Semi-Weakly Continuity of Maps in Bitopological Spaces

Ali Younis Shakir
Department Of Electric , Institute Of Babylon,
Ali_younis595@yahoo.com

Abstract

The author study in this paper, properties of semi-weakly continuous of maps in bio topological spaces.

Keywords: Semi-Weakly Continuous maps ,bitopological space.

الخلاصة

فـــي هـــذا البحـــث تـــم دراســـة خـــواص الـــدوال شـــبه ضـــعيفة الاســـتمرارية فـــي الفضـــاءات ثنائيـــة التبولـــوجي .

الكلمات المفتاحية: الدوال شبه ضعيفة الاستمرارية , فضاء ثنائي التبولوجي .

1- Introduction

let X be at topological space and A be a subset of X. A subset A is said to be semi open (s-open) (Levine , 1963) if $A \subset A^{-o}$. The complement of a s-open set is called semi closed (s-closed). The union of all s-open sets of X contained in A is called the semi interior of A denoted by A^{s}. The intersection of all s-closed sets sets of X containing A is called the s-clouser of A and denoted by A^{-s}. The family of all s-open sets in a space X is denoted by S.O (X). Let Y be any space and T_1, T_2 are topological spaces on. The space (Y, T_1, T_2) is called bio topological space (Kelly , 1963) . For a set $A \subset Y$ the T_i -closare denoted by $A^{-(i)}$ and the T_i – interior denoted by $A^{o(i)}$ of A for i =1,2.

2- Semi-weakly continuous

We study the semi-weakly continuous in bio topological space, the following results were reached by forcing the definition of semi open in semi weakly continuous.

Definition 2 -1.

Let X be a topological space. A map f: $X \to Y$ is said to be T_1-semi weakly continuous with respect to T_2 at a point $x_0 \in X$ if for every T_1-open set V of Y containing $f(x_0)$ there exists s-open set U in X containing x_0 such that $f(U) \subset V^{-s(2)}$ where $V^{-s(2)}$ the T_2-closure.

Theorem 2 - 2

A mapping f: $X \to Y$ is T_1-semi weakly continuous with respect to T_2 iff for every T_1-open set V in Y,

$f^{-1}(V) \subset [f^{-1}(V^{-s(2)})]^{os}$

Proof

Let $x \in X$ and V an T_1-open set containing f(x), then:

$x \in f^{-1}(V) \subset [f^{-1}(V^{-s(2)})]^{os}$. Put $U = [f^{-1}(V^{-s(2)})]^{os}$.

Conversely, let V be any T_1-open set of Y and x $\in f^{-1}$(V).

Then there exists a s-open U in X such that x $\in$ U and f (U) $\subset (V^{-s(2)})$ and hence x $\in[f^{-1}(V^{-s(2)})]^{os}$. This proves that

$f^{-1}(V) \subset [f^{-1}(V^{-s(2)})]^{os}$.

Definition 2 – 3.

Let (X,P) be a topological space and let (Y, T_1, T_2) be bio topological space. Let f: $X \to Y$ be a function. A function g: (X, P) $\to$ (X×Y, P×T_2) defined by g(x) = (x, f(x)) for every x $\in$ X, is called the graph function of f, where P×T_2 is the product topology on X × Y.

The following result gives elementary relation between a function and its graph function.

Theorem 2 - 4

Let f: $X \to Y$ be a mapping and g: $X \to X \times Y$ be the graph mapping of f, give by g(x)=(x, f(x)) for every point x $\in$ X.

If g is T_1-semi weakly continuous with respect to T_2, then f is T_1-semi weakly continuous with respect to T_2.

Proof

Let x$\in$X and V be any T_1-open set containing f(x). Then X×V is an (P×T_2)-open set in X×Y containing g(x). Since g is T_1-semi weakly continuos with respect to T_2, there exists s-open set U containing x such that g(U) $\subset(X\times V)^{-s}$. It follows from Lemma4 (Noir , 1978) , that $(X\times V)^{-s} \subset X\times V^{-s(2)}$. Since g is the graph mapping of f, we have f(U) $\subset V^{-s(2)}$. This shows that f is T_1-semi weakly continuous with respect to T_2.

Definition 2 – 5.

A topological space (Y, T_1, T_2) is called pairwise Hausdroff (Kelly , 1963) , if for all points x,y $\in$, x≠y, there exist disjoint sets U$\in T_i$, V$\in T_j$, i≠j =1,2 such that x$\in$U and y$\in$V.

We have the following result.

Theorem 2 – 6.

Let (X,P) be a topological space and (Y, T_1, T_2) be a pairwise Hausdroff bio topological space. If f:X→Y is T_1-semi weakly continuous with respect to T_2, then the graph G(f) of the map f is s-closed in the space (X×Y , P×T_2).

Proof

Let f be T_1-semi weakly continuous with respect to T_2. Let (x,y) $\notin$ G(f), then y≠f(x) and there exist disjoint sets U$\in T_2$, V$\in T_2$ such that f(x)$\in$U and y$\in$V. By Vu we denote the union of all sets V$\in T_2$ for which above holds with the set U. Moreover from theorem1 it follows

x$\in[f^{-1}(U^{-s(2)}]^{os}$. Thus X×Y / G(f)= $\cup$ {$[f^{-1}(U^{-s(2)}]^{os}$ ×Vu : u$\in T_1$}, since $[f^{-1}(U^{-s(2)}]^{os}$ ×Vu is s-open set in (X×Y, P×T_2) and the union of s-open sets is s-open it implies that G(f) is a s-closed set in the space (X×Y, P×T_2).

Theorem 2 – 7.

Let (X,P) be a topological space and (Y, T_1, T_2) be a bitopological space. If f:X→Y is T_1-semi weakly continuous with respect to T_2, and A is an T_1-open subset of Y containing f(x). Then f:X→A is T_1-semi weakly continuous with respect to T_2.

Proof

Let $x \in X$ and let V be an T_1-open subset A containing f(x). Since A is T_1-open in Y, then V is T_1-open in Y, therefore, there exist s-open set U in X containing x such that $f(U) \subset V^{-s(2)}]^{os}$. Then $f:X \rightarrow A$ is T_1-semi weakly continuous with respect to T_2.

Theorem 2 – 8.

Let (X,P) be a topological space and (Y, T_1, T_2) be a bio topological space. If $f : X \rightarrow Y$ is T_1-semi weakly continuous with respect to T_2, and A is open in X, then the restriction $f \setminus A:A \rightarrow Y$ is T_1-open -semi weakly continuous with respect to T_2.

Proof

Let $x \in A$ and V be T_1-open set of Y containing f(x). Since f is T_1-semi weakly continuous with respect to T2, there exist s-open set U in X containing x such that $f(U) \subset V^{-s(2)}$. Since A is open in X, by lemma1 of [3] $x \in A \cap U \in S.O(A)$ and $f|A(A \cap U) = f(A \cap U) \subset f(U) \subset V^{-s(2)}$. It follows that $f|A$ is T_1-semi weakly continuous with respect to T_2.

Theorem 2 – 9.

Let f be a map of a topological space X into a bio topological space (Y,T_1,T_2). If for every non-empty closed set $M \subset X$ the restriction
$f|M : M \rightarrow Y$ is T_1-semi weakly continuous with respect to T_2, then f is
T_1-semi weakly continuous with respect to T_2.

Proof

Let us a ssume that f is not T_1-semi weakly continuous with respect to T_2 at a point $x_o \in X$. There exist a T_1-open set V of Y containing $f(x_o)$ such that $f(U) \not\subset V^{-s(2)}$ for each $U \in S.O(X)$ containing x_o. Let $M=(X /f^{-1} (V^{-s(2)}))$. Evidently $x_o \in M$. If W is s-open containing x_o in M,then for every non-empty s-open in M set $W_1 \subset W$ we have $f(W_1) \subset V^{-s(2)}$. It implies $f|M$ is not T_1-semi weakly continuous with respect to T_2 at a point x_o.

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