda) < 0$, it is asymptotically stable. If $\mathrm{Re}(\lambda) > 0$, it is unstable. If $\mathrm{Re}(\lambda) = 0$, inconclusive. Furthermore, the trace and determinant of the Jacobian can be used to determine the type of the equilibrium point in two-dimensional systems [2]. Your results are then visualized using phase pictures. From an analytical point of view, a very effective method for analyzing nonlinear initial value problems is to rewrite the differential equation in an equivalent integral form. For a problem of the type:

If Y is a solution to the differential equation given by Y' = f(x, Y), with the initial value Y(x0) = y0, then Y(x) is equal to y0 + the integral from x0 to x of f(t, Y(t)) dt. The key to Picard iteration is that the formula for the sequence of successive approximations is given by Yn+1(x) = y0 + integral from x0 to x of f(t, Yn(t)) dt, where Y0(x) = y0, n ≥ 0. If f is continuous and satisfies a Lipschitz condition, this sequence will converge to the exact solution. This is the basis of the existence and uniqueness theorem [3], demonstrating not just the concept but its practical application.

Regarding the numerical aspect, traditional solvers like explicit Runge-Kutta and linear multistep methods are widely used in approximating the solutions for nonlinear ordinary differential equations. The MATLAB ODE solvers include several solvers for stiff and non-stiff systems, which is a standard approach for solving nonlinear ordinary differential equations [4]. In particular, Runge-Kutta-based solvers are widely used for generating reference solutions, which are then compared with other approximation schemes.

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Although there is substantial literature on nonlinear dynamics, equilibrium analysis, integral iterative approach, and numerical solvers, relatively few works have been done on combining existence analysis, integral iterative approach, and direct numerical approach with traditional Runge-Kutta solvers in an integrated manner for cubic-type nonlinear systems. The present research is motivated by this need.

The problem being addressed in the present study concerns analytical and numerical investigations of the existence of the solution for nonlinear differential equations in which the nonlinearity of the equations takes the form of polynomials. U(0) = u0, u'(t) = -alpha u(t) + beta u3(t).

The objective of this work has three parts. First, the reformulation of the nonlinear equation in terms of an integral form, which can be used for iteration. Second, the analysis of the equilibrium points, which are solutions of:

-alpha u + beta u3 = 0,

and to classify their stability using the derivative criterion:

lambda = f'(u*).

If lambda < 0, the equilibrium is stable.

If lambda > 0, the equilibrium is unstable.

Third, to perform a numerical experiment that compares an integral-type iterative scheme that is derived from an integral equation with a reference solution obtained by a Runge-Kutta method.

The main contributions of this study may be summarized as follows: (i) development of an integral-type iterative approximation that is derived directly from an integral equation form of the equation under study, and (ii) a systematic classification of the stability of equilibrium points with a phase portrait analysis. A direct numerical comparison of the integral-type iterative scheme and the standard MATLAB solution will be made, with an emphasis on whether the results converge or diverge. In this paper, a unifying framework for linking existence, stability, and numerical solutions of nonlinear differential equations will be introduced.

This study attempts to tackle the issue related to examining the existence, stability, and numerical characteristics of solutions to nonlinear differential equations with nonlinear terms having cubic powers. While there is no shortage of analytical as well as numerical methods available for nonlinear systems, few works can be found that consider a combination of integral formulation, Picard iteration, stability analysis, and numerical verification for cubic nonlinear differential equations. Moreover, the behavior of the integral iterations numerically for stable and unstable regimes needs to be studied further. Thus, the goal of this paper is to examine the nonlinear differential equation $u'(t) = -\alpha u(t) + \beta u^3(t)$.

The Contribution findings from this study include:

- Derivation of the cubic nonlinear differential equation into an alternative integral formulation which is suitable for the Picard iteration method.
- Analysis of the dynamics of equilibrium and stability characteristics of the nonlinear model.
- Comparison of the results obtained from integral iterative analysis with those of the numerical solution by the MATLAB ODE45 solver.

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RELATED WORK

Some researchers examined the existence and uniqueness of solutions for nonlinear fractional differential equations by utilizing the fixed-point method and iterative analysis. In particular, Hallaci et al. [5] discussed the existence and uniqueness of solutions of nonlinear fractional differential equations on an unbounded domain by making use of Banach's contraction principle in weighted Banach space. The authors further emphasized the significance of analytical approach in analyzing nonlinear dynamics.

The importance of the fixed-point theory and the reformulation of the problem as an integral equation for the existence of the solution has again been emphasized by the recent developments in the theory of nonlinear and fractional differential equations. The existence and uniqueness of the solution for a fractional nonlinear integro-differential equation with variable coefficients and functional boundary conditions have been investigated by [6]. The problem has been reformulated as an implicit integral equation in terms of the Mittag-Leffler function, and Banach's contraction principle, Krasnoselskii's fixed point theorem, and the inverse operator have been used. The results have been illustrated by presenting some examples [6].

The existence and uniqueness of solutions to a nonlinear fractional differential equation with functional boundary conditions were investigated by [7] in a previous study. The author has used Leray-Schauder's fixed point theorem along with Banach's contraction principle. The implicit integral equation, which involves the Mittag-Leffler function, has been used to represent the results. The applicability of the method to various types of nonlinear fractional differential equations has been shown by the author [7].

Proportional fractional integro-differential equations of a new type with boundary conditions of the functional type have been studied by [8]. Ulam-Hyres'stability, uniqueness, and existence of the solutions were proved. Babenko's method and the inverse operator were employed to transform the equation to an equivalent one. Leray-Schauder's alternative and Banach's contraction principle were also applied to the equation [8].

From an integral transform technique viewpoint, proposed the Sumudu transform technique with a hybrid technique for solution development for both integer and fractional-order Des [7], including those with variable delay terms that scale with the independent variable. The article thus proves that integral transforms can be used to model complex dynamical systems and abnormal phenomena, which is a testament to their flexibility and applicability [9].

In terms of numerical analysis, proposed an explicit two-step peer method with Runge-Kutta stability for non-stiff ODEs [10]. The numerical technique has local truncation error estimation and variable step size and order, with validation carried out by comparison with MATLAB's ode45 function, thus proving that numerical validation is crucial for any emerging computational technique [10].

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Preliminaries

This part discusses the mathematical ideas that play a role in understanding and analyzing nonlinear differential equations, stability analysis, and numerical approximation techniques. Initial Value Problem We look at the nonlinear differential equation along with the initial condition for determining the trajectory of the solution. The concept of an initial value problem is quite common when dealing with nonlinear dynamical systems and mathematical models [11]. An initial value problem may be written as follows:

$$u'(t) = f(t, u(t))$$

The problem of the value of the solution at the initial point is known as the initial value problem for ordinary and partial differential equations with a particular value of the solution at some point in the solution domain [12].

Ordinary differential equations have the dependent variable, derivatives of the dependent variable, and the independent variable [12].

Integral Equation

If an equation includes an unknown function under an integral sign, it is termed an integral equation [12]. The nonlinear differential equation in the study may be written as follows:

$$u(t) = u_0 + \int_0^t [-\alpha u(s) + \beta u^3(s)]ds$$

Integral representation serves as the foundation of constructing approximate iterative methods and studying their convergence properties.

Picard Iteration and Convergence

One method of solving the differential equations is based on the Banach contraction principle which is also referred to as Banach's fixed-point theorem [12]. Moreover, iterative methods help to find approximate solutions which eventually converge to the exact one [12].

Convergence of iterative techniques can be considered from the point of view of satisfying the Lipschitz condition [12].

Lipschitz condition means that the function satisfies the following condition, where constant L is called Lipschitz constant [12].

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STABILITY ANALYSIS

Analysis of stability helps to examine nonlinear systems' behavior in the long run. Stability analysis determines whether or not minor perturbations will cause bounded solutions around the equilibrium point [11].

Condition for stability can be expressed as:

$$|u(t) - \tilde{u}(t)| \leq \max |\Phi(t) - \widetilde{\Phi}(t)|$$

Nonlinear systems are defined as being asymptotically stable if their solutions tend towards their equilibrium point in the limit as t goes to infinity [11]. The asymptotic stability property is denoted mathematically by:

$$\lim_{t\to\infty} \quad |u(t) - \tilde{u}(t)| = 0$$

Numerical Stability and Verification

Numerical methods are often applied to find an approximation to a system of nonlinear differential equations where an analytical solution may not be easy to obtain [11]. Numerical stability techniques aid in obtaining a solution whose qualitative properties match those of the actual system [11].

Numerical stability criterion is given by:

$$|u_n - \widetilde{u_n}| \leq \max |\Phi(t) - \widetilde{\Phi}(t)|$$

In the same way, the expression for numerical asymptotic stability is given by:

$$\lim_{n\to\infty} \quad |u_n - \widetilde{u_n}| = 0$$

Numerical simulations are widely adopted in determining the stability properties of nonlinear systems and comparing approximation schemes [11].

Error Analysis

Error analysis is essential in assessing the accuracy of numerical solutions that are not exact. The absolute error for the exact and numerical solutions can be computed in the following manner [12]:

$$|r_n| = |u - u_n|$$

Maximum error remainder is an indicator used to assess the accuracy of numerical solutions [12]:

$$MER_n = \max|R_n(t)|$$

Lyapunov theory is a well-known method used in examining the stability characteristics of nonlinear dynamical systems [13]. Nonlinear dynamical systems can be represented by the following dynamics system:

$$\dot{x} = y$$
$$\dot{y} = -\frac{b}{m}y - g\sin x$$

Lyapunov function is stated mathematically as [13]:

$$V(x) = \frac{1}{2}ml^2\dot{\theta}^2 + mg(l - l\sin\theta)$$

The rate of change of the Lyapunov function is given by:

$$\dot{V}(x) = -b\dot{\theta}^2$$

To obtain the local stability behavior of the system, we can introduce a small perturbation as follows [13]:

$$u(t) = x(t) - 0$$
$$v(t) = y(t) - 0$$

Linearizing this system results in:

$$\dot{u} = f(0,0) + u\frac{\partial\dot{x}}{\partial x} + v\frac{\partial\dot{x}}{\partial y}$$

$$\dot{v} = f(0,0) + u\frac{\partial \dot{y}}{\partial x} + v\frac{\partial \dot{y}}{\partial y}$$

The Jacobian matrix corresponding to the linearized system is given by [13]:

$$J = \begin{bmatrix} 0 & 1 \\ -g & -\frac{b}{m} \end{bmatrix}$$

These mathematical concepts are vital in analyzing the characteristics of nonlinear systems.

METHODOLOGY

For numerical implementation, the parameter values were selected as α = 1 and β = 0.5 with initial condition u(0)=0.1. The computational interval was chosen as t ∈ [0,10], and six Picard iterations were applied in MATLAB. The ODE45 solver was used as the reference numerical solution for comparison purposes.

- **Problem Description**

In this study, we consider the nonlinear differential equation implemented numerically in MATLAB:

$$u'(t) = -\alpha u(t) + \beta u^3(t)$$

subject to the initial condition

$$u(0) = u_0$$

This equation was selected because it represents a nonlinear dynamical system with multiple equilibrium points and allows stability investigation.

- **Reference Numerical Solution (ODE45)**

The first step in the methodology was computing a reference numerical solution using MATLAB's built-in Runge–Kutta solver (ODE45).

The numerical solver approximates the solution of

$$u'(t) = f(t,u)$$

where

$$f(t,u) = -\alpha u + \beta u^3$$

This serves as a baseline to compare the proposed integral-based iterative method against.

- **Integral-Based Iterative Scheme**

In order to develop an alternate numerical solution to the problem, the differential equation was converted to its integral form:

$$u(t) = u_0 + \int_0^t \left(-\alpha u(\tau) + \beta u^3(\tau)\right) d$$

Based on the integral form of the problem, an iterative numerical solution was developed:

$$u_{n+1}(t) = u_0 + \int_0^t \left(-\alpha u_n(\tau) + \beta u_n^3(\tau)\right) d$$

Numerical evaluation of the integral was accomplished using MATLAB's cumulative trapezoidal rule function cumtrapz. This process was continued for a set number of iterations to achieve the solution:

$$u_{iter}(t) = u_N(t)$$

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- **Error Quantification**

In order to compare the difference in solution between the integral iterative solution and the solution obtained using the ODE45 solver, two error norms were calculated.

Maximum Error

$$E_\infty = \max_{t\in[0,T]} |u_{ode}(t) - u_{iter}(t)|$$

L2 Error

$$E_2 = \left(\int_0^T |u_{ode}(t) - u_{iter}(t)|^2 dt\right)^{1/2}$$

This method will act as a standard for comparison with the integral-based iterative method proposed.

- **Equilibrium Points**

To derive an alternate approximation for the solution of the differential equation numerically, the equation was rewritten in integral form:

$$-\alpha u + \beta u^3 = 0$$

which gives

$$u(-\alpha + \beta u^2) = 0$$

Hence

$$, u_1 = 0 \text{ and } u_{2,3} = \pm\sqrt{\frac{\alpha}{\beta}}$$

- **Stability Analysis**

Local stability was examined by linearizing the system around each equilibrium point.
The derivative of the nonlinear function is

$$f'(u) = -\alpha + 3\beta u^2$$

The eigenvalue at each equilibrium is

$$\lambda = f'(u^*)$$

Stability classification follows:

- If $\lambda < 0$, the equilibrium is stable.
- If $\lambda > 0$, the equilibrium is unstable.

RESULTS

- **Introductory Statement**

To test the performance of the integral-based iterative scheme, numerical experiments are performed to compare the results with those obtained by using the ODE45 solver in the MATLAB environment.

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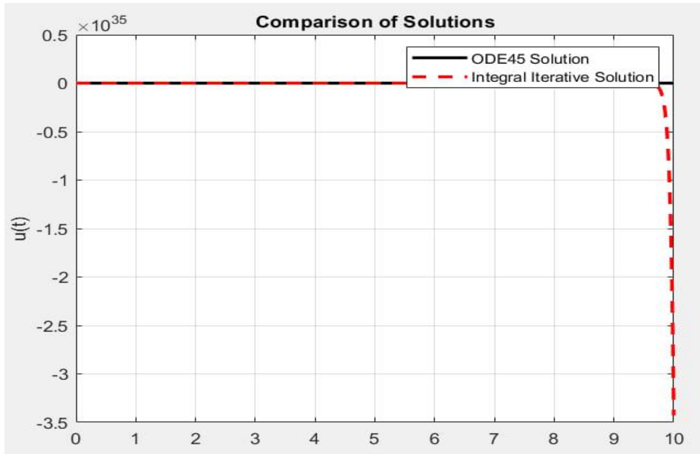

Figure 1. Comparison between ODE45 solution and integral iterative solution.

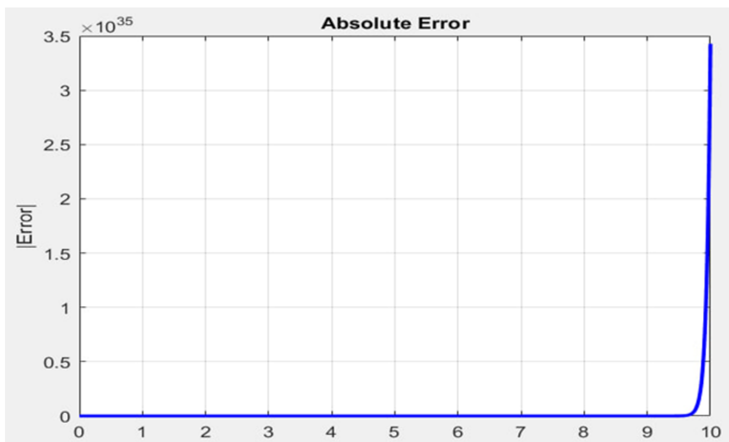

Figure 2. Absolute error between the two numerical solutions.

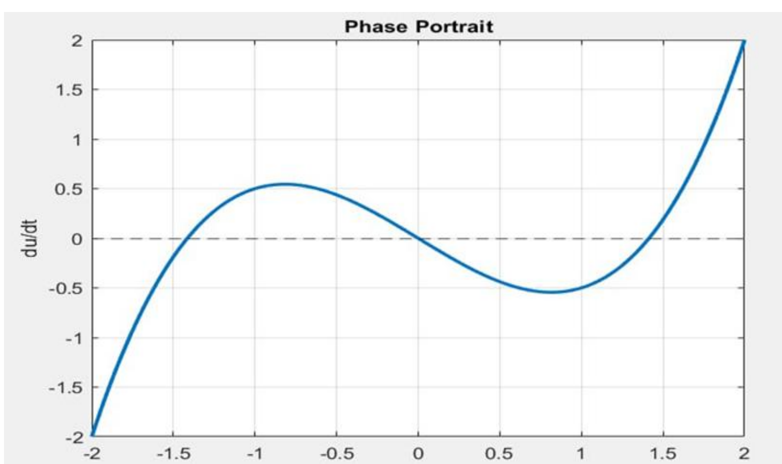

Figure 3. Phase portrait of the nonlinear system.

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The obtained numerical results indicate that the integral iterative approximation agrees with the ODE45 reference solution only during the early stages of computation. However, as time increases, the cubic nonlinear term amplifies the solution magnitude and causes numerical divergence in the iterative approximation. Therefore, the reported large error values represent numerical instability rather than successful convergence of the iterative method.

Table 1. Numerical Results and Stability Classification for the Nonlinear System.

α	β	Iterations	L2 Error	Max Error	Equilibrium Points	Stability
1	0.5	6	5.94×10^{34}	3.42×10^{35}	0, ±1.414	0 Stable, ±1.414 Unstable

DISCUSSION

The numerical experiments reveal important insights into the behavior of the proposed integral-based iterative scheme when applied to the nonlinear cubic differential equation.

$$u'(t) = -\alpha u(t) + \beta u^3(t)$$

- **Stability Structure and Phase Behavior**

The phase portrait clearly illustrates the qualitative structure of the system. The equilibrium points

$$u_1 = 0$$

$$u_{2,3} = \pm\sqrt{\frac{\alpha}{\beta}}$$

Divide the state space into distinct dynamical regions. The derivative

$$f'(u) = -\alpha + 3\beta u^2$$

confirms that

$$u_1 = 0 \; \textit{is asymptotically stable since} \; f'(0) = -\alpha < 0$$

$$u_{2,3} = \pm\sqrt{\frac{\alpha}{\beta}} \; \textit{are unstable since}$$

$$f'\left(\pm\sqrt{\frac{\alpha}{\beta}}\right) > 0$$

This classification is fully consistent with the geometric interpretation of the phase line, where trajectories move toward zero and diverge from the nonzero equilibria.

- **Numerical Comparison and Divergence Behavior**

The comparison between the ODE45 reference solution and the integral iterative solution shows a dramatic divergence near the final time interval.

The reported errors:

- L2≈5.94×1034L_2 ≈ 5.94 × 10 34 L2≈5.94×1034
- E∞≈3.42×1035E ∞ ≈ 3.42 × 1034 E∞≈3.42×1035

indicate severe instability of the iterative scheme for the selected parameters and iteration count.

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It should be noted, however, that the plots also indicate severe instability of the iterative scheme for the given parameters and iteration count.

From the absolute error plot, we can conclude that the difference grows exponentially for values of $t \approx = 10$. This, in fact, shows that the integral iteration method enhances the cubic nonlinearity when the solution enters the unstable regime.

Interpretation of Divergence

It should be noted that the instability observed does not contradict the existence of the theory. In fact, the integral iteration method guarantees the solution's existence if the solution satisfies the Lipschitz condition. However, the numerical implementation of the iteration method depends on several factors, namely:

- Step size,
- Iteration count,
- Basin of attraction of the stable equilibrium point,
- Strength of the cubic nonlinearity.

In fact, the solution enters the regime of the βu^3 nonlinearity, and numerical blow-up occurs.

LIMITATIONS

The main drawback in the current study is that numerical simulations have been carried out only for a particular set of parameters. Future studies can consider more ranges of parameters and different iterative approaches for better numerical stability.

CONCLUSIONS

In this research, the existence and stability, along with numerical aspects of solutions of the cubic nonlinear differential equation were examined through an integral iterative approach and MATLAB numerical simulation. In the case of equilibrium analysis, it was found out that the equilibrium at $u = 0$ is asymptotically stable while the others are unstable. The transformation of the nonlinear differential equation to the integral equation allowed us to construct Picard-type iterative solutions. Through numerical simulation against ODE45 solution, it was shown that our iterative method worked quite well in the stable domain; however, in the case where the cubic nonlinear term is predominant, our method suffered severe numerical divergence. Hence, one can conclude that this iterative integral method can be used for theoretical local stability analysis, while numerical stability is heavily dependent on the parameters involved, iterations performed, and time duration.

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Recommendations and Future Work

On the basis of numerical behavior, several avenues of research are proposed:

1. Adaptive step size control for the integral iteration.
2. The inclusion of damping terms for the iteration.
3. An analytical study of the convergence region to ensure that parameters are chosen so that the solution remains bounded
4. Extension to multidimensional nonlinear systems.
5. Comparison of the integral iteration with other numerical methods like implicit Runge-Kutta and stiff codes.
6. Investigation of bifurcation behavior with varying parameters α = alpha, β = beta.

Conflict of interests.

There are non-conflicts of interest.

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